Statistical Cooperative Power Dispatching in Interconnected Microgrids

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***Abstract*—Load demand management in the context of grid-connected microgrids is the scope of this paper. This issue is formulated as a power dispatching problem between distributed power sources with the objective of grid operational cost minimization. Under the assumption of time-varying demands and supplies at individual microgrids, a cooperative power dispatching algorithm of interactions among microgrids is proposed for power sharing within the grid. This is done through a communication infrastructure in the grid and a set of defined parameters known as purchase prices at individual microgrids. As a result of this algorithm, power flows within the grid are regulated to smooth power generations at microgrids despite the stochastic load demands. Numerical results demonstrate the effectiveness of the proposed load management scheme in comparison with no power sharing scheme in the grid operational cost point of view. Moreover, optimal solution of the power problem verifies these results.**

***Index Terms*—Load demand management, microgrid, optimization, power dispatching.**

I. INTRODUCTION

ELECTRICAL power generation in the form of distributed generation (DG) is a well-known structure for on-site power supplement [1]. This structure includes the application of small generators, typically ranging in capacity from 5 KW to 10 MW, at or near the end-user to provide the power needed. In comparison with centralized and conventional models of power generation, the DG offers several advantages from the perspective of both sources and end-users. It reduces the power generation and transmission costs and results in less electrical losses. Because of distributed structure, the system is more reliable in terms of maintenance and service as well as is more flexible in using fuels and renewable energy sources. Moreover, the new installation and capacity development is more convenient. These advantages can be realized as a result of technical improvement in the development of small-scale generating units.

The *smart power grid* is an interesting concept of integration DG systems into a grid [2]. This grid uses information and communication technology to enhance the grid flexibility and reliability, and enable the incorporation of various components such as renewable resources and distributed micro-generators. A unit of the grid, known as *microgrid* (MG), is a group of generators and loads connected to the grid in multiple points. As a considerable capability, each MG can operate in autonomous (isolated from the main grid) and grid-connected modes. The performance measure in the autonomous mode is the reliability of stand-alone operation. However, in the grid-connected mode the MG operates while connected to the main grid. This is especially characterized by the fact that each MG can sell a portion of its generated power to the grid at a point of connection and at the same time is a able to purchase a portion of its demand from the grid at another point of connection. As a result of power sharing in this mode, load demand supplement is guaranteed in all time by the grid.

Load demand management is a critical issue in the smart grids with power sharing capability [3], [4]. It controls the power dispatching between MGs with the aim of establishing a balance between power supply and demand in a cost-efficient manner. The objective in this balance is to alleviate peak loads at individual MGs and accordingly avoids major expenditures in power utilities. In contrast to the autonomous mode where load management results in shifting peak loads to off-peak loads [5]–[7], peak loads of a MG in grid-connected mode can be handled by the means of power sharing throughout the grid. Considering stochastic demands and maximum allowed supplies in MGs, an immediate question is how to perform power dispatching and to set interactions in the grid. Due to power generation and transmission costs, this issue raises the economic exploitation of the resources within the grid [8]. The outcome of power dispatching could be interesting in this perspective.

In this paper, load demand management of an electric network of interconnected MGs is formulated as a power dispatch optimization problem. Real-time pricing is employed as a motivation for interactions between the MGs. The objective is to minimize the network operational cost and at the same time to satisfy the stochastic demands within the MGs in average. With the solution of this problem, a cooperative power dispatching algorithm between MGs is proposed under the assumption of a communication infrastructure within the grid. The core parameter in this algorithm is a defined dynamic *purchase price* per a unit of power at each MG. Considering their demands and supplies, the MGs progressively update and broadcast their prices throughout the grid. Every MG adaptively regulates its transactions with the rest of the grid by taking into account its realized demand as well as already announced prices from the other MGs. This strategy results in a semi-distributed and reliable load management within the grid in a cost-efficient manner.

The paper is organized as follows. Section II presents an introduction on MGs structure and existing control levels. System

