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# Adaptive Price-Based Power Flow in Next Generation Electric Power Systems

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**Abstract**—Demand load management in the context of grid-connected microgrids is the scope of this paper. This issue is formulated as a power flow problem between distributed power sources with the objective of grid operational cost minimization. Under the assumption of time-varying demand loads at individual microgrids, a distributed algorithm of interactions among microgrids is proposed for power sharing within the grid. Numerical results demonstrate the outperformance of the proposed load management scheme in comparison with no power sharing scheme in terms of the grid operational cost.

## I. INTRODUCTION

Electrical power generation in the form of *distributed generation* (DG) is a well-know structure for on-site power supplement [1]. This structure includes the application of small generators, typically ranging in capacity from 5KW to 10 MW, at or near the end-user to provide the power needed. In comparison with centralized and conventional models of power generation, the DG offers several advantages from the perspective of both sources and end-users. Because of distributed structure, the system is more reliable in terms of maintenance and service as well as is more flexible in using fuels and renewable energy sources.

The *smart power grid* is an interesting concept of integration DG systems into a grid [2]. This grid uses information and communication technology to enhance the grid flexibility and reliability. A unit of the grid, known as *microgrid* (MG), is a group of generators and loads connected to the grid in multiple points. As a considerable capability, each MG can operate in autonomous (isolated from the main grid) and grid-connected modes. The performance measure in autonomous mode is the reliability of stand-alone operation. However, in grid-connected mode the MG operates while connected to the main grid. This is especially characterized by the fact that each MG can sell a portion of its generated power to the grid at a point of connection and at the same time is a able to purchase a portion of its demand from the grid at another point of connection.

Demand load management is a critical issue in the smart grids with power sharing capability [3], [4]. It controls the power flow between MGs with the aim of establishing a balance between power supply and demand in a cost-efficient manner. The objective in this balance is to alleviate peak loads at individual MGs and accordingly avoids major expenditures

in power utilities. In contrast to the autonomous mode where load management results in shifting peak loads to off-peak loads [5], peak loads of a MG in grid-connected mode can be handled by the means of power sharing throughout the grid. Considering stochastic demand loads in MGs, an immediate question is how to perform power flow and to set interactions in the grid.

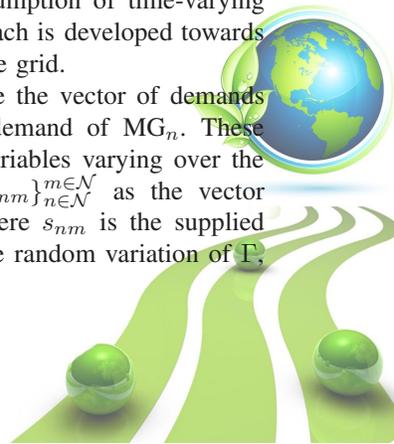
In this paper, demand load management of an electric network of interconnected MGs is formulated as a power flow optimization problem. The objective is to minimize the network operational cost and at the same to provide the stochastic demand loads within the MGs in average. With the solution of this problem, a distributed power flow algorithm is proposed under the assumption of a communication infrastructure within the grid. Considering their demand loads, the MGs progressively update their interactions throughout the grid. This strategy results in a distributed and reliable load management within the grid in a cost-efficient manner.

The paper is organized as follows. System model and problem formulation are presented in Section II. The problem solution and distributed load management algorithm are proposed in Sections III. Numerical results are given in Section IV and the paper is concluded in Section V.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a distributed power smart grid consisting of a set  $\mathcal{N} \triangleq \{n : n = 1, \dots, N\}$  of MGs interconnected through a power transmission infrastructure and a communication network, as shown in Fig. 1. Within each MG, assume a power source that can be shared within the grid following a certain strategy set by the administration (grid market operator). In a deregulated environment, power flow within the grid is certainly affected by two main factors: power generation cost at individual MGs and power transmission cost between any two MGs. Here, under the assumption of time-varying demands of MGs, a statistical approach is developed towards the total cost minimization within the grid.

