A two-level model for the participation of microgrids in energy and reserve markets using hybrid stochastic-IGDT approach

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**ABSTRACT**

In this paper, the decision making problem of microgrids (MGs) in simultaneous participation in the energy and reserve markets under uncertainty is investigated using a two-level framework. In the first level, the uncertainties of wind speed, solar radiation, probability of calling reserve, energy and reserve market prices, and demand are modeled using their probability distribution functions regarding which the operation problem of MG operator (MGO) is formulated as a two-stage stochastic optimization approach. The expected total cost and the amount of the provided reserve by the MGO for the reserve market are the output decisions of this problem which are considered as the parameters in the second level problem. The uncertainty of the accepted reserve by the market regarding the required reserve of the system and the behavior of the market players are modeled using the information gap decision theory (IGDT) approach as the second level problem. Therefore, the risk of the MGO is controlled using the conditional value at risk (CVaR) and IGDT risk-aversion parameters. Applying the proposed model on the 15-bus modified MG and 40-bus real test system shows the optimal decisions of the MGO in both markets to manage its uncertainties. Moreover, the sensitivity analysis is done to investigate the behavior of the MGO with changing the risk aversion parameters.

1. Introduction

1.1. Motivation and aim

The electrical energy systems have a large share in producing the carbon dioxide (CO\(_2\)) emissions. Without any revisions in the electrical energy generation sources, the emission production by these systems will increase in the future to meet the developing electrical energy demand. To solve this challenge, renewable energy sources (REs) have used as the cleaner production sources to supply the electrical energy demand so that many countries consider the RESs in the future power generation portfolios scenarios. For example, the goal of California is achieving to a 50% renewable portfolio standard by 2030 [1]. However, individual integration of these resources in the electrical energy systems creates several operational challenges for the transmission and distribution system operators. To reduce them, the RESs can be integrated with distributed generators (DGs) and energy storages (ESs) to meet the local demand under the concept of micro-grids (MGs). Therefore, the MGs facilitate the integration of the RES in the electrical distribution networks and play an important role in the future electrical energy system to meet the demand with low emission pollution, low energy losses, and low investment and operation costs.

The MGs participate in the energy markets to purchase the required energy from the market or to sell the extra energy to it. On the other hand, equipping the MGs with the fast-response energy resources gives the ability to them to provide the reserve to the independent system operator (ISO). In some markets, for example Ontario electricity market [2], the joint optimization is done for the energy and reserve markets where the bids and offers in the energy market and offers in the reserve market are evaluated at the same time. In such markets, an appropriate decision making framework is required for the market players such as MGs to determine their optimal decisions in both markets simultaneously. Regarding the small capacity of the MGs in comparison with other market players, they act in these markets as the price-taker players. Participation of the renewable energy-based MGs in both energy and reserve markets has the two main uncertainties. The first one is related to the output power of RESs, probability of calling reserve, energy and reserve market prices and demand which have impact on the decisions of the MGs in both markets and the second one is related to the market clearing process which is described in detail as follows.

The amount of purchased energy by the MGs from the energy market is considered as the first block of the demand curve. On the
other hand, the amount of sold power by them to the energy market is considered as the first block of the supply curve. Therefore, all the bids/others of the MGs in wholesale energy market are accepted by the ISO. This is while all amount of the reserve suggested by the MGs to the reserve market may not be accepted because of the amount of the required reserve of the system at that time and the behavior of other market players. Therefore, the uncertainty related to the amount of the reserve which will be accepted by the ISO has the important impact on the decisions of the MGs.

These uncertainties, i.e. the output power of RESs, probability of calling reserve, energy and reserve market prices, demand, and the amount of reserve should be modeled in the decision making problem of the MGs in the energy and reserve markets which is the aim of this paper.

1.2. Literature review and contributions

The operation problem of the MGs considering their participations in wholesale energy and reserve markets has investigated in many studies. In [3], an optimal energy dispatch for a grid-connected MG including distributed energy resources (DERs) and demand response (DR) is proposed. In [4], the non-linear DR programs are utilized in the energy management problem of a grid-connected MG. The optimal scheduling of the MG’s resources is determined using an intelligent technique based on recurrent neural network with the aid of Ant-Lion Optimizer (ALO) algorithm in [5]. The authors of [6] proposed an energy management model for a grid connected MG with responsive loads and DERs which is solved using Genetic algorithm. In [7], a short-term generation scheduling of a grid-connected MG is formulated for participating in day-ahead markets in the presence of DR programs. In [8], a novel energy scheduling system based on a rolling horizon strategy is implemented for a renewable-based real island MG. In these studies [3–8], the operation problem of the MGs is modeled when they participate in only the energy markets.

Optimal offering strategy of a low voltage renewable-based grid-connected MG in the energy and reserve markets is investigated in [9]. The authors of [10] proposed a new techno-economic framework for operation problem of the MGs considering energy and reserve markets as well as to provide the novel reliability services. In [11], the operation problem of the MGs in grid-connected and islanding modes is modeled in which the required spinning reserve is specified to improve the reliability of isolated MG. A double-layer including schedule and dispatch

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layers is proposed in [12] for the MG energy management in both island and grid-connected modes in which the errors between forecast and real-time data are managed using the coordination control of the two layers. This approach uses an adequate reserve capacity in the schedule layer, then allocating it in the dispatch layer to deal with the indeterminacy of uncontrollable units. In [13], a grid-connected MG energy and spinning reserve optimization method is applied in the operation problem in which the provided reserve by sources is not sold to the reserve market and regulated within the MG. In [14], a novel fuzzy system-grey wolf optimization based method is presented for optimal energy management and reserve as well as ES sizing in a standalone MG. In [15], a comprehensive analysis for a hybrid MG energy management, consisting of DR and internal power market, is done. In [16], a co-optimization approach for energy and reserve in a stand-alone MG is formulated. The problem of MGs is modeled in these studies [9–16] considering the participation of the MGs in both energy and reserve markets.

