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NONLINEAR AND LINEAR ROBUST CONTROL OF SWITCHING POWER CONVERTERS

Hassan Bevrani

West Regional Electric Co.
Research & Standards office
P.O. Box 67144
Kermanshah, Iran
Fax: +98-831-832653

M. Abrishamchian, N. Safari-shad

K. N. Toosi University of Technology
Department of Electrical Engineering
P.O. Box 16315-1355
Tehran, Iran
Fax: +98-21-862066

ABSTRACT

Switching power converters are nonlinear and time variant systems. The classical controller design for a switching power converter based on the linear model derived by one of conventional modelling methods. However the parameter and load variations and input unregulated voltage are not taken into consideration simultaneously in a controller design.

In this paper, we use "input-output feedback linearization" technique for nonlinear control issue, and "Kharitonov, H∞ and μ-synthesis" techniques for robust control issue to design of nonlinear and linear robust controllers to access both satisfactory stability and satisfactory performance, even in case of uncertainties and disturbances, for typical fifth-order DC-DC Switching power converter.

The stability and performance of the converter and the effectiveness of the proposed controllers are demonstrated by some simulated results. Making use of simulation results, varies comparisons between these controllers and a variety of other controllers are then made, exhibiting the high efficiency of these methods compared to conventional methods.

1 INTRODUCTION

Suggestion of the idea of switching power converters returns to the 1970's. The need for small, but high-quality, power supplies is recognized as the motivation for the idea. In spite of significant developments in the field and a variety of regulator generations, the issue is still to the utmost importance and technologies continue to demand power supplies with lower cost, weight and size, and higher efficiency, regulation and stability. Vast amount of literature on the subject shows the great extent of research on various aspects of these converters such as modeling and control of Switching power converters, design of high-frequency transformers and drive of power switches [1]. In practice, these systems are controlled using classical compensators and controllers which are commonly based on root locus and Bode and Nyquist diagrams, within which current-mode, voltage-mode and feed forward controllers are the most common ones. Parallel with developments in theories of nonlinear control and optimal control, controllers relying on these theories have been designed in the last two decades; for instance, sliding-mode, neural-network, Lyapunov-based and linear-quadratic controllers. In this paper, along the above direction of research, PWM DC-DC switching power converters are adequately investigated from a control point of view. Together with simulations and application of different methods on typical switching converter, significant advantages and disadvantages of any method are also mentioned, so that this paper may be utilized as suitable guide for designers and analysts of switching power converters.

In this paper, we make use of the input-output feedback linearization and powerful theories of linear robust control, and propose new directions in the control switching power converters. The nonlinear technique of input-output feedback linearization is a successful approach to the control of these system because the nature of switching regulators is severely nonlinear and the linear models achieved Jacobian linearization are operating-point dependent. This claim will be verified by simulation results. The switching power converters have a variable structure and subject to types of uncertainties, so the robust control methodologies such as Kharitonov, H∞ and μ which are based on a · more complete· model of system (by taking into consideration the uncertainties and disturbances) seem to be helpful.

2 MODEL DESCRIPTION

For simplicity and clarity, all of the control methods mentioned here are applied to a typical power converter whose schematic is depicted in figure 1. This is boost converter and is used as charger and (high) power regulator. Besides the ordinary applications, it is used for supplying the inverter connected to an AC motors [2].

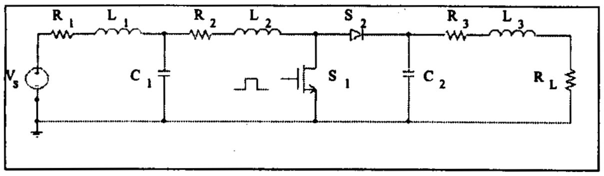

Fig. 1 : Typical power converter circuit

The following numerical values of parameters are used here [3] :

$V_s = 3$ kv ; $L_1 = 16$ mH ; $C_1 = 2.4$ mF
$V_{ref} = 4.3$ kv ; $L_2 = 24$ mH ; $C_2 = 1.2$ mF
$R_1 = R_2 = R_3 = 0.25\ \Omega$; $L_3 = 2$ mH ; $R_L = 10\ \Omega$