Let  $\Gamma \triangleq \{\gamma_n : n = 1, \dots, N\}$  be the vector of demands within the grid, where  $\gamma_n$  is the demand of  $MG_n$ . These demands are assumed as random variables varying over the time. Moreover, consider  $\mathbf{S} = \{s_{nm}\}_{n \in \mathcal{N}}^{m \in \mathcal{N}}$  as the vector of power flows within the grid, where  $s_{nm}$  is the supplied power by  $MG_n$  to  $MG_m$ . Due to the random variation of  $\Gamma$ ,



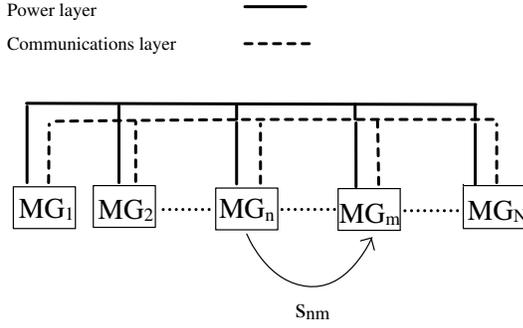


Fig. 1. Smart power grid

these flows should be regulated adaptively over the time. Our design objective is to minimize the grid mean operational cost, including power generation and transmission costs, such that demands  $\Gamma$  to be satisfied. This objective can be formulated as the following problem

$$\min_{\mathbf{S}} \sum_{n=1}^N \mathbb{E}_{\gamma_n} \left[ C \left( \sum_{m=1}^N s_{nm} \right) + \sum_{m=1}^N \mu_{nm} s_{nm} \right] \quad (1)$$

$$\text{s.t.} \quad \sum_{n=1}^N \mathbb{E}_{\gamma_n} [s_{nm}] \geq \gamma_m \quad \forall m \in \mathcal{N} \quad (2a)$$

$$s_{nm}^{\min} \leq s_{nm} \leq s_{nm}^{\max} \quad \forall n, m \in \mathcal{N}. \quad (2b)$$

where  $\mathbb{E}_{\gamma_n}$  denotes the expectation with respect to  $\gamma_n$ .  $C(\cdot)$  is the cost for power generation that is assumed to be a convex and differentiable function. On the other hand, the transmission cost is assumed to be linear function with rate  $\mu_{nm}$  as the cost per unit of power transmitted from  $\text{MG}_n$  to  $\text{MG}_m$ . Constraints (2a) satisfy MGs demands in average and constraints (2b) restrict the power flow levels within an upper and lower bounds.

This problem is convex and can be solved using several convex optimization techniques [6]. However, this requires the availability of  $\Gamma$  a priori for the whole time in the scope of the problem. This knowledge is not always available. Alternatively, we are interested in solving this problem progressively over the time, when each  $\gamma_n$  is realized at each time instant  $t$ .

### III. DISTRIBUTED LOAD MANAGEMENT

The most significant challenge in the solution of problem (1)–(2) is due to the coupling expectations. We form the Lagrangian function as

$$L(\mathbf{S}, \Lambda) = \sum_{n=1}^N \mathbb{E}_{\gamma_n} \left[ C \left( \sum_{m=1}^N s_{nm} \right) + \sum_{m=1}^N \mu_{nm} s_{nm} \right] \quad (3)$$

$$- \sum_{m=1}^N \lambda_m \left( \sum_{n=1}^N \mathbb{E}_{\gamma_n} [s_{nm}] - \gamma_m \right)$$

and the corresponding dual function as

$$D(\Lambda) = \inf_{\mathbf{S}} \{ L(\mathbf{S}, \Lambda) : (2b) \} \quad (4)$$

where  $\Lambda = \{ \lambda_m \geq 0 \}_{m \in \mathcal{N}}$  is the set of Lagrange multipliers. Accordingly, the dual problem is

$$\max_{\Lambda \geq 0} D(\Lambda). \quad (5)$$

Thanks to the decomposable form of  $L(\mathcal{L}, \Lambda)$ ,  $D(\Lambda)$  in (4) can be obtained if each  $\text{MG}_n$  solves problem