The main problem of the MG operator (MGO) is to model the uncertain parameters in its decision making problem. These parameters including the output power of RESs, load consumption and energy and reserve market prices. Therefore, the risk-based optimization approaches are needed for the MGOs to model the risk of these parameters on their decisions. The strategies proposed in the mentioned information for controlling the uncertainties are described in details as follows.

The uncertain parameters are not considered in the operation problem of the MGs in [3,6,12] and the problem is modeled as a deterministic optimization approach. The authors of [5,8,11,14,15] have developed probabilistic and stochastic optimization approaches in which different scenarios are generated to model the uncertain parameters and the results of the MGs decisions in each scenario are determined by the proposed approach. However, they are not proposed any risk management approaches to control the risk-level of the MG in the decisions. In [5], a recurrent neural network is employed to overcome the uncertainty and prediction error. In [8], a neural network for two day-ahead electric consumption forecast is designed to model the uncertainty. In [9], the uncertainties of energy and reserve prices are modeled using the lognormal PDF and the Latin Hypercube Sampling method is applied to generate appropriate scenarios. In [10], a stochastic approach is considered based on Monte Carlo simulation for the calculation of the dynamic price signals to incentive MGs for the provision of reliability services. In [11], a two-stage stochastic programing approach is used for optimal scheduling of a resilient MG considering the uncertainties of wind energy, electric vehicles (EVs), and real-time market prices. In [14], a fuzzy system-grey wolf optimization method is introduced to tackle the RESs uncertainties (solar irradiation, wind speed and tidal speed). In [15], the uncertainties of load, wind speed, and energy prices are modeled using an stochastic linear programming approach.

The authors of [4,16] used conditional value-at-risk (CVaR) index as an appropriate tool for the risk management regarding which the decision-making of MGOs is modeled as a risk-based two-stage stochastic programming model to model the uncertainties of RESs and load consumption. In [7,13], information gap decision theory (IGDT) approach is employed to investigate the risk level of MGO’s decisions. In [7], the new IGDT approach is used for modeling the market price uncertainty to create the step-wise bidding strategy of a MG. In [13], an improved IGDT-based system spinning reserve robust optimization model is developed to maximize the maximum allowance of system uncertainty while taking the operation cost, system operation mode, and RESs penetration rate into consideration.

Although the effective studies are done on the optimal operation of the MGs in the energy and reserve markets considering the uncertainties, there are two main research gaps which should be addressed. These gaps are:

- In the most studies which model the participation of the MGs in the reserve market, the reserve market price is only considered as the uncertain parameter. While the amount of the MGs' accepted reserve by the ISO is an important uncertain parameter regarding the uncertainties of the total reserve required by the ISO and the uncertain behavior of other reserve market players.
- In the most studies, the risk management of the MGs are modeled using only one parameter. In fact, the risk of MGs in decisions regarding all uncertain parameters are only controlled by the one risk parameter. Meanwhile, the different risk modeling approaches should be employed by the MGs to manage the risk of the different parameters on their decisions.

In this paper, a new decision-making framework for the participation of the MGs in the energy and the reserve markets is proposed in which these gaps are also addressed. In this framework the uncertainties are modeled in the problem of the MG for the parameters with known probability distribution functions (PDFs), i.e. wind speed, solar radiation, probability of calling reserve, energy and reserve market prices, and demand, as well as for the parameter with unknown PDF, i.e. the amount of accepted reserve. Moreover, two risk aversion parameters are proposed to control the effect of these uncertainties on the decisions of the MG in the markets. The proposed framework is mathematically modeled as a two-level problem using an hybrid stochastic-IGDT approach. In the first level, the risk-based two-stage stochastic approach is employed in which the uncertainties of parameters with known PDFs are modeled in the problem of the MG. In this level, scheduling the MG’s resources and the optimal decisions of the MG in both markets are determined. The expected total cost and the amount of the accepted reserve are the output decision variables of this level which are considered as the parameters in the second level problem. In

![Fig. 1. The information flow in the participation problem of the MG in the energy and reserve markets.](image-url)
First level problem: Modeling uncertainties of RESs, demand, reserve calling probability, energy and reserve market prices

Step 1: Modeling the uncertainties using generating different scenarios

Step 2: Modeling the decision making problem of the MG as a two-stage stochastic problem
   First-stage decision variables (independent of scenarios):
   - Optimal scheduling of DGs, ILs, Optimal reserve scheduling of ESs, the amount of reserve provided for the market
   Second-stage decision variables (depend on scenarios):
   - Power exchange with grid, Optimal energy scheduling of ESs, power flow results

Step 3: Required input data:
   - Forecast energy and reserve market prices, technical and economic data of DERs and MG's network data

Step 4: Set CVaR parameters

Step 5: Solving the proposed model and obtaining the value of the expected total cost and the amount of reserve to the market

Second level problem: Modeling uncertainty related to the amount of accepted reserve of the MG by the ISO

Step 1: Modeling the MG problem using the IGDT approach considering the expected total cost and the amount of reserve as the reference value and uncertain parameter of the IGDT model

Step 2: Set the IGDT risk-aversion parameter

Step 3: Solving the IGDT model based on the level of risk-aversion

Output decision variables:
- Optimal scheduling of DERs, optimal participation in energy and reserve markets

Fig. 2. The proposed two-level model for decision making problem of the MG under uncertainties.
parameters.
• Modeling the uncertainty related to the accepted reserve of the MG in the reserve market using the IGDT approach.

The rest of the paper is organized as follows. Section 2 clarifies the problem description. The problem is formulated in Section 3. The numerical results are presented in Section 4 and Section 5 concludes the paper.