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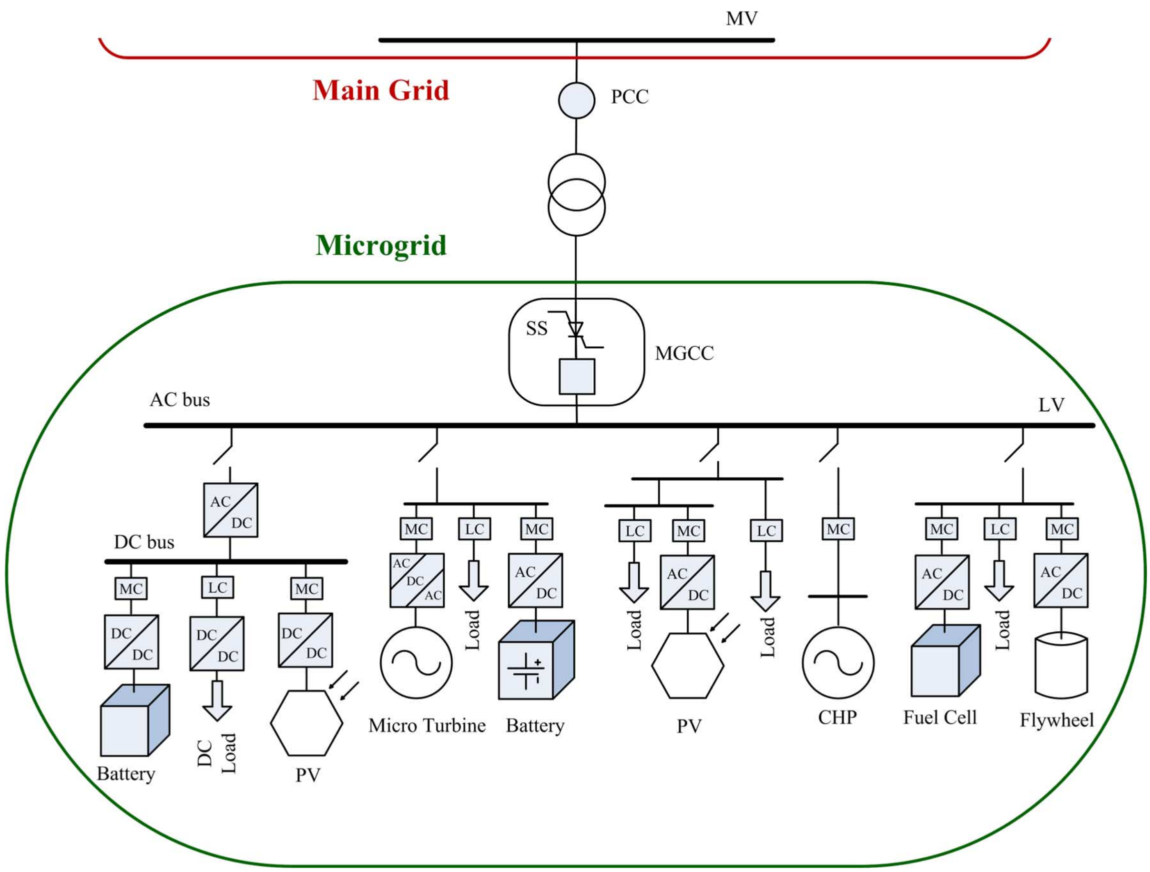

Fig. 1. Simplified structure of a MG connected to the main grid.

model and problem formulation are presented in Section III. The problem solution and cooperative power dispatching algorithm are proposed in Section IV. Numerical results are given in Section V and the paper is concluded in Section VI.

II. MG Controls

A. MG Structure

A MG is an interconnection of domestic distributed loads and low voltage distributed energy sources, such as microturbines, wind turbines, photovoltaics (PVs), and storage devices. A simplified MG architecture is shown in Fig. 1. This MG consists of a group of radial feeders as a part of a distribution system. Each feeder has a circuit breaker and a power flow controller commanded by the central controller or energy manager. The circuit breaker is used to disconnect the correspondent feeder (and associated unit) to avoid the impacts of severe disturbances throughout the MG. The MG can be connected to the distribution system by a point of common coupling (PCC) via a static switch (SS). The static switch is capable to island the MG for maintenance purposes or when faults or a contingency occurs. A MG central controller (MGCC) facilitates a high level management of the MG operation by means of technical and economical functions. The microsource controllers (MCs) control the microsources and the energy storage systems. Finally, the controllable loads are controlled by load controllers (LC).

The microsources and storage devices use power electronic circuits to connect to the MG. Usually, these interfaces depending on the type of unit are ac/ac, dc/ac, ac/dc, and ac/dc/ac power electronic converters/inverters. As the MG elements are mainly power-electronically interfaced, the MG control depends on the inverter control.

For increasing reliability in the conventional power systems, the MG systems must be able to have proper performance in both connected and disconnected modes. In connected mode, the main grid is responsible for controlling and maintaining power system in desired conditions and, the MG systems act as real/reactive power injectors. But in disconnected mode, the MG is responsible for maintaining the local loads and keeping the frequency and voltage indices at specified nominal values [9]–[11].

B. MG Control Loops

Control is one of the key enabling technologies for the deployment of a MG system. A MG has a hierarchical control structure with different layers. It requires the effective use of advanced control techniques at all levels. In islanded mode, to cope with the variations, to response to load disturbances for performing active power/frequency regulation, and to reactive power/voltage regulation, the MGs need to use proper control loops. A general scheme for operating controls in a MG is shown in Fig. 2.

Similar to the conventional power systems [12], a MG can operate using various control loops. The control loops in MGs can be mainly classified in four control levels: local, secondary, central/emergency, and global controls. The *local control* deals with initial primary control such as current and voltage control loops in the microsources. The *secondary control* ensures that the frequency and average voltage deviation of the MG is regulated towards zero after every change in load or supply. It is also responsible for inside ancillary services. The *central/emergency control* is performed by the MGCC which interfaces between the MG and other MGs as well as higher distribution networks (such as main grid). This control level covers all possible emergency control schemes and special protection plans to maintain the MG stability and availability in the face of contingencies. The *global control* coordinates the MGCC units in an interconnected MGs network. The global control as a centralized control allows the MG operation at an economic optimum and organizes the relation between the MG and distribution network as well as other connected MGs.

