# Robust stabilizer feedback loop design for a radio-frequency amplifier

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# Robust Stabilizer Feedback Loop Design For A Radio-Frequency Amplifier

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***Abstract*—In this paper, a robust proportional-integral-derivative (PID) feedback compensator is designed for a low power radio-frequency (RF) amplifier, using Kharitonov's theorem. The robust D-stability concept is used to achieve robust performance by clustering the characteristic polynomial equation roots of the closed loop system in a specified angular region. Then, the controller is realized and the obtained results demonstrate a desirable amplification over the operating frequency band. It is shown that the designed feedback compensator guarantees the robust stability and robust performance for a wide range parameter variation.**

***Keywords*—**Kharitonov's theorem, RF amplifier, PID, Robustness, D-stability.

## I. INTRODUCTION

AMPLIFIERS are among the basic building blocks of an electronic radio-frequency (RF) communication system. The purpose of the RF amplifier is to boost the power of the incoming signal relative to all the other signals picked up by the antenna to a level which can be used in the frequency changer. A second function of the RF amplifier is to act as a matched load to the antenna so that the antenna signal is not reflected at the interface leading to the loss of efficiency [1].

The RF amplifier, in theory, should have a fat amplitude response in their pass bands. But in practice, this is not so. The result is that the signal emerging from the RF amplifier has some variation of amplitude with respect to frequency. It must be removed if distortion is to be avoided [1].

There are a variety of approaches to stabilizing an RF amplifier [2, 3]. A stability approach could be achieved from a potentially unstable transistor by making sure that the chosen amplifier terminating impedances remain inside the stable regions at all frequencies as determined by the stability circles. Another method would be to load the amplifier with an additional shunt or series resistor on either the generator or load side. If the condition for unconditional stability is achieved for this expanded transistor model, then optimization can be performed for the other circuit elements to achieve the desired gain and bandwidth [2].

A third approach that is sometimes useful is to introduce an external feedback path that can neutralize the internal feedback of the transistor. The most widely used scheme is the shunt–shunt feedback circuit which is also used in this work. Negative feedback can widen the bandwidth of amplifier and improve the matching at its input and output. However, the gain of the amplifier is reduced and it may adversely affect the noise figure unless another relatively low-noise amplifier is added in the forward path before the original amplifier [4].

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In the present work a robust technique based on Kharitonov's theorem [5], is used to obtain an admissible proportional-integral-derivative (PID) feedback compensator which guarantee satisfactory operation of the system under realistic operating conditions. The D-stability concept is used to fine tuning of the designed PID loop parameters.

The above technique has been applied to a typical low power amplifier applicable for amplification of TV signals in the Very High Frequency (VHF) range. The resulting feedback system is shown to achieve robust performance among the all operating frequencies.

## II. TEST RF AMPLIFIER CIRCUIT

An *ac* schematic diagram of the RF amplifier case study is shown in Fig. 1. This amplifier designed to be quickly installed between two coaxial cables to amplify received TV signal in appropriate amplitude value. Both input and output impedances are compatible with 75 Ω cables. The circuit parameters and operating data are considered as the same as given in [6].

The T1 transistor has the main role in the signal amplification, and T2 is working as an emitter follower to create an enough bandwidth. Here, the feedback bias is conventional one, and is determined by R2 and R4. Replacing this feedback structure with a robust PID compensator circuit is the main design objective.

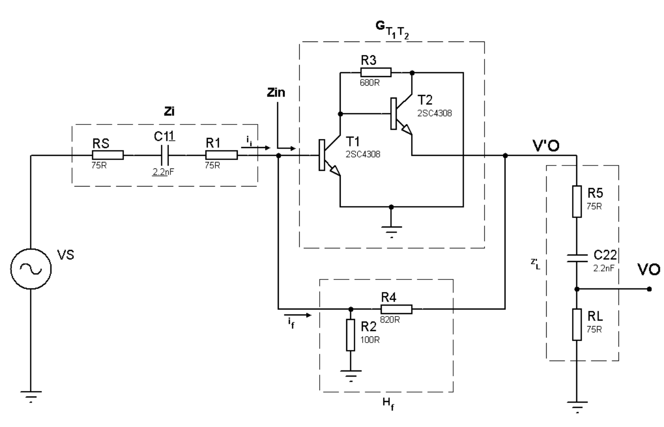



Fig. 1 An *ac* schematic diagram of the test RF amplifier

The total gain of this RF amplifier is fixed at 22dB. Because of the high frequency limit of used transistors (less than 2GHz), the described RF amplifier in Fig. 1 works well up to 150 MHz. The total current consumption of this amplifier circuit is around 20mA. A view of implemented circuit and its real response (amplification behavior) to a sinusoidal input signal are shown in Fig. 2.