2. Problem description

In this paper, the operation problem of a grid-connected MG is modeled where it participates in the energy and reserve markets, simultaneously. The MG consists of DGs, RESs, ESs, and interruptible loads (ILs) and can trade energy with the main grid as well as can provide reserve for the system as shown in Fig. 1. Since the MGO deals with several uncertainties in its decision making problem, two main approaches are proposed to consider the uncertainties in the operation problem of the MG regarding which the problem is modeled in two levels as described in Fig. 2. In the first level, the uncertainties of the demand, the calling reserve probability, the energy and reserve market prices, and the output power of RESs are modeled using their PDFs. For this purpose, a large number of scenarios, i.e. 24,000 scenarios, are generated according to the normal PDF for demand, energy and reserve market prices and considering Weibull and irradiance as PDFs of wind speed, and solar radiation, respectively. These scenarios are reduced to 15 scenarios regarding the proposed approach in [17]. To control the effect of uncertain parameters on the decisions of the MG with notice to the generated scenarios, the problem is modeled as a risk-based two-stage stochastic approach in the second step where the decision variables of each stage are described in Fig. 2. At the third step, as shown in Fig. 1, the MGO receives the technical data and bids/offers from the DERs as well as it forecasts the energy and reserve market prices. In the next step, the MGO sets the CVaR index parameters and then solves the problem with the aim of minimizing the expected total cost, considering the technical constraints of DERs and MG’s network. After solving the problem, the optimal expected total cost and the optimal reserve scheduling of the MGO are taken into account as the parameters in the second-level.

In the second level, the uncertainty related to the amount of accepted value of the MG’s reserve is modeled using the IGDT risk-based approach. The optimal value of the expected total cost and the amount of reserve extracted from the first-level are considered as the reference value and the uncertain parameter of the IGDT approach in the first step. In the second step, the risk-aversion parameter related to the proposed IGDT model is set with the purpose of indicating the risk-level of MGO’s decision-making regarding which the operation problem of the MG is solved as the third step. Then, the MGO decides on its optimal bids/offers to the wholesale energy and reserve markets with controlling the uncertainties in its decisions using the two risk-aversion parameters.

3. Problem formulation

In this section, the operation problem of the MG is modeled in the first and the second levels as follows.

3.1. First level problem

In the first level, the MG operation problem is formulated as a risk-based two-stage stochastic model in which the first-stage variables are independent of scenarios whereas the values of second-stage variables depend on occurring the scenarios. The details of this model is presented as follows.

3.1.1. Expected total cost

The objective function of the MGO is modeled as follows:

\[
ETC = \sum_{s=1}^{S} \rho_s TC_s
\]  

\[
TC_s = \sum_{i=1}^{T} \left[ \sum_{n=1}^{N_G} \left( C_{DG}^{nDG} F_{DG}^{nDG} + C_{DG}^{nBM} R_{DG}^{nBM} \right) + \sum_{n=1}^{N_L} \left( C_{EM}^{nEM} R_{EM}^{nEM} \right) + \sum_{i=1}^{NL} \left( p_{i}^{Grid-in} - p_{i}^{Grid-out} \right) - C_{EM}^{Rm} R_{EM}^{Rm} \right] \quad \forall \; s \in S
\]

\[
P_{i,t,s}^D = \left( \sum_{n=1}^{NL} \left( p_{i,n,t,s}^{Grid-out} - p_{i,n,t,s}^{Grid-in} \right) \right) \quad \forall \; i, t, s
\]

where Eq. (1) describes the expected total cost and Eq. (2) presents the total cost of the MGO in each scenario consisting of six terms. The first term is related to the operation cost of the fossil-fuel based DGs and their respective cost in invoking reserve with related probability. The second term illustrates the cost of load interruption and its invoked reserve with related probability. The third term describes the cost/revenue of/from the power exchange with the main grid. The fourth term models the revenue from selling power to the consumers (the amount of demand consumption in each scenario at each bus is equal to the difference between the initial load and the amount of load interruption calculated as Eq. (3)). The fifth term indicates the revenue from providing reserve to the market and finally, the last term is associated with the revenue from selling the amount of invoked reserve by the MG.

3.1.2. Risk management

In this paper, the risk-averse decision-making of the MGO is modeled using the CVaR approach because of its advantages in comparison with other methods [18,19]. The MGO adds the CVaR index to the operation problem formulation and controls the effects of uncertain parameters on the results in the worst scenarios using the risk-aversion parameter (β). In the stochastic optimization approach, the CVaR at the α confidence level (α - CVaR) can be defined as the expected cost in the (1-α) × 100 percent of the worst scenarios which is presented as follows [20]:

\[
CVaR = \delta + \frac{1}{1-\alpha^{CVaR}} \sum_{i=1}^{S} \rho_i \lambda_i
\]

\[
TC_s \cdot (1 - \delta - \lambda_i) \leq 0 \quad \forall \; s \in S
\]

\[
\lambda_i \geq 0 \quad \forall \; s \in S
\]

where δ, αCVaR, S, ρi, and λi are the value at risk, confidence level, number of scenarios, probability of each scenario, and an auxiliary positive variable which is equal to value at risk minus TC in each scenario, respectively.

3.1.3. Objective function of the first-level problem

The objective function of the first-level problem is to minimize the ETC and the expected cost of (1-α)% of the worst scenarios (CVaR) proposed as follows [4,17]:

\[
\text{Minimize} \ ETC + \beta^{CVaR} CVaR
\]

where βCVaR is the risk aversion parameter. When βCVaR is equal to zero the MGO is risk neutral and with increasing βCVaR, the MGO become risk-averse.

3.1.4. Constraints of the first-level problem

• Power and reserve balance constraints
The power balance at reference bus and other buses are given in Eqs. (8) and (9). Also, the MG load (MGL) is satisfied by the main grid, micro-turbines (MTs), fuel cells (FCs), wind turbines (WTs), photovoltaic (PV) systems, ESs, and ILs. The power flow problem in the MG is modeled using the linearized equations as described in [21]. Eqs. (10)–(12) indicate the injected power to the MG's network consisting of the active power flow and the power losses between bus i and j. Eqs. (13) and (14) determine the MG current magnitude and its limitation, respectively. The upper and lower limitations of the voltage magnitude at the MG buses are shown in Eq. (15). The reserve provided by the MG in the reserve market (one of the first-stage decision variables) is met by DGs, ILs and ESs modeled by (16).