C. Global Control and Power Dispatching

Global control deals with some system-wide responsibilities for the MGs, such as interchanging power with the main grid and/or other MGs. This control, which is mainly accomplished by a market operator through the central controllers, is acting in an economical-based energy management level between the neighboring MGs similar to the existing supervisors for power exchanges and economic dispatch in a conventional multiarea power system [13]. To meet the global control objective, wide area monitoring and estimation are needed for many parameters and indices including fuel and devise storage conditions, commercial power cost and demand charge tariffs, MG reliability, real/reactive power components (power factor), predicted weather, system constraints, and load pattern.

As shown in Fig. 2, the global control center interfaces the MGCCs of the MGs as well as the distribution network (main grid), and also supervises the power flow control and market operating. This control unit controls power dispatching between the MGs to maintain close to the scheduled values. Different control options are investigated for the MGs centralized global controller in different MG projects. In the Consortium for Electrical Reliability Technology Solutions (CERTS) MG in USA [14], this controller called MG energy manager or market operator is responsible for dispatching the output power and the terminal voltage of the DGs. Similarly, in the Hachinohe demonstration project in Japan [15], economic dispatch and weekly operational planning are performed centrally. While, in the European architecture it is still known as MG central controller and has several control functions [16].

In an interconnected MGs network, identifying the optimal generation schedule to minimize production costs and to balance the demand and supply which comes from both MGs and the distribution feeder, as well as online assessment of the MGs' security and reliability are the responsibilities of the global control center (market operator). Global control together with the MGCCs supervise the MGs' market activities such as buying and selling active and reactive power to the grid and possible network congestions for transferring energy from an MG to nearby feeders of the distribution network and other MGs. They perform an energy management system (EMS) for the MGs to ensure a subset of basic functions such as load and weather forecasting, economic scheduling, overall security assessment, and demand side management.

The global control for an interconnected MGs network should be implemented through the cooperation of various MGCCs, located in all MGs, on the basis of communication and collection of information about distributed energy systems and control commands. This could be deployed by optimizing the power exchanged between different MGs, as well as the main grid, thus maximizing the MGs production depending on the market prices and security constraints.

Due to the high diversity of generation and loads, an interconnected MGs network exhibits high nonlinearities, changing dynamics, and uncertainties that may require advanced control and optimization strategies such as the used methodology in the present work to solve. The present paper is focused on the optimal power dispatching between the MGs in an electric network, as a global control issue.

III. System Model and Problem Formulation

Consider a distributed power smart grid consisting of a set $\mathcal{N} \triangleq \{n : n = 1, \ldots, N\}$ of MGs interconnected through a power transmission infrastructure and a communication network, as shown in Fig. 3. Within each MG, assume a power source that can be shared within the grid following a certain strategy set by the administration (grid market operator). In other words, the demand of each MG can be supplied partially from the internal sources and at the same time from the other sources throughput the grid. In a deregulated environment, power dispatching within the grid is certainly affected by two main factors: power generation cost at individual MGs and power transmission cost between any two MGs. Here, under the assumption of time-varying demands and supplies of MGs, a statistical approach is developed towards the total cost minimization within the grid as described below.

Let $\Gamma \triangleq \{\gamma_n : n = 1, \ldots, N\}$ be the vector of demands and $\Sigma \triangleq \{\sigma_n : n = 1, \ldots, N\}$ be the vector of maximum permitted supplies within the grid, where γ_n and σ_n are the demand and supply of MG_n, respectively. These demands and supplies are assumed as random variables varying over the time, but without any assumption on their probability distribution functions (pdf). Moreover, consider $\mathbf{S} = \{s_{nm}\}_{n\in\mathcal{N}}^{m\in\mathcal{N}}$ as the vector of power flows within the grid, where s_{nm} is the supplied power by MG_n to MG_m. By this notation, s_{nn} is clearly the produced power by MG_n used for internal consumption. Due to random variations of Γ and Σ, flows within $\mathbf{S}$ should be regulated adaptively over the time. Our design objective is to minimize the grid mean operational cost, including power generation and transmission costs, such that demands Γ to be satisfied. This objective can be formulated as the follows:

$$\min_{\mathbf{S}} \quad \sum_{n=1}^{N} \mathbb{E}_{\gamma_n}\left[C\left(\sum_{m=1}^{N} s_{nm}\right) + \sum_{m=1}^{N} \mu_{nm} s_{nm}\right] \tag{1}$$

$$\text{s.t.} \quad \sum_{n=1}^{N} \mathbb{E}_{\gamma_n}\left[s_{nm}\right] = \gamma_m \quad \forall m \in \mathcal{N} \tag{2a}$$

$$\sum_{m=1}^{N} \mathbb{E}_{\gamma_n}\left[s_{nm}\right] \le \sigma_n \quad \forall n \in \mathcal{N} \tag{2b}$$