## III. Modeling

To obtain the dynamic model of the RF amplifier circuit (Fig. 1), one may use the two-port model to calculate the circuit Y parameters. For this purpose, the modified circuit in shunt–shunt feedback configuration and associated equivalent two-port circuit are shown in Fig. 3a and Fig. 3b, respectively.

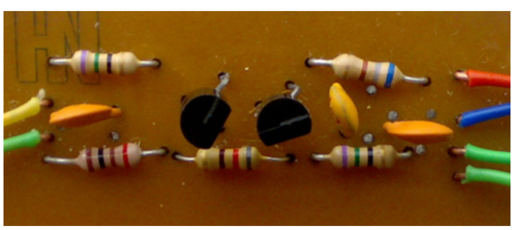
Fig. 3b illustrates a simplified feedback arrangement of the RF amplifier. The Y parameters for the composite circuit are simply the sum of the Y parameters of the amplifier and feedback two-port circuits:

$$|Y_C| = |Y| + |Y_f| \quad (1)$$

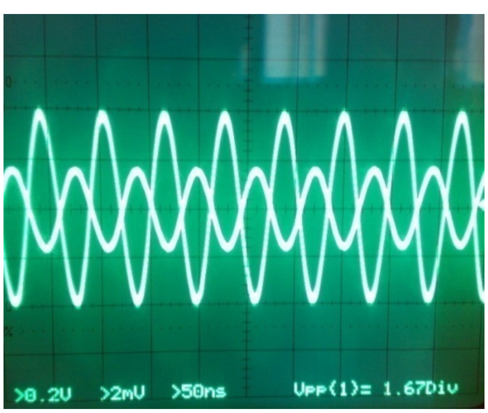
where,

$$Y_C = \begin{bmatrix} y_{C11} & y_{C12} \\ y_{C21} & y_{C22} \end{bmatrix}, Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}, Y_f = \begin{bmatrix} y_{f11} & y_{f12} \\ y_{f21} & y_{f22} \end{bmatrix}$$

When the real part Re($Y_{in}$) and/or Re($Y_{out}$) are negative the device is producing a negative resistance and is therefore likely to be unstable causing potential oscillation. However, if $y_{C11}$ is large, this part of the input impedance is lower and the device is more likely to be stable.

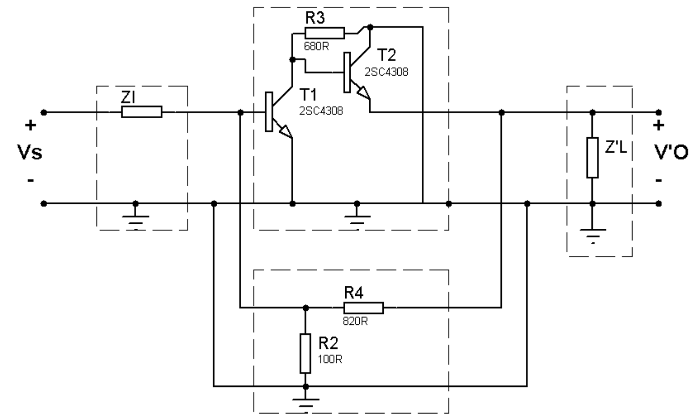


(a)

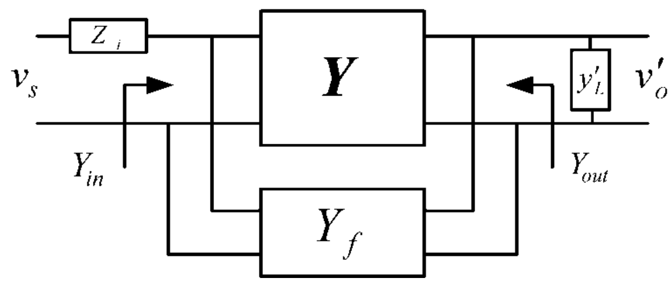



(b)

Fig. 2 Case study RF amplifier; a) Implemented circuit, b) Circuit response for a sinusoidal input




(a)




(b)

Fig. 3 RF amplifier circuit; a) Shunt–shunt feedback configuration, b) Y parameter representation

In fact placing a resistor across (or sometimes in series with) the input or output or both is a common method to ensure stability. This degrades the noise performance and it is often preferable to place a resistor only across the output.