We can obtain a continuous approximation by applying the well-known state-space averaging technique [1]. This procedure yields a system of nonlinear dynamical equations given by

$$
\begin{aligned}
\dot{x}_1 &= -\frac{R_1}{L_1}x_1 - \frac{1}{L_1}x_2 + \frac{V_s}{L_1} \\
\dot{x}_2 &= \frac{1}{C_1}x_1 - \frac{1}{C_1}x_3 \\
\dot{x}_3 &= \frac{1}{L_2}x_2 - \frac{R_2}{L_2}x_3 - \frac{(1-d)}{L_2}x_4 \\
\dot{x}_4 &= -\frac{(1-d)}{C_2}x_3 - \frac{1}{C_2}x_5 \\
\dot{x}_5 &= \frac{1}{L_3}x_4 - \frac{(R_3+R_L)}{L_3}x_5 \\
y &= x_4
\end{aligned}
\quad (1)
$$

Where the state vector is:

$$[x_1, x_2, x_3, x_4, x_5] = [i_{L_1}, v_{C_1}, i_{L_2}, v_{C_2}, i_{L_3}].$$

3 NONLINEAR CONTROL

DC-DC switching power converters have a nonlinear and time-varying nature, and the linear models are approximate and operating-point depended. Therefore, the classical linear controllers cannot maintain the desired stability and performance under a wide range of operating conditions. To the moment, various nonlinear methods have been used for control of models and DC-DC switching power converters among which may be mentioned the use of artificial neural network, sliding-mode control, one-cycle control, nonlinear PI control, nonlinear feed forward control and extended duty-cycle based control. Hence, we deal with the input-output feedback linearization technique which has been a focus of interest in the nonlinear control design issue.

The basic idea is partial or exact linearization such that the well-known methods of linear control are applicable to the new representation of the system. It is notable that the method is basically different from classical (Jacobian) linearization. Indeed, in Jacobian linearization, the simplified model is a local approximation of the original model about the operating point, however, in the input-output feedback linearization, linear equations are obtained from exact algebraic change of variable [4]. Thus, the mentioned input-output feedback linearization technique is predicted to be a useful manner in the control of switching power converters. The claim will be verified by simulation results. By applying this method, the resulting control low will be:

$$u(t) = \frac{C_2}{x_3}\left[\frac{x_5}{C_2} + \dot{y}_d(t) + k_0(y_d(t) - y(t))\right] \quad (2)$$

where, k_0 and y_d are the constant proportional gain and desired output voltage. Figure 2 shows the output voltage response and correspond duty cycle to a step input command.

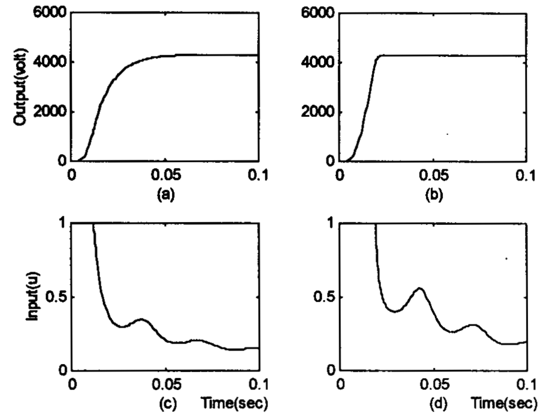

Fig. 2 : Unit step-response, a) Vo (ko=100), b) Vo (ko=1000), c) u (ko=100), d) u (ko=1000).

According to the simulation results obtained from application of the controller based on input-output feedback linearization, the closed-loop system shows satisfactory stability and performance under a wide range of variations in load and input voltage. Two important advantages of this method are : 1) more accuracy and reliability as compared with classical linearization, and 2) sensitivity of the system to be controlled to variations in load and input voltage is considerably low and the system easily adopts itself to these variations. Besides the advantages of the method, the disadvantages will be mentioned here, and for purpose of completeness, other nonlinear methods proposed in other paper will be taken into account, too. according to simulation results, major advantages and disadvantages of the input-output feedback linearization method as well as other common nonlinear control methods are those listed in table 1. "Miscellaneous methods" consist of some specific methods proposed in recent years [5-11].