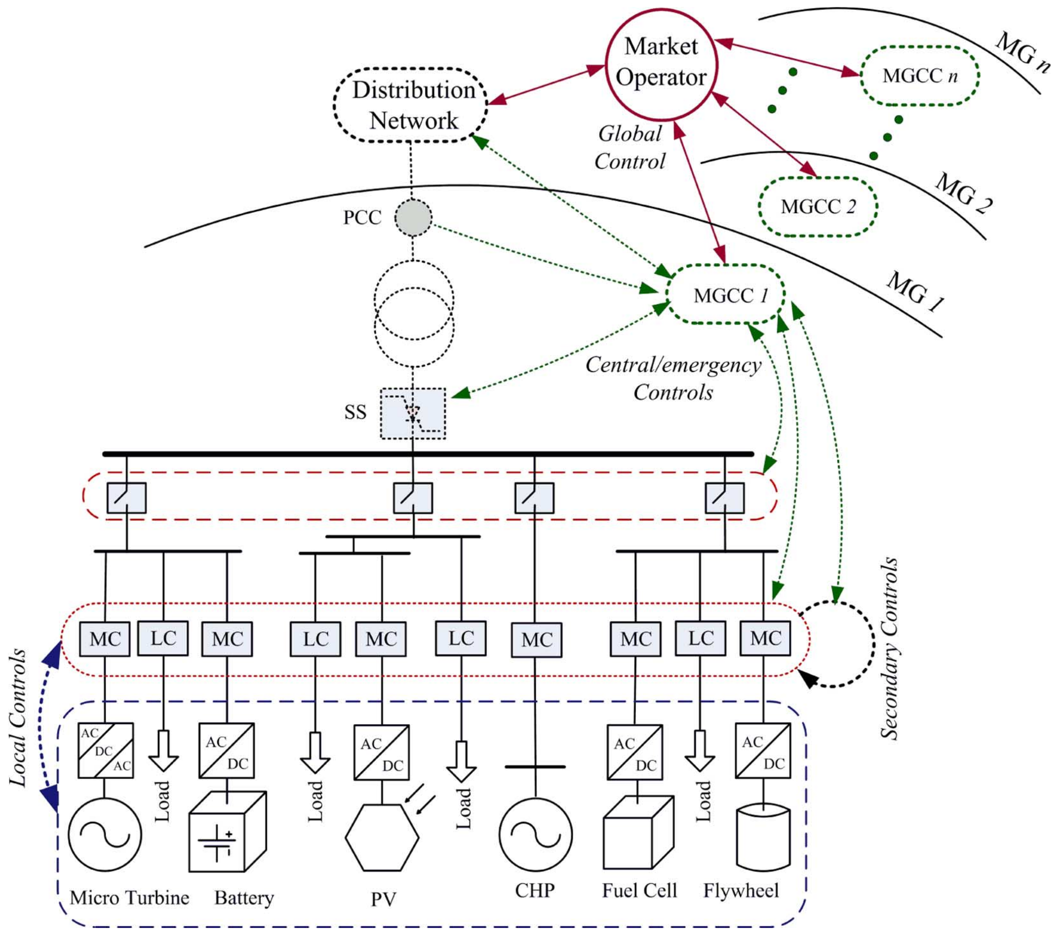

Fig. 2. A general scheme for MGs control levels.

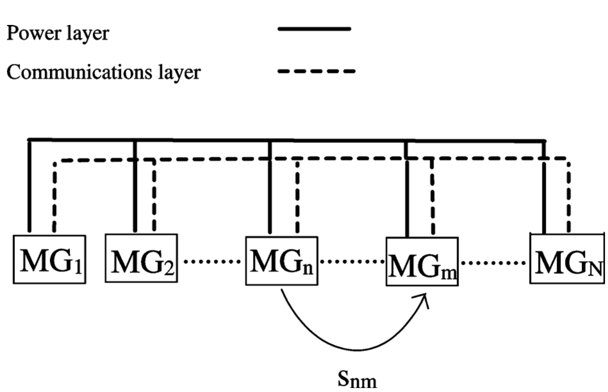

Fig. 3. Smart power grid.

where $\mathbf{S} = \{s_{nm}\}_{n\in\mathcal{N}}^{m\in\mathcal{N}}$ is the set of optimization variables and $\mathbb{E}_{\gamma_n}$ denotes the expectation with respect to γ_n. Moreover, $C(.)$ is the cost for power generation that is assumed to be a convex and differentiable function. On the other hand, the transmission cost is assumed to be a linear function with rate μ_{nm} as the cost per unit of power transmitted from MG_n to MG_m. Constraints (2a) satisfy the supply-demand balance within each MG in average and constraints (2b) restrict the power supplied by a MG_n to a maximum value σ_n.