Note that as $y_{C12}$ tends to zero this also helps as long as the real part of $y_{C11}$ is positive. The device is unconditionally stable if for all positive source and loads resistance the real part of $Y_{in}$ is greater than zero and the real part of $Y_{out}$ is greater than zero. The imaginary part can of course be positive or negative. In other words the real input and output impedance is always positive for all source and loads which are not negative resistances. Note that when an amplifier is designed the stability should be checked at all frequencies as the impedance of the matching network changes with frequency [7].

To maintain stability the Re($Y_{in}$) ≥0 and the Re($Y_{out}$) ≥0 for all the loads presented to the amplifier over the whole frequency range. The device is unconditionally stable when the above applies for all positive resistance part of source and loads. Note that the imaginary part of the source and load can be any value [7].

Considering the detailed circuit model for transistors (T1, T2), Y parameter representation (Fig. 3), and after some algebraic manipulations, the corresponding Y parameters (at nominal operating point) for the open-loop system, introduced in Fig. 3b, can be computed, as follows [3]:

$$y_{22} = \frac{3.6\times10^{-22}s^2+1.5\times10^{-12}s+2.2\times10^{-5}}{2.2\times10^{-11}s+5.6\times10^{-3}} \quad (2)$$

$$y_{12} = -\frac{s(6.1\times10^{-23}s+1.9\times10^{-14})}{2.2\times10^{-11}s+5.6\times10^{-3}} \quad (3)$$

$$y_{21} = \frac{-6.1\times10^{-23}s^2+4.6\times10^{-12}s-7.2\times10^{-2}}{2.2\times10^{-11}s+5.6\times10^{-3}} \quad (4)$$

$$y_{22} = \frac{1.2\times10^{-22}s^2-2.6\times10^{-12}s-4.5\times10^{-4}}{2.2\times10^{-11}s+5.6\times10^{-3}} \quad (5)$$

Finally, the overall transfer function (from output voltage to input voltage) for the circuit shown in Fig. 1 is:

$$G(s) = \frac{v_0}{v_s} = -\frac{1.073\times10^{8}s^2(s-6.9\times10^{10})}{(s+3\times10^{6})^2(s+2.5\times10^{8})(s+1.2\times10^{10})} \quad (6)$$

Considering (6), it is seen that G(s) is a nonminimum phase transfer function, because there are a zero in the right-half plane (RHP) that it has been created by feedback structure Y parameters. An objective to replace the existing feedback circuit by a PID compensator is to eliminate this zero in the RHP.

## IV. Feedback Synthesis Methodology

### A. RF Amplifier with PID Feedback System

PID is a well known control configuration to use in the feedback loop of various real-world dynamical systems over years. In addition to simplicity of structure, it is flexible enough to meet different control objectives [8]. The block diagram of RF amplifier circuit with PID compensator is shown in Fig. 4a. Transfer function of the feedback loop is:

$$G(s) = \frac{i_f}{\acute{v}_0} = -y_{f12} = \left(k_P + \frac{k_I}{s} + k_D s\right) \quad (7)$$

And input admittance, introduced in Fig. 4b, can be computed as follows [3]:

$$Y_{in} = \acute{y}_{11} - \frac{\acute{y}_{12}\acute{y}_{21}}{\acute{y}_{22}} \quad (8)$$

where

$$\acute{y}_{ij} = y_{ij} + y_{fij} \quad (i, j=1,2).$$

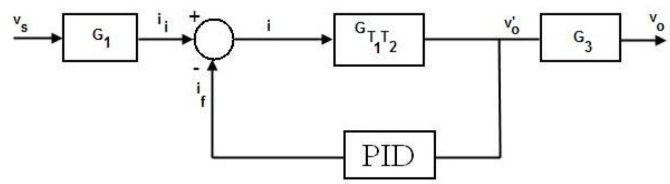
To remove zero in the RHP, and considering (8), it would be better to fix $y_{f11}, y_{f21}$ and $y_{f22}$ at zero. Finally, the $Y_f$ matrix in case of using PID feedback can be represented as follows:

$$Y_f = \begin{bmatrix} 0 & -(k_P + \frac{k_I}{s} + k_D s) \\ 0 & 0 \end{bmatrix} \quad (9)$$