$$\min_{\mathbf{S}_n = \{s_{nm}\}_{m \in \mathcal{N}}} \mathbb{E}_{\gamma_n} \left[ C \left( \sum_{m=1}^N s_{nm} \right) + \sum_{m=1}^N (\mu_{nm} - \lambda_m) s_{nm} \right] \quad (6)$$

$$\text{s.t.} \quad (2b). \quad (7a)$$

Considering this problem at time  $t$  with a given set of  $\{ \lambda_m(t) \}_{m \in \mathcal{N}}$ , it is convex and can be solved using interior point method (IPM) [6] to obtain  $\{ s_{nm}^*(t) \}_{m \in \mathcal{N}}$ . Inspecting (6), each Lagrange multiplier  $\lambda_m(t)$  can be interpreted as the marginal benefit of  $\text{MG}_n$  from selling a unit of power to  $\text{MG}_m$  at time  $t$ . In other words,  $\lambda_m(t)$  is the announced *purchase price* of  $\text{MG}_m$  in interaction with the other MGs.

Having solved (6)–(7) at all MGs, each  $\text{MG}_m$  can be aware of  $\{ s_{nm}^*(t) \}_{n \in \mathcal{N}}$ , i.e. its own produced power for internal usage as well as the incoming power flow from the rest of the grid. By this knowledge,  $\text{MG}_m$  is able to solve its portion of the dual problem in (5) using subgradient method. Beginning with an initial  $\lambda_m(0)$ , given  $\lambda_m(t)$  at time  $t$ , this MG obtains the knowledge of  $\{ s_{nm}^*(t) \}_{n \in \mathcal{N}}$  from distributed problems (6)–(7) in the grid. This MG then updates its own purchase price for time  $t + 1$  as

$$\lambda_m(t + 1) = \lambda_m(t) + \alpha \left( \gamma_m - \sum_{n=1}^N \mathbb{E}_{\gamma_n} [s_{nm}] \right)^+ \quad (8)$$

where  $\alpha$  is a step size.

#### Algorithm 1 Power flow control

- 1: Initialization:  $\lambda_m(0) = \lambda_{\text{init}} \quad \forall m \in \mathcal{N}, t = 0$ .
- 2: **while** {1} **do**
- 3:     **Phase 1: Purchase price declaration**
- 4:     Every MG  $m \in \mathcal{N}$  broadcasts purchase price  $\lambda_m(t)$ .
- 5:     **Phase 2: Power distribution**
- 6:     **for**  $n \in \mathcal{N}$  **do**
- 7:         Realize a new load  $\gamma_n(t)$  randomly.
- 8:         Using (6)–(7), obtain  $\{ s_{nm}^*(t) \}_{m \in \mathcal{N}}$ .
- 9:         Send  $s_{nm}^*(t)$  to  $\text{MG}_m$  for all  $m \in \mathcal{N}$ .
- 10:     **end for**
- 11:     **Phase 3: Purchase price update**
- 12:     Every MG  $m$  updates  $\lambda_m(t + 1)$ , using (8).
- 13:      $t = t + 1$ .
- 14: **end while**

The above described solution can be summarized as an adaptive approach in Algorithm 1. Each time slot of this algorithm consists of three phases to be run in the beginning of the time slot. In phase 1 of time slot  $t$ , each MG  $m \in \mathcal{N}$



