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***Abstract*—This paper proposes a consensus-based secondary frequency control scheme for islanded microgrids. The scheme only deals with droop equation and secondary control variables, and coincides with the hierarchical control concept. In this way, the effects of primary control level are integrated into the active power variation as a disturbance signal. Finite-gain L_2-stability of the system, from the disturbances to the system states, is then investigated. Minimum requirements for convergence of the system to the desired operating point are also extracted theoretically. The scheme is then augmented with an online Kron reduction strategy, which prevents the spanning trees in the communication network from cutting, and provides the distributed generation units with smooth plug-and-play functionality, when they get disconnected physically. The effectiveness of the proposed scheme for different case studies is validated in MATLAB/Simscape Electrical™ software environment.**

***Index Terms*—Consensus, frequency control, Kron reduction, L_2-stability, microgrids, plug and play, secondary control.**

I. Introduction

Droop-control is a decentralized way to integrate inverter-interfaced distributed generation units (DGs) into the autonomous microgrids (MGs). Despite its simple, scalable, plug-and-play (PnP), and cheap functionality, and its proper active power sharing, frequency-active power (Hz-Watt) droop-control leads to undesired frequency deviations [1]. Under the hierarchical control policy, secondary control is responsible to compensate for these deviations [2]. This compensation, however, should not disturb the droop-induced proportional Watt sharing. To this end, different secondary control schemes with centralized [3], [4] or decentralized [5], [6] or distributed [7]–[19] decision making strategies have been proposed. Among these schemes, the distributed consensus-based controllers demonstrate superior performance in terms of reliability, expandability, and accuracy [20], [21].

Literature review and motivations: Consensus-based secondary Hz-Watt control of MGs has been investigated in [10]–[19]. In [10], [11], leader-following-based frequency control and average-consensus-based active power control are proposed. Over time, the performance of these Hz-Watt controllers have been modified by using, e.g., finite-time [12], robust [13], adaptive [14], noise-resilient [15], and event-triggered [16] control techniques. In these works, however, by defining two auxiliary control inputs and by using frequency and active power feedback signals of DGs, the Hz and Watt control problems are considered as consensus of two separate multi-agent systems (MASs) with single integrator dynamics and all stability analyses are conducted based on this consideration, while they (frequency & active power) are not separate and have algebraic relation (through droop equation). In [17], similar secondary Hz-Watt controller is proposed, but for economical droop mechanism. Therein, different from the mentioned works, the systems are not considered as MASs with single integrator dynamics and the small signal stability of the whole MG is investigated. By using frequency and secondary variable feedback signals, a secondary Hz-Watt control is proposed in [18], [19] and its local stability and convergence analysis, regarding the whole MG structure and *undirected* communication networks (CNs), is investigated in [19]. However, the mathematical analyses, regarding whole MG structure, is not always necessary, in particular for modifying the performance of the existing works from a cyber-standpoint. Therefore, providing systematic modeling and stability approaches for applications, only dealing with droop equation, secondary control and cyber-side (communication layer), and considering *directed* CNs seems to be effective.

In secondary controlled microgrids, once a DG gets disconnected, it is not an active agent anymore and cannot cooperate with other DGs (agents) to achieve consensus. Thus, it is conventionally assumed that once a DG gets unplugged, its corresponding communication links are all interrupted and the neighboring DGs do not receive any kind of data from it, until the DG returns back to the MG (e.g., see PnP case-studies in [12]–[16], [19]). However, this may weaken the connectivity of the CN or even dissolve it (e.g., under a directed ring-structured CN). Therefore, it is effective to propose a strategy which preserves the connectivity of the CN without any communication topology variation, in case of temporary physical disconnection of a DG (plugging-out).

Contributions: Motivated by the above statements, a consensus-based secondary Hz-Watt control is presented herein. Salient features of this paper are as follows.