This problem is convex and can be solved using several convex optimization techniques [17]. However, this requires the availability of Γ and Σ *a priori* for the whole time in the scope of the problem. This knowledge is not always available. Alternatively, we are interested in solving this problem progressively over the time, when each γ_m and σ_n are realized at each time instant t, to come up with an adaptive load management scheme within the grid.

IV. Cooperative Power Dispatching

The most significant challenge in the solution of problem (1) and (2) is due to the coupling expectations. The solution would be straightforward if we decouple the demand load constraints. This motivates the incorporation of (2a) into the objective function and form a Lagrangian function as

$$L(\mathbf{S},\Lambda) = \sum_{n=1}^{N} \mathbb{E}_{\gamma_n}\left[C\left(\sum_{m=1}^{N} s_{nm}\right) + \sum_{m=1}^{N} \mu_{nm}s_{nm}\right] - \sum_{m=1}^{N} \lambda_m \left(\sum_{n=1}^{N} \mathbb{E}_{\gamma_n}[s_{nm}] - \gamma_m\right) \quad (3)$$

and the corresponding dual function as

$$D(\Lambda) = \inf_{\mathbf{S}} \{L(\mathbf{S},\Lambda) : (2b)\} \quad (4)$$

where $\Lambda = \{\lambda_m\}_{m\in\mathcal{N}}$ is the set of Lagrange multipliers. This dual function provides a lower bound on the optimal solution of

(1) and (2) [17]. The best lower bound is surely achieved by the corresponding dual problem as

$$\max_{\Lambda} \quad D(\Lambda). \tag{5}$$

Prior to solve the problem in the dual domain, we first need to evaluate $D(\Lambda)$. We rewrite $L(\mathbf{S}, \Lambda)$ as

$$\begin{aligned} L(\mathbf{S}, \Lambda) & \\ = \sum_{n=1}^{N} \mathbb{E}_{\gamma_n} \Bigg[C \left(\sum_{m=1}^{N} s_{nm} \right) & \\ + \sum_{m=1}^{N} \left(\mu_{nm} - \lambda_m \right) s_{nm} \Bigg] & \\ + \sum_{m=1}^{N} \lambda_m \gamma_m. & \end{aligned} \tag{6}$$

Thanks to the decomposable form of $L(\mathbf{S}, \Lambda)$, $D(\Lambda)$ in (4) can be obtained if each MG_n solves

$$\min_{\mathbf{S}_n = \{s_{nm}\}_{m \in \mathcal{N}}} \quad \mathbb{E}_{\gamma_n} \Bigg[C \left(\sum_{m=1}^{N} s_{nm} \right) + \sum_{m=1}^{N} \left(\mu_{nm} - \lambda_m \right) s_{nm} \Bigg] \tag{7}$$

$$\text{s.t.} \quad \sum_{m=1}^{N} \mathbb{E}_{\gamma_n} \left[s_{nm} \right] \le \sigma_n. \tag{8}$$

Considering this problem at time t with a given set of $\{\lambda_m(t)\}_{m \in \mathcal{N}}$, we observe that it is convex and can be can be solved using interior point method (IPM) [17] to obtain $\{s_{nm}{}^*(t)\}_{m \in \mathcal{N}}$, the optimal solution. Inspecting (7), each Lagrange multiplier $\lambda_m(t)$ can be interpreted as the marginal benefit of MG_n from selling a unit of power to MG_m at time t. In other words, $\lambda_m(t)$ is the announced *purchase price* of MG_m in interaction with the other MGs.

Having solved (7) and (8) at all MGs, each MG_m can be aware of $\{s_{nm}{}^*(t)\}_{n \in \mathcal{N}}$, i.e., its own produced power for internal usage as well as the incoming power flow from the rest of the grid. By this knowledge, MG_m is able to solve its portion of the dual problem in (5) using subgradient method. Beginning with an initial $\lambda_m(0)$, given $\lambda_m(t)$ at time t, this MG obtains the knowledge of $\{s_{nm}{}^*(t)\}_{n \in \mathcal{N}}$ from distributed problems (7) and (8) in the grid. This MG then updates its own purchase price for time $t+1$ as

$$\lambda_m(t+1) = \lambda_m(t) + \alpha \left(\gamma_m - \sum_{n=1}^{N} \mathbb{E}_{\gamma_n} \left[s_{nm} \right] \right) \tag{9}$$

where $\gamma_m - \sum_{n=1}^{N} \mathbb{E}_{\gamma_n} [s_{nm}]$ is the subgradient of $D(\Lambda)$ with respect to λ_m and α is a step size.