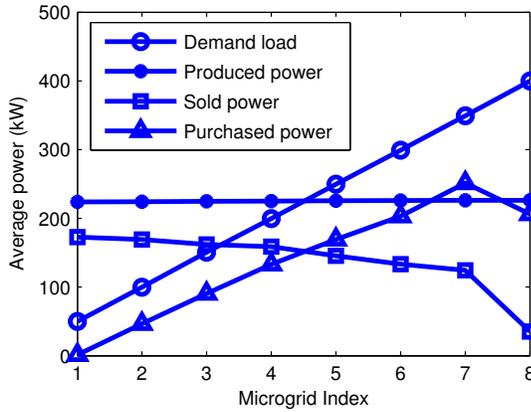


Fig. 2. Average demand loads, and produced, sold and purchased powers

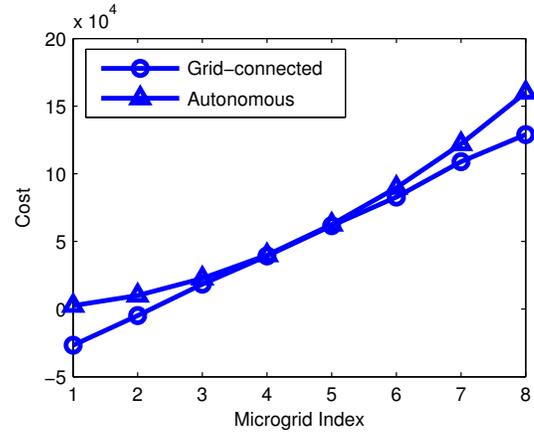


Fig. 3. Cost comparison between grid-connected and autonomous modes

broadcasts its purchase price  $\hat{\lambda}_m(t)$  of this time slot to the rest of the grid. In phase 2, each MG  $n \in \mathcal{N}$  takes its realized demand  $\gamma_n(t)$  and announced prices  $\{\hat{\lambda}_m(t)\}_{m \in \mathcal{N}}$  into account to come up with optimal power flows  $\{s_{nm}^*(t)\}_{m \in \mathcal{N}}$  during slot  $t$ . Phase 3 starts when all MGs are aware of their incoming power flows. In this phase, all MGs update their purchase price for the next time slot. The algorithm proceeds progressively over time.

#### IV. NUMERICAL RESULTS

As an example, consider a power grid consisting of  $N = 8$  small MGs starting from MG<sub>1</sub> and ending with MG<sub>8</sub>. The distance between any two neighbor MGs is the same and is noted as one hop. The transmission price per unit of power between any two MGs is assumed to be the number of hops between them. Moreover, power generation function is assumed to be a square function,  $C(\cdot) = (\cdot)^2$ . Demand loads are assumed to follow normal random variables with mean  $\Omega = \{50 : 50 : 400\}$  KW per unit of time and standard deviation  $\sigma = 5$  KW, i.e.  $\Gamma \sim \{\mathcal{N}(50n, 5)\}_{n \in \mathcal{N}}$ . Maximum permitted power flow from MG <sub>$n$</sub>  to each MG <sub>$m$</sub>  is set as  $s_{nm}^{\max} = 50n$  KW and the minimum power flow is  $s_{nm}^{\max} = 0$  KW for all  $n, m \in \mathcal{N}$ .

The simulation is run for 200 time slots; each slot with realization of a new demand value per MG. The average *demand* load, *produced*, *sold* and *purchased* powers of individual MGs are shown in Fig. 2. While demand increases linearly with MG indexes in accordance with assumed values in  $\Omega$ , produced powers are approximately the same for all MGs. Remarkably, the sum of produced and purchased powers at each MG is equal to the sum of demand plus sold power. Purchased power increases with demand whereas sold power decreases. Exceptionally, the decrease of purchased power at MG<sub>8</sub> is possibly due to its location in the network topology, which has high power transmission cost. In a logical statement, low demand MGs sell power to high demand ones.

We compare the average operational cost in both grid-connected and autonomous modes in Fig. 3. To this end, the cost in autonomous mode is obtained from the square power

production function. Furthermore, the cost in grid-connected mode is the production cost plus purchased cost minus the revenue from selling power to the other MGs. As shown, grid-connected mode achieves lower cost for low demand and high demand MGs. The decrease in the cost of low demand MGs is the result of selling power to high demand ones. In particular, the cost of MG<sub>1</sub> and MG<sub>2</sub> even get negative as a result of high revenue from selling power that compensates their production cost. This outcome, also, decrease the cost of high demand MGs as they purchase a portion of their demand from the load demand ones. In summary, as a numerical indicator, proposed grid-connected demand load management achieves 20% cost decrease in comparison with stand-alone operation. In overall, this power sharing scheme transforms the parabolic cost curve to a linear one as shown in Fig. 3.

#### V. CONCLUSION

Demand load management with the aim of operational cost minimization in distributed smart grids have been investigated. It was shown that this objective could be achieved by collaboration between MGs using a communication infrastructure and defining a set of parameters known as purchase prices. It was shown that power sharing in grid-connected mode results in lower price than stand-alone operation.

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