Table 1 :
Comparisons between nonlinear controllers

Nonlinear Controller	Advantages	Disadvantages
I/O feedback linearization	• Accurate and fast tracking, • No oscillation and minimum overshoot at the starting point (satisfactory transient response), •Simply digitally realizable.	• Due to appearance of unstable zero dynamics, not easily applicable to certain types of models, • Tracking speed in inverse proportion to feed forward gain of linear Controller.
Neural network [12]	• Good tracking accuracy	• Overshoot and rather severe oscillation at the starting point (at the time of learning of network), • Due to complicated structure, not economical for ordinary applications.
Sliding-mode control [3]	• Fast tracking	• Not economical for ordinary applications, • Difficulty in determining the coefficients of sliding functions for high-order regulators.
Lyapunov theory [3]	• Fast tracking	• Difficulty in determination of optimal weighting matrices, • Existence of small ripples on the output voltage in steady state, • Difficulty in physical realization because of initial response of switching elements to fast changes in control input.
Miscellaneous Methods [5-11]	• Good tracking accuracy, Simple structure (for the most part)	• Applicable to a limited number of regulators

4 LINEAR ROBUST CONTROL

To design a linear controller for switching power converters, first the nominal operating conditions and then a class of linear equations or a circuit model for system description are derived. Having this (Circuit or mathematical) model at hand, one can use different methods for syntheses of linear controllers. In classical control methodologies, to obtain the desired gain and phase margins, Bode and Nyquist diagrams as well as root locus are usually used. Majority of these methodologies assume a rather accurate modeling of the system to be controlled. The models are commonly derived through linearization of nonlinear equations about the operating point. Analysis and synthesis are based on these models and in case of variations in model parameters, the results are no longer necessarily reliable. Although the effects of variations, in system parameters on the closed-loop system are investigated in a few design methods, however, the design methods presented up to here have not been aimed at robustifying control system against these variation. In this section, our objective is to design linear controllers to not only meet nominal stability and nominal performance requirements, but also guarantee the "robust stability" and "robust performance" in switching power converters. This objective is not reachable using classical controllers. Robust stability and performance amounts to satisfactory stability and performance for a given set of deviations to the nominal model. In switching power converters, these deviations and uncertainties are, in part, due to changes in element are circuit characteristics, especially load, and in part, due to errors in modeling (high-frequency dynamics and unmodeled elements) and linearizing. In this section, an attempt is made to provide the stability and performance robustness for DC-DC switching regulators H∞, μ and Kharitonov's robust control theories.

These theories have proved successful for control of power-electronic systems, especially Switching power converters. The main capability of these theories is in possibility of controller design based on a "more complete" model of system which considers uncertainties, too. This fact is of great importance knowing that Switching power converters have a variable structure and are subject to type of uncertainties (structured type) and disturbances.

By using μ-toolbox [13], after model linearization, uncertainty description and performance weights, we obtained a 6th-order μ-optimal controller. The state space realization of the (reduced order) controller has the following form, [13] :

$$\begin{aligned} \dot{x} &= A_k x + B_k u \\ y &= C_k x + D_k u \end{aligned} \qquad (3)$$

Figure 3 shows the output voltage regulation in case of variations in line voltage. figure 4 shows the step-response of the output when the load is 400 ohms.

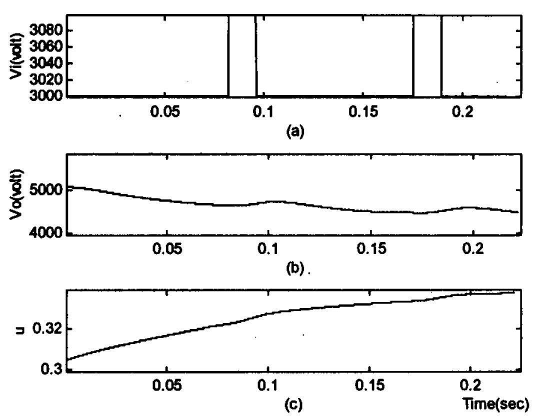

Fig. 3 : Output voltage regulation using μ-based controller, a) Line voltage, b) Output voltage, c) Duty cycle.