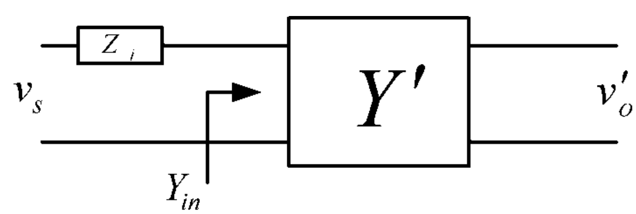
A practical circuit for PID compensator represented in (9), is shown in Fig. 5. Considering (Fig. 5), the PID parameters can be easily computed [3]:

$$\begin{cases} k_P = \frac{1}{R_P} \\ k_I = \frac{1}{L_I} \\ k_D = C_D \end{cases} \quad (10)$$

The simplified two-port model in the presence of a PID compensator is shown in Fig. 4b. Using the simplified transistors *ac* model [3], the closed-loop admittance matrix can be calculated as follows:




(a)




(b)

Fig. 4 RF amplifier circuit with PID feedback; a) Shunt–shunt PID feedback configuration, b) Simplified two-port model

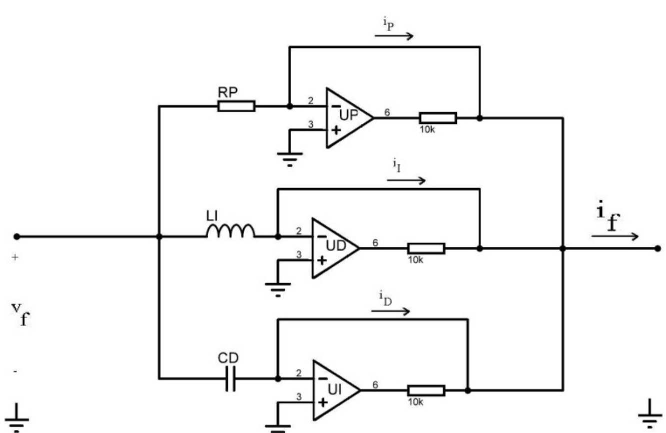



Fig. 5 A circuit for PID feedback compensator

$$Y' = \begin{bmatrix} \frac{es}{cs+g} & -\frac{k_D s^2+k_P s+k_I}{s} \\ \frac{k}{cs+g} & \frac{C_{22}s}{\delta_2 s+1} \end{bmatrix} \quad (11)$$

where

$$e = g_{m1}c_{\mu 1} + gc_{\mu 1}, g = g_3, c = c_{\mu 1}$$

$$k = -g_{m1}g_{m2}, \delta_2 = C_{22}(R_L + R_5)$$

The $g_{mi}$, $c_{\mu i}$ (i=1, 2) are the transistor parameters, and $g_3 = {}^{1}/_{R_3}$.

Then, the corresponding transfer function for the closed loop circuit from the output voltage signal to the input voltage signal can be easily computed.

$$G(s) = \frac{v_o}{v_s} = -\frac{q_3 s^3}{r_4 s^4+r_3 s^3+r_2 s^2+r_1 s+r_0} \quad (12)$$

where,

$$q_3 = C_{11}k\delta_1, r_0 = kk_I$$

$$r_1 = (t_s+\delta_2)kk_I + kk_P$$

$$r_2 = (t_s\delta_2)kk_I + (t_s+\delta_2)kk_p + kk_D$$

$$r_3 = (t_s\delta_2)kk_P + (t_s+\delta_2)kk_D + C_{22}(e + C_{11}g)$$

$$r_4 = (t_s\delta_2)kk_D + C_{22}(t_s e + C_{11}c)$$

here, $\delta_1 = C_{22}R_L$, and $t_s = (R_s + R_1)C_{11}$.

### B. Kharitonov's Theorem

We proceed to design a robust feedback loop using the Kharitonov based synthesis approach. In this paper, our main focus is concentrated on robust stability and robust performance in presence of load and parameters variation.