\[ P_{\text{DG,in}} - P_{\text{DG,out}} + \sum_{i=1}^{NDG} P_{\text{DG},i} + \sum_{i=1}^{NWT} P_{\text{WT},i} + \sum_{k=1}^{NES} P_{\text{ES},k} + \sum_{j=1}^{NL} P_{\text{IL},j} \quad \forall \ i \in \text{NB}, \ i = 1, \ t \in T, \ s \in S \]  

\[ P_{\text{DG},i} = P_{\text{Flow},i} + P_{\text{Loss},i} \quad \forall \ i \in \text{NB}, \ i \neq 1, \ t \in T, \ s \in S \]  

\[ P_{\text{Flow},i} = \frac{R_{i,j} + (V_{i,t,s}^2 - V_{j,t,s}^2)}{Z_{i,j}} \quad \forall \ i, j \in \text{NB}, \ t \in T, \ s \in S \]  

\[ P_{\text{Loss},i} = J_{i,j} \quad \forall \ i, j \in \text{NB}, \ t \in T, \ s \in S \]  

\[ V_{i,t,s} \leq V_{i,t,s} \quad \forall \ i \in \text{NB}, \ t \in T, \ s \in S \]  

\[ R_{N}^{DL} = \sum_{n=1}^{NDG} R_{n,t}^{DL} + \sum_{n=1}^{NL} R_{n,t}^{DL} + \sum_{k=1}^{NES} R_{k,t}^{DL} \quad \forall \ t \in T \]  

• DG constraints

Eqs. (17)–(19) are used to model the limitations of the fossil-fuel based DGs where sum of the amount of the energy and reserve provided by the DGs is limited between their minimum and maximum output power [1,22].

\[ P_{n,t}^{DL} + R_{n,t}^{DL} \leq P_{n,t}^{DG} \quad \forall \ n \in \text{NDG}, \ t \in T \]  

\[ P_{n,t}^{DL} \geq 0 \quad \forall \ n \in \text{NDG}, \ t \in T \]  

\[ R_{n,t}^{DL} \geq 0 \quad \forall \ n \in \text{NDG}, \ t \in T \]  

• IL constraints

Eqs. (20)–(22) express the ILs constraints in the MG’s problem. The demand response program based on the bid mechanism is provided in which consumers interact with MGO to curtail the certain part of their loads [23,24].

\[ P_{nl,t}^{DL} + R_{nl,t}^{DL} \leq P_{nl,t}^{IL} \quad \forall \ n \in \text{NL}, \ t \in T \]  

\[ P_{nl,t}^{DL} \geq 0 \quad \forall \ n \in \text{NL}, \ t \in T \]  

\[ R_{nl,t}^{DL} \geq 0 \quad \forall \ n \in \text{NL}, \ t \in T \]  

• RESs constraints

The upper/lower bound of the output power obtained from WT and PV in each scenario is as follows [4,16]:

\[ P_{l,t}^{WT} \leq P_{l,t}^{PV} \leq P_{l,t}^{WT} \quad \forall \ l \in \text{NWT}, \ t \in T, \ s \in S \]  

\[ P_{m,t}^{PV} \leq P_{m,t}^{PV} \leq P_{m,t}^{PV} \quad \forall \ m \in \text{NPV}, \ t \in T, \ s \in S \]  

• Energy storage constraints

![Diagram](https://www.tarjomano.com/order)
Eqs. (25)–(32) describe the operational modelling of the ES to provide energy and reserve [1,22]. Eq. (25) defines the charging/discharging power of the ES. Eqs. (26)–(28) indicate the limitations of discharging and charging power, and the stored energy of the batteries, respectively. Eqs. (29)–(31) show the limitations of the batteries to provide energy and reserve. Moreover, the state of charge of the battery is illustrated in (32).

\[
P_{\text{ES,Ch}}^{k,t,s} - P_{\text{ES,Disch}}^{k,t,s} = P_{\text{grid,in}}^{k,t,s} - P_{\text{grid,out}}^{k,t,s} \quad \forall \ k \in \text{NES}, \ t \in T, \ s \in S
\]

\[
P_{\text{k,t,s}}^{\text{ES,Disch}} \leq P_{\text{k,t,s}}^{\text{ES,Disch},\max} \quad \forall \ k \in \text{NES}, \ t \in T, \ s \in S
\]

\[
P_{\text{k,t,s}}^{\text{ES,Ch}} - P_{\text{k,t,s}}^{\text{ES,Ch},\max} \leq P_{\text{k,t,s}}^{\text{ES,Ch}} - P_{\text{k,t,s}}^{\text{ES,Ch},\min} \quad \forall \ k \in \text{NES}, \ t \in T, \ s \in S
\]

\[
S_{\text{ES,Ch}}^{k,t,s} + S_{\text{ES,Disch}}^{k,t,s} \leq S_{\text{ES}}^{k,t,s} \quad \forall \ k \in \text{NES}, \ t \in T, \ s \in S
\]

\[
P_{\text{k,t,s}}^{\text{ES,Disch}} = P_{\text{k,t,s}}^{\text{ES,Ch}} + P_{\text{k,t,s}}^{\text{ES,Disch}} - P_{\text{k,t,s}}^{\text{ES,Disch}} \quad \forall \ k \in \text{NES}, \ t \in T, \ s \in S
\]

\[
\text{Power exchange constraints}
\]
The amount of energy and reserve exchange with the main grid is restricted by (33) and (34).

\[ 0 \leq P_{\text{grid-in}}^{\text{dis},t} + R_{\text{dis}}^{\text{dis},t} \leq P_{\text{grid-out}}^{\text{dis},t} \quad \forall t \in T, s \in S \]  

(33)

\[ 0 \leq P_{\text{grid-in}}^{\text{dis},t} \leq P_{\text{grid-out}}^{\text{dis},t} \quad \forall t \in T, s \in S \]  

(34)

### 3.2. Second level problem

In the first level problem, the MGO decides on optimal power trading with the energy market and determines the optimal reserve which can provide to the reserve market. However, with notice to the behavior of other reserve market players and also the required reserve of the system, the accepted reserve from the MGO by the ISO faces with the uncertainty. Therefore, the amount of reserve should be considered as an uncertain parameter into the MG operation problem. Since this parameter cannot be modeled with a known PDF in the operation problem of the MGO, an IGDT-based approach is used to model this type of uncertainty.

