

# Locational Load Shedding Marginal Pricing

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**Abstract**—This paper presents the Locational Load Shedding Marginal Pricing (LSMP), a useful index for market-based real time intelligent load shedding problems. This parameter is well-described by introducing the load shedding program as an optimization problem very similar to, but in inverse vision, to Economic Dispatch (ED) problem. The load shedding is formulated as an optimal non-linear constrained problem. The LSMPs are calculated for some test systems and the simulation results are demonstrated as a function of active power. The LSMP curves that are finally shown demonstrate the practical advantages of the proposed index for real-time intelligent load shedding schemes.

**Keywords**— Load Shedding; Electric Market; LSMP; Intelligent Load Shedding;

## I. INTRODUCTION

Following a large generation loss disturbance in a power system, it may be needed to shed some amount of system load to maintain power system stability and to assure supplying the remaining loads [1-6]. Referring to these literatures, some parameters should be determined for a load shedding scheme, such as: decision indices (based on voltage or frequency), number of load shedding steps, load shedding delays, and amount of load shedding in each step. Power system economic is an important issue that should be considered in the load shedding strategies. This issue may affect load shedding decisions considerably. To decrease penalty costs that a generation company should pay because of their expected energy that is not supplied, in addition to the above parameters, some other factors should be considered in a load shedding scheme. The load shedding may cause a load service interruption from customers that have bought electricity through some contracts; and, curtailing their loads makes significant penalty prices for generation companies. Therefore, minimizing total penalty costs is another aspect which affects load shedding decisions. Actually, the total amount of load shedding should be optimally distributed among the system loads.

When a generation company cannot supply their expected energy following a disturbance, it should pay penalty to its non-served consumers, because of breaking the contract [6]. There are some non-interruptible loads in a real power system and also the important features of the different loads are not the same. In a restructured power system, customers can buy some special services from ancillary service markets to assure their reliability and to be protected against shedding in the emergency conditions [7]. Buying these services makes a right for the customers to receive more penalty payments from generation companies when their loads are curtailed. In a restructured environment, spinning reserve is also bought through ancillary service contracts. Coordination between the amount of spinning reserve allocation and load shedding can reduce total penalty costs that generation companies should pay in the emergency conditions [7]. Power systems usually allot a portion of their loads as interruptible loads to curtail them in the emergency conditions triggered by a large generation loss disturbance, suddenly load increasing, and even in the annually load peak durations [8]. Considering the above points, it is noteworthy that shedding penalty price of loads differs from one load to another and this difference is significantly grown up in the restructured environments. Therefore, an optimal load shedding algorithm that considers the daily or hourly penalty prices (like bid prices) is needed to significantly reduce the expected energy not-supplied (EENS) punishments.

Furthermore, interconnecting wind turbines into power systems as distributed, non-dispatchable, and uncertain predictable energy sources, can affect load shedding amount at each bus because of their impacts on the real power transferring. Therefore, the real-time wind speed should be considered. For each wind speed value a fixed amount of power can be obtained from a wind farm. The wind power should be considered as a fixed generator at a PQ bus.

In the present paper, at first, the impact of line capacity limits on optimal load shedding amount at each bus is clearly emphasized. Then, the formulation of an optimal load shedding is described as a constrained nonlinear minimization

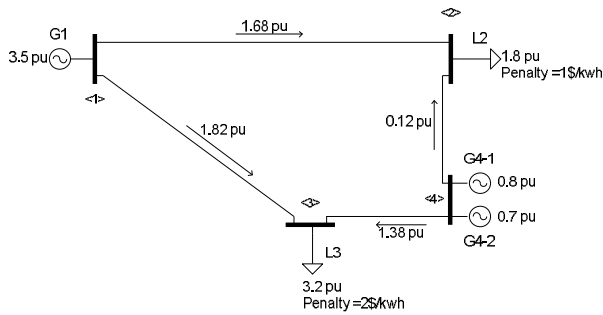


Fig. 1. Test system-normal configuration.

problem. The LSMP index is proposed and finally the application of this index is demonstrated through some simulations. Case studies are implemented on a power system considering restructured environments aspects. The proposed methodology is tested on 4 and 9-bus test systems.

## II. PROBLEM DESCRIPTION AND FORMULATION

### A. Impacts of Transmission Line Constraints on the Optimal Load Shedding

In this section, we consider an optimal load shedding scheme similar to an economic dispatch (ED) problem. Transmission line congestion is one of the important problems that may limit the contracts, considerably. The load shedding results can also be influenced by these limitations. By means of a simple 4-bus test system, the impacts of transmission line capacity limits on the load shedding amount at each bus is studied here. The examples presented in this section are similar to the examples in reference [9], while in [9] this methodology is used for an ED problem rather than a load shedding problem.