The gradient iteration (9) is efficient to find the optimal powers. A key knowledge we need in (9) is the pdf of every γ_n and σ_m, only with which we can evaluate the expected values. Assumption of known pdf of Γ and Σ may be reasonable for theoretic studies. However, the importance of practical load management schemes motivates the optimal strategy by *learning* the demands on-the-fly. Interestingly, a stochastic gradient iteration can be developed to solve (5) without the PDF of γ_n and σ_m *a priori*. To this end, consider dropping $\mathbb{E}_{\gamma_n}$ from (9), to devise online iterations for *adaptive* decisions based on per slot realization $\gamma_m(t)$ as

$$\hat{\lambda}_m(t+1) = \hat{\lambda}_m(t) + \alpha \left(\gamma_m(t) - \sum_{n=1}^{N} s^*_{nm}(t) \right) \tag{10}$$

where hats are to stress that these iterations are stochastic estimates of those in (9). Provided that load demand process of all MGs is stationary and ergodic, the stochastic gradient iteration (10) and the ensemble gradient iterations (9) are a pair of primary and averaged systems [18]. Convergence of such a stochastic gradient iteration can be established statistically, provided that α is small enough. Such a proof is provided in the Appendix.

The above described solution can be summarized as a statistical cooperative power dispatching (SCPD) in Algorithm 1. It is cooperated in that all MGs participate in decision making and signalling. Each time slot of this algorithm consists of three phases to be run in the beginning of the time slot. In phase 1 of time slot t, called purchase price declaration, each MG $m \in \mathcal{N}$ broadcasts its purchase price $\hat{\lambda}_m(t)$ of this time slot to the rest of the grid, course via the communication infrastructure. Following this declaration, in phase 2 called power distribution, each MG $n \in \mathcal{N}$ takes its realized demand $\gamma_n(t)$ and announced prices $\{\hat{\lambda}_m(t)\}_{m \in \mathcal{N}}$ into account to come up with optimal power flows $\{s^*_{nm}(t)\}_{m \in \mathcal{N}}$ during slot t, using (7) and (8). This MG reasonably let each MG_m to know about $s^*_{nm}(t)$. Phase 3 starts when all MGs are aware of their incoming power flows. In this phase, all MGs update their purchase price for the next time slot, i.e., $\hat{\lambda}_m(t+1)$, using (10). The algorithm proceeds progressively over time. The signalling overhead of SCPD is limited to broadcasting purchase prices as well as notifying MGs of the supplied power by a certain MG.

Algorithm 1 SCPD algorithm

Initialization: $\hat{\lambda}_m(0) = \lambda_{\text{init}} \quad \forall m \in \mathcal{N}, t = 0.$
while $\{1\}$ **do**
  Phase 1: *Purchase price declaration*
  Every MG $m \in \mathcal{N}$ broadcasts purchase price $\hat{\lambda}_m(t)$.
  Phase 2: *Power distribution*
  for $n \in \mathcal{N}$ **do**
    Realize a new load $\gamma_n(t)$ randomly.
    Using **(7) and (8)**, obtain $\{s^*_{nm}(t)\}_{m \in \mathcal{N}}$.
    Send $s^*_{nm}(t)$ to MG_m for all $m \in \mathcal{N}$.
  end for
  Phase 3: *Purchase price update*
  Every MG m updates $\hat{\lambda}_m(t+1)$, using **(10)**.
  $t = t + 1$.
end while

V. Numerical Results

As an example, consider a power grid consisting of $N = 8$ small MGs starting from MG_1 and ending with MG_8. The

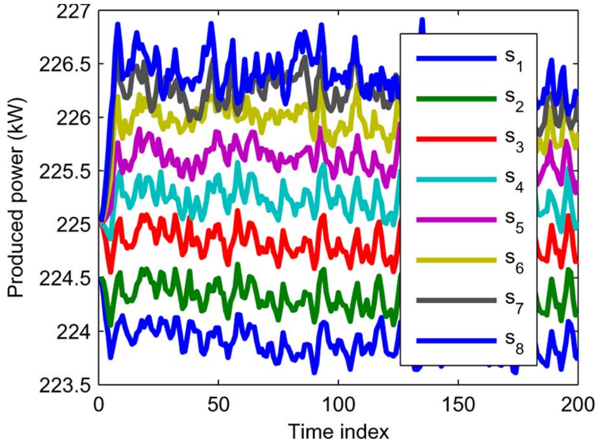

Fig. 4. MGs produced powers over time.

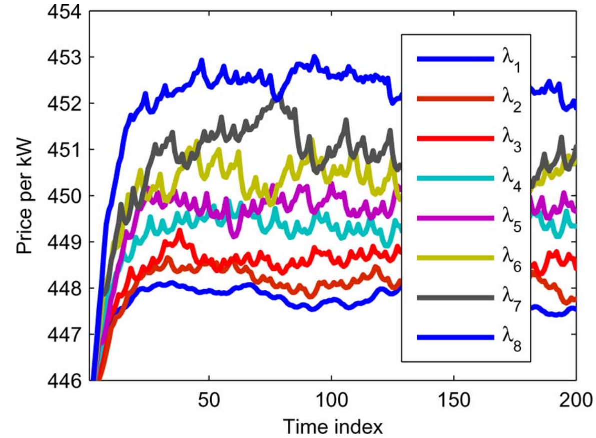

Fig. 5. Announced power prices over time.

distance between any two neighbor MGs is the same and is noted as one hop. The transmission price per unit of power between any two MGs is assumed to be the number of hops between them. Moreover, power generation function is assumed to be a square function, $C(.) = (.)^2$. Demands are assumed to follow normal random variables with mean $\Omega = \{50 : 50 : 400\}$ KW per unit of time and standard deviation $\sigma = 5$ KW, i.e., $\Gamma \sim \{\mathcal{N}(50n, 5)\}_{n\in\mathcal{N}}$. Supplies are also assumed to follow normal random variables with mean 400 KW per unit time and standard deviation 5 KW for all MGs, i.e., $\sigma_m \sim \mathcal{N}(400, 5)$ for $m \in \mathcal{N}$.