#### 3.2.1. IGDT Background

In this paper, to model the MG's decision on the amount of reserve, the IGDT approach is employed which it is described in details in this sub-section. Generally, the optimization problems are described as follows [25,26]:

\[ f = \min (f(X, \gamma)) \]  

(35)

\[ H(X, \gamma) \leq 0, \quad G(X, \gamma) = 0 \]  

(36)

\[ \gamma \in U \]  

(37)

where, \( \gamma \) is the uncertain parameter, U describes the set of uncertain parameters, and X is the set of problem decision variables. Mathematical definition of the uncertain parameters is as follows:

\[ \Gamma = \Gamma(X, \alpha) = \left\{ \gamma: \left| \gamma - \bar{\gamma} \right| \leq \alpha \right\}, \quad \alpha > 0 \]  

(38)

where \( \bar{\gamma} \) is the amount of forecast value of the uncertain parameter and \( \alpha \) is the maximum deviation of the uncertain parameter from its forecast value which it is called as the uncertainty radius. A commonplace

### Table 3

The results of MGO decisions considering the CVaR risk aversion parameter.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^{\text{CVaR}} )</td>
<td>447.937</td>
<td>218.509</td>
<td>200.155</td>
<td>204.117</td>
<td>262.448</td>
<td>288.106</td>
<td>318.830</td>
</tr>
<tr>
<td>( \beta^{\text{CVaR}} = 0.5 )</td>
<td>434.986</td>
<td>672.707</td>
<td>790.847</td>
<td>814.776</td>
<td>874.649</td>
<td>896.105</td>
<td>922.626</td>
</tr>
<tr>
<td>( \beta^{\text{CVaR}} = 1 )</td>
<td>3.167</td>
<td>1.903</td>
<td>2.243</td>
<td>2.443</td>
<td>2.868</td>
<td>3.137</td>
<td>3.460</td>
</tr>
<tr>
<td>( \beta^{\text{CVaR}} = 100 )</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
</tr>
<tr>
<td>( \text{CVaR} )</td>
<td>806.932</td>
<td>818.881</td>
<td>836.809</td>
<td>723.981</td>
<td>723.981</td>
<td>723.981</td>
<td>723.981</td>
</tr>
<tr>
<td>ETC (€)</td>
<td>-235.988</td>
<td>-235.706</td>
<td>-235.223</td>
<td>-234.909</td>
<td>-234.909</td>
<td>-234.909</td>
<td>-234.909</td>
</tr>
</tbody>
</table>

### Table 4

The results of MGO decisions considering the IGDT risk aversion parameter.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^{\text{IGDT}} )</td>
<td>447.937</td>
<td>218.509</td>
<td>200.155</td>
<td>204.117</td>
<td>262.448</td>
<td>288.106</td>
<td>318.830</td>
</tr>
<tr>
<td>( \beta^{\text{IGDT}} = 0.5 )</td>
<td>434.986</td>
<td>672.707</td>
<td>790.847</td>
<td>814.776</td>
<td>874.649</td>
<td>896.105</td>
<td>922.626</td>
</tr>
<tr>
<td>( \beta^{\text{IGDT}} = 1 )</td>
<td>3.167</td>
<td>1.903</td>
<td>2.243</td>
<td>2.443</td>
<td>2.868</td>
<td>3.137</td>
<td>3.460</td>
</tr>
<tr>
<td>( \beta^{\text{IGDT}} = 100 )</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
<td>497.520</td>
</tr>
<tr>
<td>( \text{CVaR} )</td>
<td>806.932</td>
<td>818.881</td>
<td>836.809</td>
<td>723.981</td>
<td>723.981</td>
<td>723.981</td>
<td>723.981</td>
</tr>
<tr>
<td>ETC (€)</td>
<td>-235.988</td>
<td>-235.706</td>
<td>-235.223</td>
<td>-234.909</td>
<td>-234.909</td>
<td>-234.909</td>
<td>-234.909</td>
</tr>
</tbody>
</table>
A strategy is to consider the Eqs. (35)–(37) and assume that the uncertain parameter is equal to the amount of its forecast value as described as follow. Hence, it is known as the base model.

\[ f_b = \min(f(X, \gamma)) \]  
\[ H(X, \gamma) \leq 0 \quad G(X, \gamma) = 0 \]  

Using the Eqs. (39) and (40) the base value of objective function is obtained [26]. In other words, the optimum value of the objective function considering the uncertain parameter is exactly equal to its forecast value. If the uncertain parameter value is different from its forecast value, the decision makers will deal with the two strategies as follow [25,26]:

- **Risk averse strategy**: This strategy is related to a situation that the uncertain parameter may have undesirable effect on the objective function. Therefore, this strategy tracks to find the maximum uncertainty radius for the worse case. This strategy makes robust objective function which resist against the uncertain parameters deviation. Mathematical formulation of this strategy is expressed as follow:

\[ \hat{\beta} (C_b) = \max \alpha \]  
\[ H(X, \gamma) \leq 0 \quad G(X, \gamma) = 0 \]  
\[ C_b = f_b (X, \gamma) \times (1 + \beta^{IGDT}), \quad 0 \leq \beta^{IGDT} \leq 1 \]  
\[ \gamma = (1 + \alpha) \tau \]  

- **Risk taker strategy**: The uncertain parameters may not have undesirable effect on the objective function. Therefore, this strategy tracks to find the minimum uncertainty radius for better objective function than its forecast value. Mathematical formulation of this strategy is as follow:

\[ \tilde{\beta} (C_b) = \min \alpha \]  
\[ H(X, \gamma) \leq 0 \quad G(X, \gamma) = 0 \]  
\[ C_b = f_b (X, \gamma) \times (1 - \psi^{IGDT}), \quad 0 \leq \psi^{IGDT} \leq 1 \]  
\[ \gamma = (1 - \alpha) \tau \]  