A simple 4-bus test system is considered as a test system to clarify the above discussions. The configuration of the test system and its line data are shown in Fig. 1 and Table 1, respectively. Losses have been neglected and it is assumed that in-service units are not contributed to the frequency control purposes. Therefore, there is not any available reserve throughout the power system. Furthermore, assume that  $L3$  has bought reliability services from ancillary service markets and its related penalty price is two times larger than  $L2$  (2\$/KWh vs. 1\$/KWh).

Following  $G4-1$  loss disturbance, two cases have been tested:  $a)$  when lines are unconstrained and  $b)$  when they are constrained. Fig. 2 shows the optimal load shedding solution for case  $a$ . When line capacity limits are not considered, it is clear that optimal solution is load shedding at cheapest

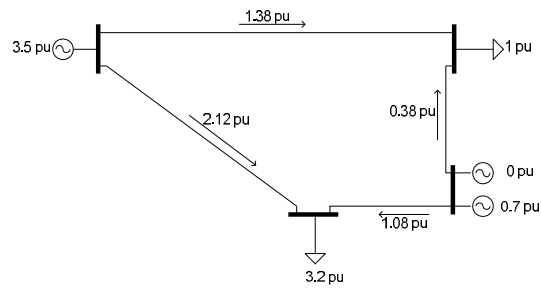


Fig. 2. Optimal load shedding when lines are not constrained

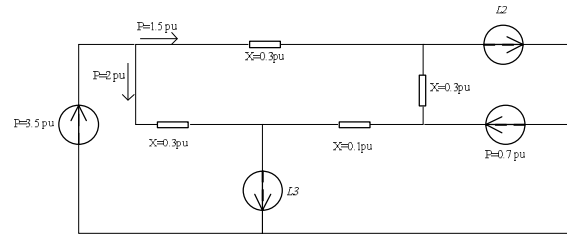


Fig. 3. DC Model for 4-bus test system

customer(s). Analyzing the results in Fig. 2, it is noteworthy that load shedding only at bus 2 is not the reliable solution; and it may lead to the cascading events if line 1-3 is constrained.

In the second case, lines are constraint. Therefore, to avoid line thermal limit violation, it is necessary to shed load at expensive customer(s), too. Consider line 1-3 transfers maximum possible amount of power (2pu), and calculate the amount of  $L2$  and  $L3$ . Fig. 3 illustrates the equivalent circuit (DC Model) of this case that helps us to calculate load shedding amounts.

$$\begin{cases} Eq.1 : 1.5 \times 0.3 + (1.5 - L_2) \times 0.3 + (1.5 - L_2 + 0.7) \times 0.1 = 2 \times 0.3 \\ Eq.2 : L_2 + L_3 = 3.5 + 0.7 \end{cases}$$

$$\rightarrow \begin{cases} L_2 = 1.3 pu \\ L_3 = 2.9 pu \end{cases} \rightarrow \begin{cases} \Delta L_2 = 0.5 pu \\ \Delta L_3 = 0.3 pu \end{cases}$$

Where,  $L_i$  and  $\Delta L_i$  are acceptable load and load shedding amount at bus  $i$ , respectively. From the above solution, it can be seen that transmission line constraints may affect the optimal load shedding solution and also, the load shedding penalties may increase.

### B. Optimal Load Shedding Formulation by AC Analysis

The objective function of an optimal load shedding problem is to minimize the total load shedding penalties in the presence of some constraints. Considering voltage constraints, power transferring limits, generator reactive power constraints and, the other operating and reliability constraints is possible by an AC analysis. An optimal load shedding problem follows the formulation of an ED problem in an optimal power flow (OPF) form. The adjustable variables of such minimization problem are angle of bus voltages ( $V_a^i$ ), magnitude of bus

TABLE I  
TEST SYSTEM BRANCH DATA

from	To	X (pu)	Limits (pu)
1	2	0.3	2
1	3	0.3	2
2	4	0.3	2
3	4	0.1	2

voltage ( $V_m^i$ ), generator reactive power ( $Q_G^j$ ) and load shedding amounts at different buses ( $\Delta P_L^k$ ). For the problems that are solved to coordinate load shedding with other schedules such as reserve allocation and load restoration problems, the generator active power outputs are also considered as adjustable variables. But in a single load shedding problem, if a fast spinning reserve will be available through the power system, it can be added to the output of the generators. Therefore, the generator active powers are considered as fixed variables in our optimization problem. The optimization formulation can be written as follows:

$$o.f : \min \sum_{i=1}^{N_L} C^i \Delta P_D^i \quad (1)$$

s.t :