Based on Khartonov's theorem, every polynomial such K(s),

$$K(s) = a_0 s + a_1 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \cdots \quad (13)$$

with real coefficients is Hurwitz if and only if the following four extreme polynomials are Hurwitz [7]:

$$K_1(s) = a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + \cdots$$

$$K_2(s) = a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + \cdots$$

$$K_3(s) = a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + \cdots$$

$$K_4(s) = a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + \cdots$$

(14)

The "-" and "+" show the minimum and maximum bounds. For the problem at hand (12), the order of closed-loop system is 4, and the closed loop characteristic polynomial can be rewritten as follows.

$$K(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \quad (15)$$

where

$$a_0 = r_0 = kk_I, \ a_1 = r_1 = (t_s+\delta_2)kk_I + kk_P$$

$$a_2 = r_2 = (t_s\delta_2)kk_I + (t_s+\delta_2)kk_p + kk_D$$

$$a_3 = r_3 = (t_s\delta_2)kk_P + (t_s+\delta_2)kk_D + C_{22}(e + C_{11}g)$$

$$a_4 = r_4 = (t_s\delta_2)kk_D + C_{22}(t_s e + C_{11}c)$$

Based on Khartonov's theorem, to check the stability of closed loop amplifier, it is need to test the Hurwitz criteria for the following four polynomials:

$$K_1(s) = a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4$$

$$K_2(s) = a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4$$

$$K_3(s) = a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4$$

$$K_4(s) = a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4$$

(16)

### C. PID Design

Here, the $R_{L_n}$, $R_{5n}$ and $C_{22n}$ are considered as uncertain parameters. The variation domains are assumed as given in (17); while the nominal values are fixed at $R_{L_n} = 75\Omega$, $R_{5n} = 75\Omega$, $C_{22n} = 2.2nF$.

$$\begin{cases} R_L \in [50, 100] \\ R_5 \in [50, 100] \\ C_{22} \in \{2.2nF \pm 30\%\} \end{cases} \quad (17)$$

As has mentioned, the problem of checking the Hurwitz stability of the family for the present RF amplifier can be reduced to that of checking the Hurwitz stability of four polynomials (16). This procedure after some manipulations results a set of inequalities, which are satisfied for some values of $k_P$, $k_D$ and $k_I$. These values are graphically shown in Fig. 6. Variation posture of PID feedback system pole-zero map for acceptable values of PID parameters (Fig. 6) is shown in Fig.7.

The basic geometry associated with the zero exclusion condition [5], is fully demonstrated in Fig. 8 for $0 < \omega < 2 \times 10^8$ rad/sec, while the PID parameters have been fixed at $(k_p, k_I, k_D) = (8 \times 10^{-4}, 1000, 1 \times 10^{-12})$.

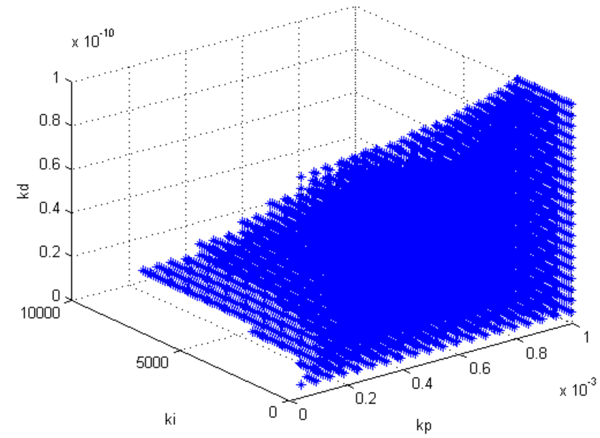



Fig. 6 Acceptable values of $k_P$ , $k_D$ and $k_I$ , to make a stabilizing PID feedback system