### 3.2.2 Objective function of the second-level problem

In this paper, the risk-averse strategy is employed to model the effect of the uncertainty of the reserve in the decision-making problem of the MGO. For this purpose, the operation problem of the MGO is modeled as Eqs. (49)-(53) where the uncertainty radius is maximized regarding the risk-level of the MGO \( \beta^{IGDT} \) as modeled in (51). It should be noted that the ETC obtained in the first-level problem is considered as the base value \( ETC_b \) as described in (50) and (51). The amount of reserve regarding the mentioned uncertainty is calculated as Eq. (52).

\[ \hat{\alpha} (C_b) = \max \hat{\alpha}^{IGDT} \]  

Subject to:

\[ ETC_b = [ETC: \text{Minimize } ETC + \tilde{\beta}^{CVaR} \tilde{\nu}] \]  
\[ ETC \leq (1 + \tilde{\beta}^{IGDT}) ETC_b, \quad 0 \leq \tilde{\beta}^{IGDT} \leq 1 \]  
\[ R_b^{TV} = (1 - \tilde{\alpha}^{IGDT}) R_b^{TV} \]  
\[ \tilde{\beta}^{TV} = 0 \]  
\[ \tilde{\alpha} = 0 \]  
\[ \tilde{\beta}^{TV} = 0 \]  

The proposed linear optimization model is solved with GAMS 24.1.2 software under CPLEX solver. A personal computer with 6 GB RAM running on Intel Corei-7 with a CPU speed of 2.00 GHz, 64 bits operating system is used to solve the model.

<table>
<thead>
<tr>
<th>Energy Reserve</th>
<th>Table 5 Results of the hybrid stochastic IGDT approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ CVaR</td>
<td>545.74 257.279 284.117 342.448 514.986 752.707 894.776 954.946 2346.946 2847.758 3275.058 3280 30.456 42.290 44.298 46.085 3049.568 2094.458 1149.247 842.636</td>
</tr>
<tr>
<td>Δ CVaR</td>
<td>545.74 257.279 284.117 342.448 514.986 752.707 894.776 954.946 2346.946 2847.758 3275.058 3280 30.456 42.290 44.298 46.085 3049.568 2094.458 1149.247 842.636</td>
</tr>
<tr>
<td>Δ CVaR</td>
<td>545.74 257.279 284.117 342.448 514.986 752.707 894.776 954.946 2346.946 2847.758 3275.058 3280 30.456 42.290 44.298 46.085 3049.568 2094.458 1149.247 842.636</td>
</tr>
<tr>
<td>Δ CVaR</td>
<td>545.74 257.279 284.117 342.448 514.986 752.707 894.776 954.946 2346.946 2847.758 3275.058 3280 30.456 42.290 44.298 46.085 3049.568 2094.458 1149.247 842.636</td>
</tr>
<tr>
<td>Δ CVaR</td>
<td>545.74 257.279 284.117 342.448 514.986 752.707 894.776 954.946 2346.946 2847.758 3275.058 3280 30.456 42.290 44.298 46.085 3049.568 2094.458 1149.247 842.636</td>
</tr>
</tbody>
</table>
4. Numerical results

To investigate the behavior of the MG in both energy and reserve markets considering the uncertainties, the proposed model is applied on the modified 15-bus MG test system [27]. Then, to show the effectiveness of the proposed model for a practical test case, it is applied on a 40-bus real system. The results of both systems are presented in details in the next sub-sections.

4.1. 15-bus test system

The modified 15-bus MG test system is shown in Fig. 3. The forecast MGL and the output power of WTs and PVs are given in Fig. 4. Note that, 80\% of entire MGL including L1-L4, L6-L9 are associated with the loads located at buses 2, 3, 8, 9, 11, 13, 14, and 15 and the remainder amount of the MGL (L5) is located at bus 5. In addition, the location of PV1, PV2, WT1, and WT2 are considered to be on buses 13, 8, 3, and 11, respectively. The forecast energy and reserve market prices related to a sample day extracted from Red Electrica De Espana S.A. market [28] indicated in Fig. 5. The probability of calling reserve and the IL incentive cost are given in Table 1 [22]. The characteristics of FCs, MTs, and the ESs are given in Table 2. Also, the limitation of the power exchange with the main grid is specified to be 200 kW. Moreover, the maximum IL is equal to 10\% of the MGL at each hour. The minimum/maximum magnitude of the bus voltage and the line current are considered 390/400 V and 0.462/−0.462 KA, respectively. The number of piecewise of linearization power flow is 40. The results are investigated in three cases to show the effectiveness of the proposed model as follows:

- Case I: The results of the first-level problem (the stochastic approach)
- Case II: The results of the second-level problem (the IGDT approach)
- Case III: The results of both first and second level problems (the hybrid stochastic-IGDT approach)

![Fig. 10. Sensitivity of ETC and reserve to risk-aversion parameters.](image1)

![Fig. 11. The 40-bus real network.](image2)
resources are sold to the main grid. Moreover, the ESs are charged at 2 and discharged in hour 24 to supply the demand. Instead of using MTs and ESs, the MGO purchases power from the grid to supply the MGLs at hours 18–19 and 21–23. On the other hand, Fig. 7 illustrates the optimal amount of reserve provided by the ILs, ESs, and DG (MT) to the system. Since the FCs have low operation cost in comparison with the energy market price and they are scheduled to sell energy to the grid, their share to provide the reserve is zero. As depicted in this figure, the significant amount of reserve is provided to the system at hours 3–6, 15, 18–19, and 21–23 because of high reserve market prices. As shown in Figs. 6 and 7, MT is the main resource to provide the reserve for the system.