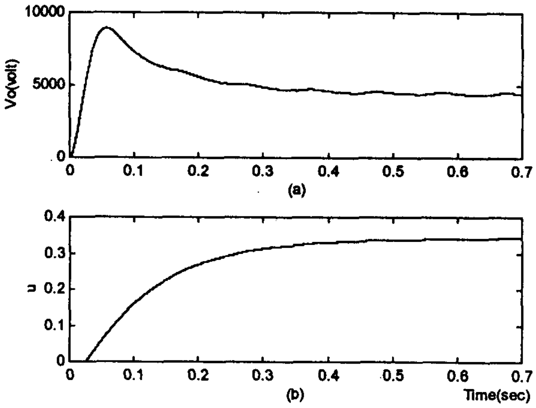

Fig. 4 : Unit step-response using μ-based controller whit RL=400 ohms, a) Output voltage, b) Duty cycle.

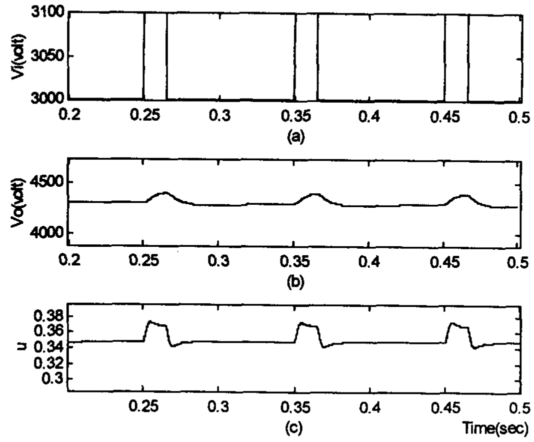

Fig. 6 : output voltage regulation, using H∞-based controller a) Line voltage, b) Output voltage, c) Duty cycle.

In the next step, the S-domain transfer function of the resulted H∞-based control-ler is obtained as [14] :

$$K(s)=\frac{(-35.1732)(s+45.96\pm j21.06)(s+100000)}{(s+19612)(s+12758)(s+8336.6)(s+0.0066)} \quad (4)$$

Figure 5, shows the output voltage of converters in case of variation in load. figure 6, shows the voltage regulation in case of variation in line voltage.

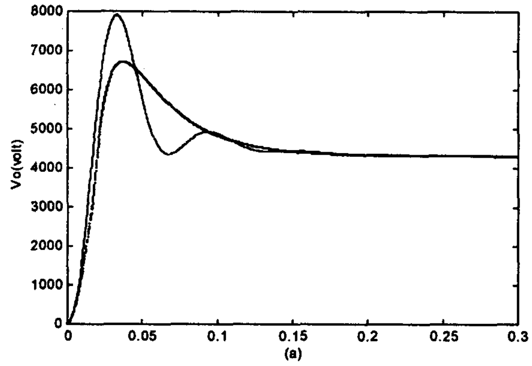

Fig. 5 : output voltage, using H∞-based controller with RL=10 ohms (dash) and RL=250 ohms (solid).

Finally, by using Kharitonov's synthesis, we obtained a simple PI-controller [14], that the closed-loop step response given in figure 7.

The basic geometry associated with the zero exclusion condition ([15]) is more fully demonstrated in figure 8 for $0<\omega<10^3$ rad/sec, where $R_L \in [10\ ;600]\ \Omega$ and a constant $\pm 10\%$uncertainty is assumed for C_2 and L_2.