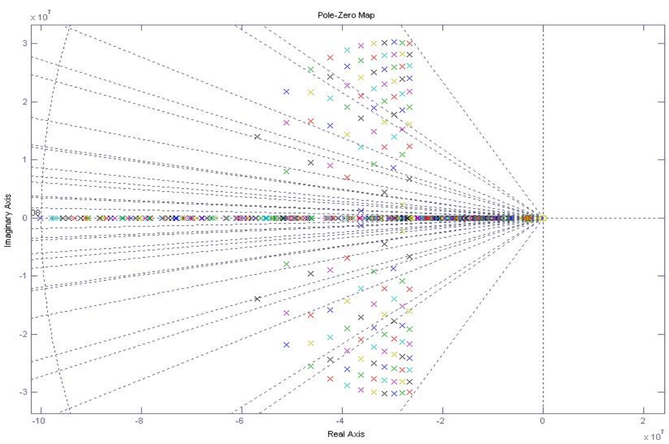
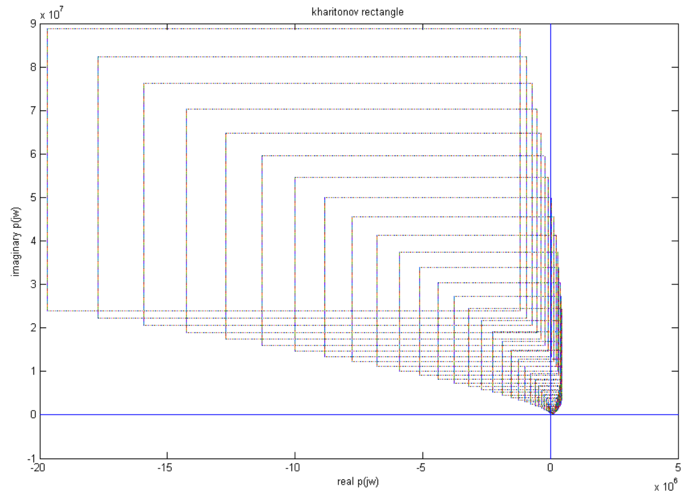
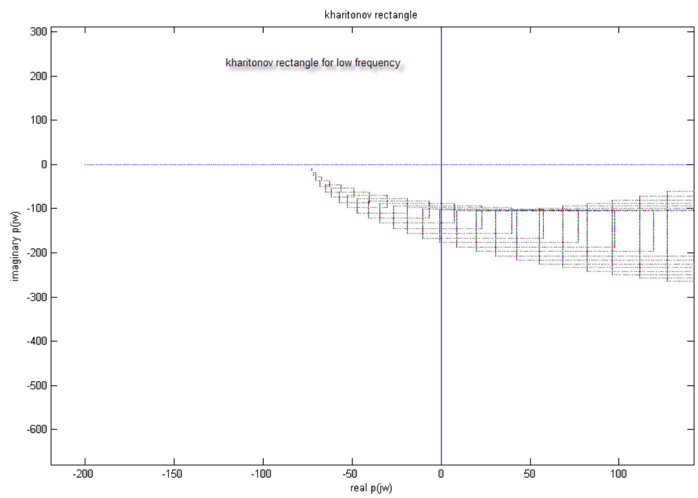



Fig. 7 PID feedback system pole-zero map for acceptable values of PID parameters




(a)




(b)

Fig. 8 Motion of the Kharitonov's rectangle for $0 < \omega < 2 \times 10^8$ rad/sec. The magnified view around origin is shown in figure (b).

## V. Application of D-Stability Concept

To ensure both stability robustness and specified performance robustness, it is important to guarantee that the roots of the characteristic equations for a linear time-invariant system under parameter perturbations remain in a specific region.

The locations of the roots of the characteristic equations for linear systems determine some performance specifications such as transient stability, damping and the speed of the time response. These specifications can be assured by the placement of the roots of the characteristic equations in an appropriate region (*D*) in the roots plane. *D-stability* investigates the robustness problem of the characteristic equation's roots clustering in specific regions for dynamical systems with parameter perturbations.

Locating the roots inside the left-half plane (LHP) guarantees the stability of the system, placing the roots inside the left-sectors in the LHP guarantees a minimum damping ratio for the roots, and clustering the roots inside the shifted LHP guarantees a maximum settling-time for the time response of the system [5]. To have the combined effects of the left-sector and the shifted LHP, one may choose a special region in the LHP as the root assignment region.

Various performance specifications for a RF amplifier can be achieved by the placement of the closed loop roots of its characteristic polynomial in appropriate regions. The main aim is to ensure that the roots of the family of characteristic polynomials lie inside D. However, a set of feedback compensators rather than a unique feedback usually satisfies this requirement. One may select a compensator among all D-stabilizing compensators, following introducing an additional objective.

Considering the assumed parameter perturbation (17), the characteristic equation of the closed-loop system formed by (15) in the feedback configuration of Fig. 4a, has a parametric uncertainty structure. Holding the maximum bandwidths, robust performance in the presence of parameters variation and decreasing the sensitivity to the disturbances are considered as main objectives to calculate D-stability margin of the present RF amplifier. For the example at hand, according to the required desirable performance, the D region for the characteristic polynomial roots, as shown in Fig. 9, is defined such that

$$D: \begin{cases} \text{real}\{p_n\} \in [-5,-1] \times 10^6 \text{ rad/sec}; \quad n = 1,2,3 \\ \text{real}\{p_4\} < -880 \times 10^6 \text{ rad/sec} \\ \text{imaginary}\{p_n\} = 0; \quad n = 1,2,3,4 \end{cases} \qquad (18)$$

and, Fig. 10 shows the parametric space for D-stabilizing PID parameters.