The effect of risk-aversion parameter ($\beta$) on the ETC and the decision variables is presented in Fig. 8 and Table 3. When the risk parameter changes from 0 to 100, the ETC increases (minus ETC decreases) and the CVaR decreases (minus CVaR increases) as revealed in Fig. 8. In other words, the MGO utilizes the CVaR index to control the expected cost of the worst scenarios. This strategy will be carried out by making the risk-based decisions to reschedule the MG’s resources. Therefore, the risk-averse ($\beta > 0$) MGO changes the first-stage decisions with the aim of achieving less CVaR through controlling the results of the MG operation problem in the worst scenarios. When the value of the risk-aversion parameter increases, the MGO increases its dependency on the first stage decisions where it uses DGs and ILs to provide energy and DGs, ILs, and ESs to provide the reserve. According to Table 3, increasing the CVaR risk-aversion parameter from 0 to 1, increase the amount of reserve provided by the DGs (MTs) and the sold/purchased power to/from the main grid. This behavior of the MGO leads to an increase in the provided reserve to the market. Of note that, if the risk-aversion parameter increases to a considerable amount (for instance, 100), the MGO's decisions on the first stage changes to achieve more efficient outcomes. Regarding Table 3, it is clear that, the MGO wants to use DGs for producing the required energy instead of providing the reserve. Moreover, the purchased power from the main grid decreases as risk-aversion increases.

**Case II**

In this sub-section, the IGDT-based operation model without considering the CVaR index is solved which the obtained results are illustrated in Table 4 and Fig. 9. For this purpose, the results of the first level problem are determined with $\beta_{CVaR} = 0$ and then the second level problem is solved considering the value of $\beta_{IGDT}$. The results which are shown in Fig. 9 illustrate that the uncertainty radius increases as the amount of the risk-aversion parameter changes from 0 to 0.3. As a result, when the risk-aversion parameter regarding the risk-level of the MGO’s decisions increases, the ETC of the MG operation rises. For instance, regarding $\beta_{CVaR} = 0$ and $\beta_{IGDT} = 0.3$, the total cost is $-165.191$ € and the maximum uncertainty radius is $\alpha = 0.957$. Thus, increasing the radius uncertainty leads to a reduction in the amount of reserve. The MGO, consequently, decides to use the surplus power of the DGs (MT) to meet the demand and to sell more energy to the energy market. According to Table 4, decreasing the amount of reserve from 2924.568 kW to 120.014 kW, increases the sold power to the main grid from 434.986 kW to 922.626 kW. Also, due to the operation cost of the ESs, the considerable part of their capacities is used to provide reserve. As a matter of fact, the amount of uncertainty radius has a significant effect on the MGO's decisions.

**Case III**

In this sub-section, the effect of uncertainties on the decisions of the MG is investigated using the hybrid stochastic-IGDT approach. At first, the two-stage stochastic problem is solved regarding the different amount of $\beta_{CVaR}$ and then the model is resolved for each value of $\beta_{IGDT}$. The behavior of the MGO based on utilizing the proposed hybrid
According to Table 5, when $\beta_{IGDT} = 0$, the behavior of the MGO depends only on the CVaR risk-aversion parameter and increasing that, decreases the amount of the scheduled energy of the DGs and Is which leads to providing more reserve to the reserve market as the first stage decision. Moreover, when $\beta_{CVaR} = 0$, the MGO decides to decrease the amount of power from DGs and Is to provide reserve and it sells the extra energy to the energy market.

On the other hand, considering both $\beta_{IGDT}$ and $\beta_{CVaR}$ in the decision making process of the MGO changes its behavior. For instance, regarding $\beta_{IGDT} = 0.05$, with an increase in $\beta_{CVaR}$, the MGO prefers to utilize more energy from DGs and Is to sell it to the energy market. Furthermore, increasing the amount of the discharging power of the Es, decreases the purchased power from the energy market. Also, in the case of $\beta_{IGDT} = 0.2$ and $\beta_{CVaR} = 100$, the MGO has the most risk-aversion level in its decisions. In fact, the MGO would provide the minimum amount of the reserve besides the maximum tendency to the first stage variables (DGs as well as Is) for energy management of the MG.

The sensitivity of the ETC and the amount of reserve to both the risk-aversion parameters is investigated in Fig. 10. As shown in this figure, with increasing the risk aversion parameter of the IGDT approach ($\beta_{IGDT}$), the amount of uncertainty radius increases which leads to decreasing the amount of reserve and the minus ETC. For example, with the same $\beta_{IGDT}$, when $\beta_{CVaR}$ is equal to 100, the MGO would be more risk-averse to solve the second-level problem (IGDT approach) in comparison with the case where $\beta_{CVaR}$ is equal to 0.5. Also, the maximum uncertainty radius for $\beta_{CVaR} = 100$ and $\beta_{CVaR} = 0.5$ is 0.737 and 0.734, respectively. Thus, in the lower $\beta_{CVaR}$, the amount of reserve is 810.816 kW, while it is 789.761 kW for the higher $\beta_{CVaR}$.

The results show that in the proposed hybrid stochastic-IGDT approach, the MGO has an opportunity to affordably participate in both the energy and the reserve markets. This means that, specifying both $\beta_{IGDT}$ and $\beta_{CVaR}$ in a high level of risk-aversion, the amount of provided reserve and the sold power to the grid become 789.761 kW and 830.978 kW with an acceptable ETC of $-187.927$.

### 4.2. 40-bus real network

To show the effectiveness of the proposed model for the real networks, a low voltage distribution network of Kurdistan Province Electricity Distribution Company (KPEDC) in Iran is employed in this sub-section as shown in Fig. 11. The data of this network is given in Appendix. The other input data is the same as used in the previous subsection. The location of PV1, PV2, WT1, and WT2 are considered to be on buses 31, 23, 16, and 7, respectively. The results of applying the proposed model on this network are shown in Table 7 and Fig. 12.

As shown in Table 6, the Disco has the same behavior described in case III to manage the uncertain parameters using the hybrid approach. It is clear that, the Disco prefers to have more dependency on the first-stage decisions besides participating more effectively in both the energy and the reserve markets. When both of the risk-aversion parameters ($\beta_{CVaR}$ and $\beta_{IGDT}$) increase, the most efficient results (e.g. the accepted reserve of 797.764 kW and the ETC of $-182.282$) would be achieved to deal with the minimum cost related to the unexpected behavior of the uncertainties.

According to Fig. 12, with the same $\beta_{IGDT}$, when $\beta_{CVaR}$ equals 0.5 or 100, the maximum uncertainty radius becomes 0.717 and 0.725, respectively. Thus, in the lower $\beta_{CVaR}$, the amount of reserve is 818.212 kW, while it is 797.764 kW for the higher $\beta_{CVaR}$.