The simulation is run for 200 time slots; each slot with realization of a new demand and new maximum permitted supply values per MG. Produced powers at individual MGs by SCPD are shown in Fig. 4. The s_n in this figure represents the produced power of MG_n. Despite the diverse power demands Ω, the produced powers are close to each other. In comparison with their demands, low demand MGs produce higher power and high demand ones produce lower power. This is due to the fact that low demand MGs tend to sell power whereas high demand ones tend to purchase power. Fig. 5 illustrates Lagrange multipliers by which MGs interact within the grid. As previously mentioned, each $\lambda_n(t)$ in SCPD algorithm can be interpreted as the price that MG_n announces at time slot t to pay for a unit of power from any other MG in the system. As shown, even though all prices are initialized with the same value, they statistically converge to different levels during the simulation. The higher is the demand, the higher is the announced price for power purchasing. The curves in this figure are interpreted relative to each other, i.e., power flow direction within the grid is from MGs with low prices to ones with high prices. In other words, a MG with low purchase price in comparison with others is an indication of power selling and vice-versa.

The average *demand*, *produced*, *sold* and *purchased* powers of individual MGs in SCPD algorithm are shown in Fig. 6. While demand increases linearly with MG indexes in accordance with assumed values in Ω, produced powers are approximately the same for all MGs, in compliance with Fig. 4. Remarkably, the sum of produced and purchased powers at each MG is equal to the sum of demand plus sold power. This reveals the energy balance within the grid. Purchased and sold power

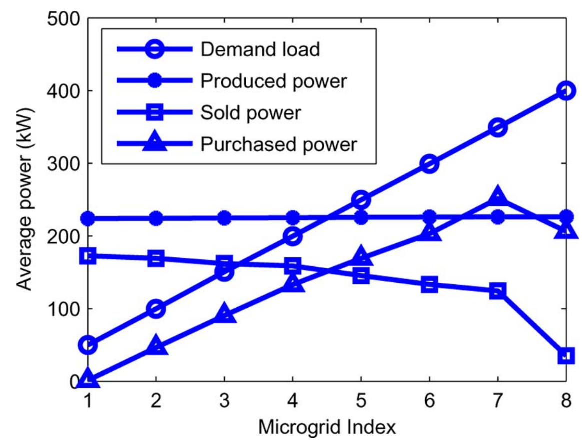

Fig. 6. Average load demands, and produced, sold and purchased powers.

curves vary in opposite directions versus the MG index, i.e., purchased power increases with demand whereas sold power decreases. Exceptionally, the decrease of purchased power at MG_8 is possibly due to its location in the network topology, which burdens high power transmission cost. In a logical statement, low demand MGs sell power to high demand ones.

As mentioned in Section I, grid-connected mode is expected to offer some advantages in comparison with autonomous mode. We investigate this statement by comparing the average operational costs at individual MGs in these two modes, shown in Fig. 7. To this end, the cost in autonomous mode is obtained from the square power production function. Furthermore, the cost in grid-connected mode implemented by SCPD algorithm is the production cost plus purchased cost minus the revenue from selling power to the other MGs. As shown, grid-connected mode achieves lower cost for low demand and high demand MGs. The decrease in the cost of low demand MGs is the result of selling power to high demand ones. In particular, the cost of MG_1 and MG_2 even get negative as a result of high revenue from selling power that compensates their production cost. This outcome, also, decrease the cost of high demand MGs as they purchase a portion of their demand from the low demand ones. This is accomplished by the means of high announced purchased prices in Fig. 5. Furthermore, the purchased and sold powers of moderate demand MGs are mostly the same.

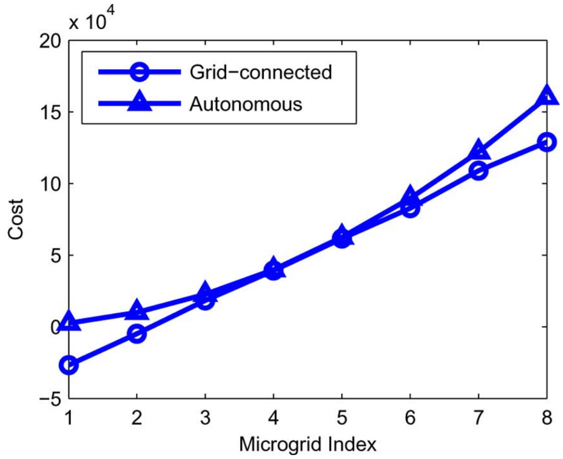

Fig. 7. Cost comparison between grid-connected SCPD and autonomous modes.