Non-equality constraints:

$$V_{\min}^i \leq V^i \leq V_{\max}^i \quad (2)$$

$$Q_G^{i,\min} \leq Q_G^j \leq Q_G^{j,\max} \quad (3)$$

$$\Delta P_{D,\min}^k \leq \Delta P_D^k \leq \Delta P_{D,\max}^k \quad (4)$$

$$-P_{\max}^{ij} \leq P^{ij} \leq P_{\max}^{ij} \quad (5)$$

$$-P_{\max}^{ji} \leq P^{ji} \leq P_{\max}^{ji} \quad (6)$$

Equality constraints:

$$\text{real} \left( V^i \times (Y_{\text{bus}} \times V)^* - (S_G^i - (S_D^i - \Delta S_D^i)) \right) = 0 \quad (7)$$

$$\text{imag} \left( V^i \times (Y_{\text{bus}} \times V)^* - (S_G^i - (S_D^i - \Delta S_D^i)) \right) = 0 \quad (8)$$

Where,

$Y_{\text{bus}}$  : system admittance matrix

$V$  : bus voltage vector

$S_G^i = P_G^i + jQ_G^i$  : generation at bus  $i$

$S_D^i = P_D^i + jQ_D^i$  : demand at bus  $i$

$\Delta S_D^i = \Delta P_D^i + j \Delta Q_D^i$  : load shedding at bus  $i$  ( $\Delta Q_D^i = PF^i \times \Delta P_D^i$ )

$PF^i$  : load power factor at bus  $i$

$P^{ij}$  : active power transferring from bus  $i$  to bus  $j$

Eqs. (2), (3) and, (4) are bus voltage constraints, generator

reactive power limits and load shedding amount limits, respectively. These constraints are linear but (5) that is line transfer constraint, is a nonlinear constraint. Equal constraints (7) and (8) are also nonlinear constraints. Therefore, we are encountered with a nonlinear constraint multi-variable optimization problem.

To solve this minimization problem, we can define it as a Lagrangian function. Then, some optimization algorithms such as sequential quadratic programming (SQP) [10], interior point techniques [11] and, trust region methods can be used to solve it.

### III. PROPOSED INDEX: LSMP

#### A. Definition

Studying the ED concepts and obtaining similar ideas to an optimal load shedding problem may lead to some interesting points about load shedding pricing aspect. Locational Marginal Pricing (LMP) is one of the related concepts which are used for calculating the bidding price at each bus. It is a useful index for ED schemes, congestion management problems, and allotting prices throughout the power system [9]. Using this pricing index, a customer knows the marginal price of delivering energy to its connected locations. This concept can be inversely used to calculate the marginal penalty price that a tripped generation company should (directly or indirectly) pay to their loads in the case of selecting them for load shedding. Here, a new pricing index is defined for the emergency conditions. We call it Load Shedding Marginal Pricing (LSMP [1]). This index is calculated at generator buses, and determines marginal amount

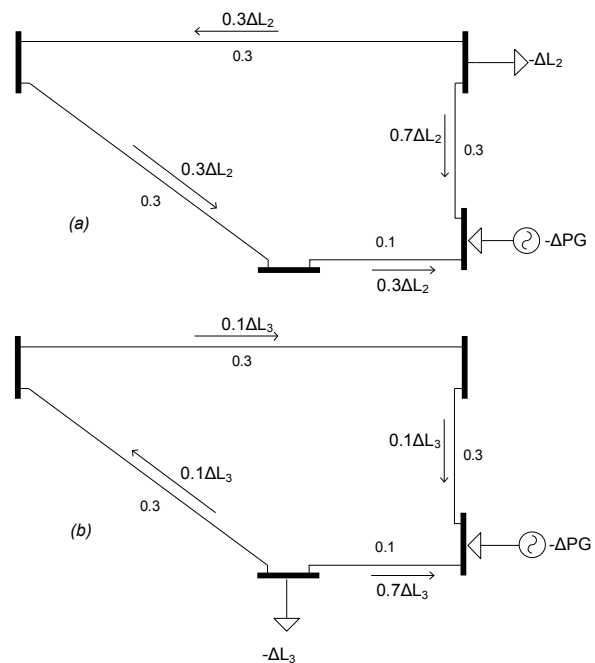


Fig. 4. Flows due to load reduction; a) at bus 2, b) at bus 3

of penalty price that a generation company should pay to the ISO for losing 1KW of its generation.