Using Fig. 10, D-stabilizing PID feedback gains cab be selected as

$$(k_p, k_I, k_D) = (5 \times 10^{-4}, 1000, 5 \times 10^{-13})$$

The amplifier frequency response according to the above PID feedback parameters is shown in Fig. 11.

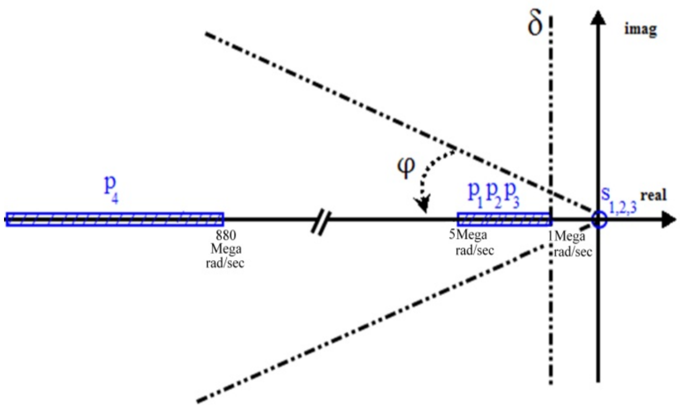



Fig. 9 Specified D region (shaded area)

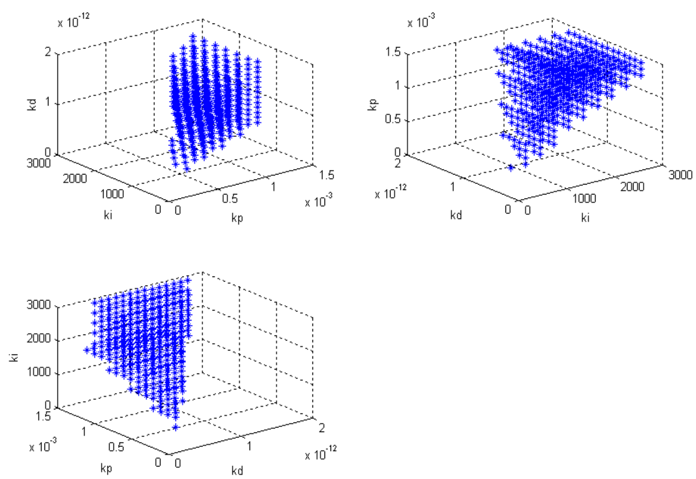



Fig. 10 The $k_P$ , $k_D$ and $k_I$ to have a D-stabilizing PID feedback system

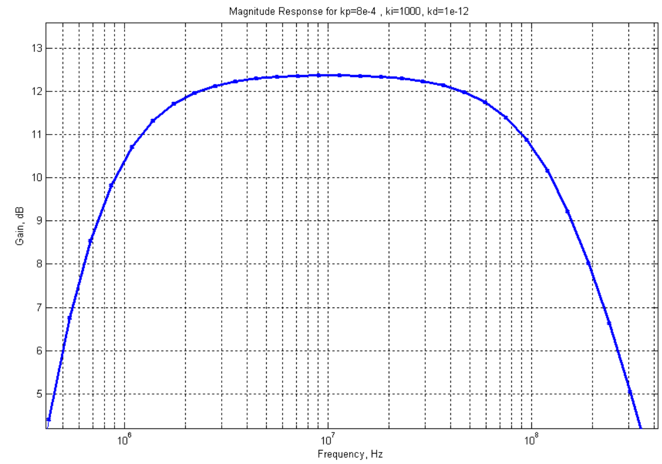



Fig. 11 Amplifier frequency response for the designed PID feedback system

## VI. Conclusion

A simple robust PID feedback loop based on Kharitonov's theorem is designed for a low power RF amplifier. The D-stability concept is used to fine tuning of PID parameters. Results show robustness against perturbation in parameters, and a desirable performance over a wide range operating conditions without degrading the system bandwidth.

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