TABLE I
OPTIMAL AVERAGE PRODUCED POWERS (KW)

MG_1	MG_2	MG_3	MG_4
223.0716	223.5716	224.0714	224.5709
MG_5	MG_6	MG_7	MG_8
225.0699	225.5689	226.0679	226.5672

In summary, as a numerical indicator, proposed SCPD in grid-connected mode achieves 20% cost reduce in comparison with stand-alone operation. In overall, this power sharing scheme transforms the parabolic cost curve to a linear one as shown in Fig. 7.

In order to evaluate SCPD algorithm in comparison with the optimal solution of (1) and (2), the problem at hand is also solved using IPM method to find the optimal solution, absolutely with given demands and maximum permitted supplies *a priori* at $t = 1$. The resulting average produced powers are shown in Table I. As seen, these values are mostly in accordance with those in Fig. 4 and average produced powers in Fig. 6. Moreover, the average operational cost of this solution is $4.0646e5$ unit in comparison with $4.0651e5$ and $5.0933e5$ units in grid-connected and autonomous modes, respectively. The performance gap between the optimal cost and that of the grid connected mode is less than 0.1%. This reasonably verifies our results with the proposed cooperative power dispatching algorithm.

VI. CONCLUSION

Load demand management with the aim of operational cost minimization in distributed smart grids have been investigated. It was shown that this objective could be achieved by a collaboration between MGs using a communication infrastructure and defining a set of parameters known as purchase prices. A natural consequence of this collaboration was to smooth the power generation within the grid. It was shown that power sharing in the grid-connected mode results in lower price than the stand-alone operation. This was due to the fact that low demand MGs revenue from purchasing power to the grid. On the other hand, high demand MGs reduces their production cost by purchasing power from the grid.

APPENDIX
CONVERGENCE OF THE STOCHASTIC ITERATION

As obtained in Section IV, the solution of dual problem (5) is obtained by stochastic iteration

$$x(t+1) = x(t) + \alpha(g(t))^+ \tag{11}$$

where $x(t) \equiv \hat{\lambda}_n(t)$ and $g(t) \equiv E_n - l_n^{t\,*}(\gamma(t))$. Let x^* be the optimal solution of x. Taking *norm-2* of $(x(t+1) - x^*)$, we get

$$\begin{aligned}\|x(t+1) - x^*\|^2 &\le \|x(t) + \alpha g(t) - x^*\|^2 \\ &= \|x(t) - x^*\|^2 + 2\alpha g(t)(x(t) - x^*) + \alpha^2 \|g(t)\|^2 .\end{aligned} \tag{12}$$

Considering the the concavity of $D(\Lambda)$, we have $D(x^*) \le D(x(t)) + \alpha g(t)(x^* - x(t)))$ [19]. This implies

$$\begin{aligned}&\|x(t+1) - x^*\|^2 \\ &\quad \le \|x(t) - x^*\|^2 - 2(D^* - D(t)) + \alpha^2 \|g(t)\|^2\end{aligned} \tag{13}$$

where $D(t) \equiv D(x(t))$ and $D^* \equiv D(x^*)$. Taking a similar recursive approach from $x(t)$ to $x(0)$ as an initial value, we derive

$$\begin{aligned}\|x(t+1) - x^*\|^2 &\le \|x(0) - x^*\|^2 \\ &\quad -2\sum_{i=0}^{t}(D^* - D(t)) + \alpha^2 \sum_{i=0}^{t} \|g(i)\|^2 .\end{aligned} \tag{14}$$

Since the left-hand side is always nonnegative, we derive

$$2\sum_{i=0}^{t}(D^* - D(t)) \le \|x(0) - x^*\|^2 + \alpha^2 \sum_{i=0}^{t} \|g(i)\|^2 . \tag{15}$$

We take the following two assumptions:
- $\|g(i)\| \le G$, for all i.
- $\|x(0) - x^*\|^2 \le R^2$.

With reference to the system model in Section III, these assumptions are reasonable and can be provided in our case. Dividing both sides of (15) by $2t$, we derive

$$\frac{\sum_{i=0}^{t}(D^* - D(t))}{t} \le \frac{R^2}{2t} + \frac{1}{2}\alpha^2 G^2. \tag{16}$$

If $t \to \infty$, by the law of large numbers we have

$$D^* - \bar{D} \le \frac{1}{2}\alpha^2 G^2 \tag{17}$$

where $\bar{D} = E[D(t)]$. Since D is a concave function, by the Jensen's inequality [17] we have $\bar{D} \le D(\bar{x})$, and accordingly

$$D^* - D(\bar{x}) \le \frac{1}{2}\alpha^2 G^2. \tag{18}$$

Finally, choosing step size α small enough, we conclude that the stochastic iteration (11) converges statistically.

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