### 5. Conclusion

A hybrid stochastic-IGDT approach is employed in this paper to model the uncertainties in the operation problem of the MGO in energy and reserve markets. For this purpose, the uncertainties of the output
The results of the proposed model are investigated in a standard test system in three cases: (1) the stochastic approach, (2) the IGDT approach, and (3) the hybrid method. Moreover, to show the capability of the proposed hybrid method of being applied on a real test case, it is applied in a 40-bus real network. The main conclusions from the results are as follows:

- The MGO doesn’t control the uncertainty related to the accepted reserve in the reserve market in the first case while it controls the effect of the other uncertain parameters on its decisions in the energy market. In this case, instead of producing the required energy, the MGO prefers to use the capacity of the resources to provide the reserve to reach a lower ETC. For example, in $\beta_{\text{VaR}} = 100$ the MGO provides 3007 kW reserve to the market with the lowest amount of ETC, i.e. $234.909\,\text{$.}

- In the second case, the MGO controls the uncertainty related to the amount of accepted reserve on its decisions. In this case, the MG’s resources are scheduled to provide more energy in comparison with the reserve. Moreover, increasing the risk-aversion parameter of the IGDT increases the radius uncertainty which consequently decreases the amount of the reserve. Therefore, the MGO provides the minimum reserve to the market, i.e. 125.014 kW with the highest ETC, $187.927\,\text{$.}

- In the first and the second case, the MGO prefers to use the resources to participate more in the reserve and the energy markets, respectively, which leads to achieving the lowest ETC with the maximum amount of reserve for the first case and the highest ETC with the minimum amount of the reserve for the second case. This is while the results of the third case reveal that the proposed hybrid stochastic-IGDT approach would provide an opportunity for the risk-averse MGO to effectively participate in both the energy and the reserve markets. In fact, a trade-off between the amount of the reserve and the ETC is obtained in this case with controlling all the uncertain parameters. For the most risk-averse MGO, the amount of the reserve is 789.761 kW with the ETC of $187.927\,\text{$.}$ Moreover, the amount of selling energy to the market in the hybrid approach is 830.978 kW which is between that of case I and II with 527.664 kW and 922.626 kW, respectively. The same behavior of the MG also occurs in scheduling the resources and participating in the markets to manage the uncertainties in the real test network.

### CRediT authorship contribution statement

Ramyar Mafakheri: Conceptualization, Methodology, Software, Data curation, Formal analysis, Writing - review & editing, Writing - original draft. Pouria Sheikholeslami: Conceptualization, Methodology, Software, Data curation, Formal analysis, Writing - review & editing, Writing - original draft. Salah Bahramara: Conceptualization, Methodology, Software, Data curation, Formal analysis, Writing - review & editing, Writing - original draft.

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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**Table 7**
The line data consisting of resistance and impedance for the real network.

<table>
<thead>
<tr>
<th>Line</th>
<th>R (Ω)</th>
<th>Z (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>0.2412</td>
<td>0.0678</td>
</tr>
<tr>
<td>2-3</td>
<td>0.4734</td>
<td>0.2745</td>
</tr>
<tr>
<td>2-4</td>
<td>0.2412</td>
<td>0.0678</td>
</tr>
<tr>
<td>4-5</td>
<td>0.4734</td>
<td>0.2745</td>
</tr>
<tr>
<td>6-7</td>
<td>0.2104</td>
<td>0.1220</td>
</tr>
<tr>
<td>8-9</td>
<td>0.3029</td>
<td>0.1756</td>
</tr>
<tr>
<td>9-10</td>
<td>0.3834</td>
<td>0.2223</td>
</tr>
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<td>10-11</td>
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</tr>
<tr>
<td>11-12</td>
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<td>0.2745</td>
</tr>
<tr>
<td>11-13</td>
<td>0.4734</td>
<td>0.2745</td>
</tr>
<tr>
<td>11-14</td>
<td>0.5042</td>
<td>0.2136</td>
</tr>
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</table>

**Table 8**
The load proportion of each bus from the whole demand of the real network.

<table>
<thead>
<tr>
<th>Load</th>
<th>% of total MGL</th>
<th>Load</th>
<th>% of total MGL</th>
<th>Load</th>
<th>% of total MGL</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.178784</td>
<td>L14</td>
<td>1.191895</td>
<td>L27</td>
<td>0.476758</td>
</tr>
<tr>
<td>L2</td>
<td>1.191895</td>
<td>L15</td>
<td>0.476758</td>
<td>L28</td>
<td>0.953516</td>
</tr>
<tr>
<td>L3</td>
<td>0.953516</td>
<td>L16</td>
<td>2.145411</td>
<td>L29</td>
<td>3.098927</td>
</tr>
<tr>
<td>L4</td>
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<td>L17</td>
<td>0.715137</td>
<td>L30</td>
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</tr>
<tr>
<td>L5</td>
<td>1.430274</td>
<td>L18</td>
<td>3.576582</td>
<td>L31</td>
<td>1.668653</td>
</tr>
<tr>
<td>L6</td>
<td>8.820024</td>
<td>L19</td>
<td>1.907032</td>
<td>L32</td>
<td>0.476758</td>
</tr>
<tr>
<td>L7</td>
<td>0.953516</td>
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<td>L33</td>
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</tr>
<tr>
<td>L8</td>
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<td>L21</td>
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<td>L34</td>
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</tr>
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<td>0.715137</td>
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<td>L13</td>
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<td>L26</td>
<td>3.814064</td>
<td>L39</td>
<td>13.17053</td>
</tr>
</tbody>
</table>
Appendix. The required data for the real test system

The resistance and impedance data of the real network lines required for the power flow problem is given in Table 7. Moreover, the load proportion of each bus from the total load of the network which is extracted from the historical data of KPEDC (http://kurdelectric.ir) is given in Table 8. Regarding the data of this table, the demand of the MG in each hour which is presented in Fig. 4 is divided among the buses of the real network.

Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ijepes.2020.105977.

References