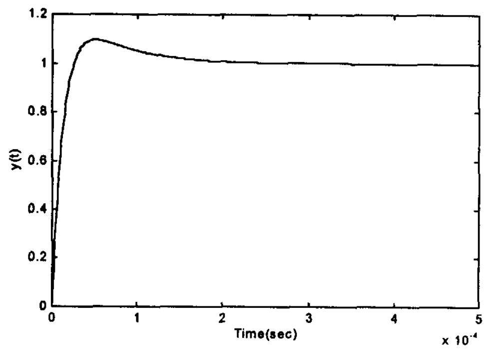

Fig. 7 : closed-loop step response

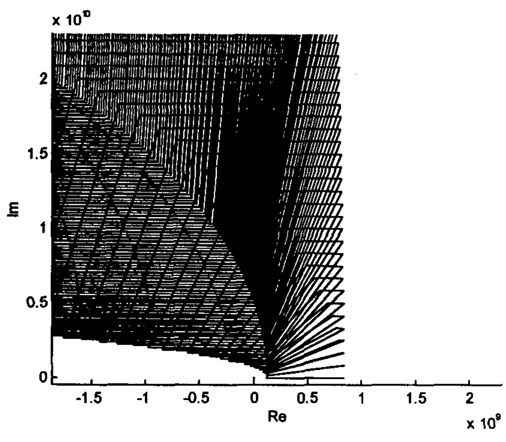

Fig. 8 : Motion of the Khritonov rectangle for $0<\omega<10^3$

Simulation results demonstrate the considerable advantages of robust controllers over other conventional controllers used for control of switching power converters (classical controllers). Table 2 shows the basic advantages and disadvantages of these two classes of controllers.

It was investigated that under a wide spectrum of uncertainties and disturbances, the H∞ and μ controllers, in addition to maintaining, ensure satisfactory performance. Via these two methods, the problem of "reducing the output impedance" is directly introduced to the synthesis, however, the H∞-based design is, to some extent, conservative and the μ-based design hardly gives low-order controllers. Simple controller and controller synthesis is the main advantage of Kharitonov's method, however, this method cannot fulfill the performance requirements as efficient as H∞ and μ controllers. This is because of the fact that reducing output impedance cannot directly be introduced to the Kharitonov's synthesis. Moreover, there exists limitations in application of these methods to some types of converters. To summarize, Kharitonov's theory for straight forward synthesis and simple resulting controller, H∞ theory for flexibility in achieving satisfactory performance and low degree of controller, and μ theory for applicability to structured uncertainties are of interest. On the other hand, limitations in application of Kharitonov's synthesis are not negligible. Further more, each of these controllers have advantages and disadvantages over one another that are summarized in table 3.

5 CONCLUSIONS

According to the simulation results obtained from application of four designed controller in this paper based on "input-output feed-back Linearization" technique, and "Karitonov, H∞ analysis and synthesis" techniques, the closed-loop system shows satisfactory stability and performance under a range of variations in converter elements, especially load and input voltage. Together with simulations, significant advantages and disadvantages of any method was also mentioned.

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Table 2:
Comparisons between classical and robust linear controllers

Linear controllers	Advantages	Disadvantages
Classical controllers	• Easy synthesis, • Simple controller and controller realization, • Less time consumption and expenditure,	• Not guaranteed stability robustness and performance robustness, because of the synthesis based on nominal conditions, • Extremely dependent upon the nominal operating-point as well as selected type of model. • Difficult or impossible access to both satisfactory stability and satisfactory performance, • Not efficient in the state space framework, • Decrease in regulator band width.
Robust controllers	• Possible consideration of uncertainties and system parameters deviations, • Possible direct introduction of "reducing output impedance" to the H∞ and μ synthesis, • Simultaneous satisfactory stability and satisfactory performance (reduced sensitivity to parameters variations, satisfactory disturbance rejection, reduced output impedance, and attenuated transfer from input to output).	• Over low frequencies, slight static errors due to lower open-loop gain (H∞ and μ), • High-order resultant controller and high realization expenditure (H∞ and μ), • Synthesis is rather difficult and time-consuming.

Table 3 :
Comparisons between linear robust controllers

Linear robust controllers	Advantages	Disadvantages
Kharitanov's controller	• Simplicity of method, • Low degree of controller.	• Limitations in application to some type to models, • Impossible direct "reducing output impedance " to the introduction of synthesis (weaker, from a performance viewpoint).
H∞ controller	• Relatively low degree of controller, • possible direct introduction of "reducing output impedance" to the synthesis.	• Difficulty in determination of weighting functions, • Synthesis conservatism, (because of considering uncertainties in the unstructured form).
μ controller	• Non conservative and accurate treatment (be cause of considering uncertainties in the structured form), • Possible direct introduction of "reducing output impedance" to the synthesis, • Stability robustness under a wider range of load variation.	• Difficulty in determination of weighting functions, • High degree of controller, • Larger setting time, • Little dissipation on the switching element at the starting point.