LSMP may also be used for reserve allocation issues, optimal load restoration problems, coordinating between amount of load shedding and reserve allocation, and coordination between optimal load shedding and restoration.

The methodology is used here for calculating the LSMP is similar to the methods used for LMP in [9]. Following loss of G4-1 in example 1, we calculate the LSMP indices. For the first case, when lines are unconstrained, LSMP will be the same as penalty price at the cheapest customer (1\$/KWh) because, for each KWh generation loss, load shedding at bus 2 is not limited. However when line constraints are considered, LSMP at bus 4 is 1.75\$/KWh that is obtained as follows: consider the solution results of example 1.b. In this example line 1-3 transfer maximum possible amount of power (2pu). In this situation, if the generation at bus 4 were to reduce by 1KW, KW would not be completely compensated by load shedding at bus 2 because of the constraint in line 1-3.

Load shedding only at bus 2 increases the power flow in line 1-3. Fig. 4.a is illustrated to clarify this point by introducing load reduction as an additional generation in the inverse direction of load at its location; and introducing generation reduction as load increasing in the opposite direction of generation at its bus.

Assume generation at bus 4 is reduced by  $\Delta PG$  and, to compensate it  $L_2$  decreases by  $\Delta L_2$ . It will change line transferring as shown in Fig. 4.a based on the line impedances. Reduction of  $L_2$  by  $\Delta L_2$  adds  $0.3\Delta L_2$  power in line 1-3. To avoid it, consider  $L_3$  is reduced by  $\Delta L_3$ . As shown in Fig. 4.b, it causes adding power transferring in line 1-3 by  $0.1\Delta L_3$ , but in the opposite direction. Therefore, marginal load shedding amounts can be achieved as follows:

$$\begin{cases} \Delta L_2 + \Delta L_3 = \Delta PG = 1KW \\ 0.3\Delta L_2 - 0.1\Delta L_3 = 0 \end{cases} \rightarrow \begin{cases} \Delta L_2 = 0.25KW \\ \Delta L_3 = 0.75KW \end{cases}$$

The load shedding marginal pricing is calculated as follow:

$$LSMP = 0.25 \times 1\$ / KWh + 0.75 \times 2\$ / KWh = 1.75 \$ / KWh$$

Lagrange multipliers associated with the  $i^{th}$  equality constraint, in an optimization problem, represent the change in the optimal objective per unit change in the related equality constraint and also the Lagrange multipliers are called shadow prices [12]. Thus, similar to the LMP in the ED problem [5],  $\lambda_i^*$ , the Lagrange multiplier of the  $i^{th}$  equality constraint (associated with  $i^{th}$  bus) in an optimal load shedding solution, refers to the LSMP at bus  $i$ . But, some points need to be mentioned.

TABLE II  
OPTIMAL LOAD SHEDDING RESULTS FOR FOUR-BUS TEST SYSTEM

	CASE (A)	CASE (B)	CASE (C)	CASE (D)
Lines are constrained	No	Yes	No	Yes
Amount of generation loss (pu)	0.8	0.8	1.7	1.7
Tripped generator's bus number	4	4	1	1
Total penalty cost (\$/KWh)	800	983.9	1700	1700
Amount of LS at bus 2 (pu)	0.8	0.616	1.7	1.7
Amount of LS at bus 4 (pu)	0	0.184	0	0
LSMP at bus 1 (\$/MWh)	1000	1000	1000	1000
LSMP at bus 4 (\$/MWh)	1000	1777	1000	1000

### B. Case Study: Optimal load shedding

For implementing the discussed optimal load shedding algorithm, OPF m-files of MATPOWER software package are modified. MATPOWER is able to use some optimization algorithms such as: SQP, interior point and, trust region algorithms. The objective function, optimization variable vector and, linear and nonlinear constraints has been modified. The load penalty prices are also added to case study data. All of the constraints introduced by (2)-(8) have been considered. Here, the interior point technique is used to solve this optimization problem. The simulation has been done on the two case studies: 4-bus and 9-bus. Two scenarios are examined for each test system: when transmission lines are unconstrained and when they are assumed to be constrained. The simulation results are shown in the Tables 2 and 3.

As illustrated in Table 2 when the generation at bus 4 is reduced and lines are unconstrained, only the load at bus 2 that is the cheapest customer is shed and, the LSMP at all generator buses is the same as penalty price of the cheapest customer. But when lines are constrained, load shedding amount at bus 2 is limited. Therefore, total penalty cost of load shedding is increased; and also, the LSMP at bus 4 is increased. However, this value is not changed at bus 1. The

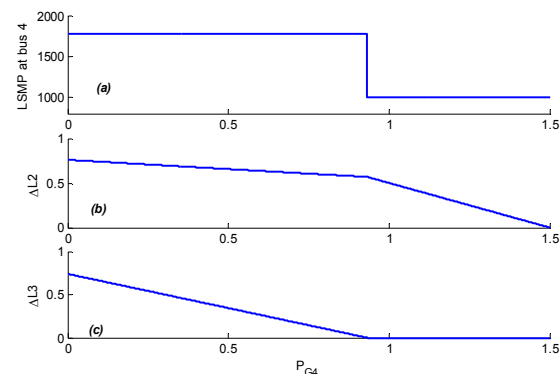


Fig. 5. a) LSMP at bus 4, b) amount of load shedding at bus 2 and, c) amount of load shedding at bus 3 in terms of different value of generation loss at bus 4

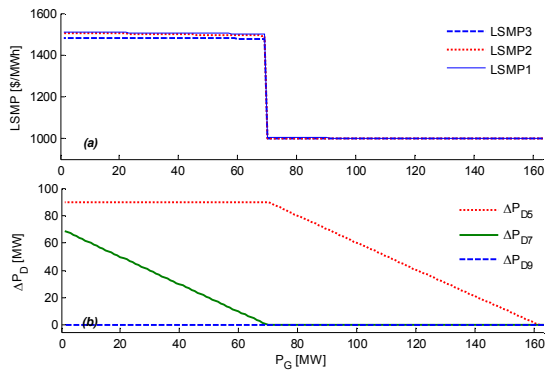


Fig. 6. a) LSMP at generator buses, b) amount of load shedding at load buses in terms of generation values at bus 2

simulation also shows that generation reduction at bus 2 and its following load shedding is not limited by line constraints.

### C. Case Study: Load Shedding Marginal Pricing and Marginal Load Shedding Amounts

In order to analyze LSMP behavior in terms of generation loss magnitudes, here the generation at a specific bus is swept; and, the LSMP values and the optimal load shedding amounts at different buses are calculated. Two power systems are studied: 4-bus and 9-bus. Fig. 5 shows the simulation results on the 4-bus test system. In this simulation the generation at bus 4 is reduced; then, an optimal load shedding algorithm is used to compensate load-generation imbalance. Through the generation reduction scenario at first, load is shed only at bus 2 that is the cheapest costumer. Therefore, LSMP is the same as penalty price of the  $L_2$ . But if the generation reduction continues, it reaches a point that the constraints of line 1-3 make  $L_3$  to be participated into the load shedding program. Thus, the LSMP increases. The simulation results of 9-bus test system are illustrated in Fig. 6 and Fig. 7 that show the unconstraint and constraint scenarios, respectively.

### CONCLUSION

Presenting an innovative load shedding pricing index, this paper provides the opportunity for electric market holders to allocate appropriate real-time penalty factors for generator buses, as well as the participation factors for load buses in potential load shedding situations. The proposed method is

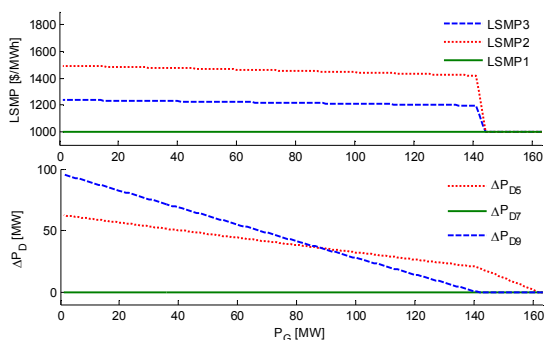


Fig. 7. a) LSMP at generator buses, b) amount of load shedding at load buses in terms of generation values at bus 2

TABLE III  
OPTIMAL LOAD SHEDDING RESULTS FOR NINE-BUS TEST SYSTEM

	CASE (A)	CASE (B)	CASE (C)	CASE (D)
Lines are constrained	No	Yes	No	Yes
Amount of generation loss (MW)	85	85	163	163
Tripped generator's bus number	3	3	2	2
Total penalty cost (\$/KWh)	82605	119749	195011	226719
Amount of LS at bus 5 (pu)	82.605	29.76	90	62.86
Amount of LS at bus 7 (pu)	0	0	70	0
Amount of LS at bus 9 (pu)	0	52.93	0	96.38
LSMP at bus 1 (\$/MWh)	1000	1352.2	1510.4	1000
LSMP at bus 2 (\$/MWh)	1000	1000	1506.7	1495.3
LSMP at bus 3 (\$/MWh)	1000	1000	1481.3	1238.2

verified by some case studies.

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