- A state representation for secondary Hz-Watt control is provided, which only deals with droop equation and the secondary control variables, while the effects of underlying control and physical layers are reflected via active power variations as disturbance signals. Hence, *i)* compared to the works in [10]–[16], this model does not

fully neglect the fast dynamics of the inner controllers, and *ii)* compared to the works in [17]–[19], the whole MGs dynamics and power flow equation is not considered. Thus, the model is suitable for secondary control designs, focusing on the cyber-side and consensus control performance, while presents a genuine state model.
- According to the represented state model, the stability analysis with finite-gain L_2-stability (from the power variations to states) approach is provided.
- Compared to [18], without investigating the whole MG structure and by considering a directed CN, the least requirements for desired system convergence are probed.
- An online Kron reduction strategy is proposed, under which, once a DG stops power injection, its communication module acts as an interface, and interconnects its neighboring DGs. This strategy helps prevent cutting down the spanning trees of the CN, which are key factors for connectivity of it. In other words, the strategy provides the DGs with a kind of smooth PnP functionality.

Paper Outline: Section II introduces the system modeling and formulation of the proposed consensus-based controller. The system stability is investigated in Section III. Convergence analysis and the proposed online Kron reduction strategy are provided in Section IV. The simulation results, performed in MATLAB/Simscape Electrical™ software environment, are included in Section V. Finally, Section VI concludes the paper.

Notation: Consider the set $\{x_i\}, \forall i|_1^n$. Then, $\text{col}\{x_i\}$, $\text{row}\{x_i\}$, and $\text{diag}\{x_i\}$ denote column and row vectors, and proper diagonal matrix with the corresponding entries x_is. $\mathbf{1}_n$ (resp. $\mathbf{0}_n$) denotes a $n \times 1$ vector of ones (resp. zeros). Given the matrix $\mathbf{M}$, $\text{rank}\{\mathbf{M}\}$, $\ker\{\mathbf{M}\}$, and $\lambda_{min}^{\mathbf{M}}$ & $\lambda_{max}^{\mathbf{M}}$ are its rank, kernel, and smallest & greatest eigenvalues, respectively. The notion of *finite-gain L_2-stability* is defined in [22].

II. System Setup

A. Communication Network (Graph Theory) [23], [24]

The CN among DGs, can be regarded as a directed graph (digraph) with DGs and communication links playing the roles of its nodes and edges, respectively. Consider the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$, where, $\mathcal{N} = \{1, ..., n\}$, $\mathcal{E} = \mathcal{N} \times \mathcal{N}$, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ are its node set, edge set, and adjacency matrix, respectively. If node i obtains data from node j, then, node j is an in-neighbor (sender) of node i, node i is an out-neighbor (receiver) of node j, $(j, i) \in \mathcal{E}$, and $a_{ij} > 0$; otherwise, nodes i and j are not neighbors, $(j, i) \notin \mathcal{E}$, and $a_{ij} = 0$. A graph is simple (loop-less) if $a_{ii} = 0$ and is undirected if $a_{ij} = a_{ji}$. Let $N_i = \{j \mid (j, i) \in \mathcal{E}\}$, $N_i^o = \{j \mid (i, j) \in \mathcal{E}\}$, $d_i = \sum_{j \in N_i} a_{ij}$, and $d_i^o = \sum_{j \in N_i^o} a_{ji}$ be the in-neighbor set, out-neighbor set, in-degree, and out-degree of node i, respectively. A graph is balanced, if $d_i = d_i^o$, $\forall i$. Laplacian matrix of $\mathcal{G}$ is $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where, $\mathcal{D} = \text{diag}\{d_i\}$. A directed path from node j to node i is a sequence of pairs, belong to $\mathcal{E}$, expressed as $\{(j, n_1), ..., (n_m, i)\}$. A graph has a spanning tree, if there is a node r (called the root node), such that there is a directed path from the root node to every other node in the graph. A graph is strongly connected, if its nodes are all root nodes.

B. Droop-Based Secondary Hz-Watt Control of Microgrids

Consider i^{th} DG's droop equation below, where the secondary control variable (correction term) Ω_i is included.

$$f_i = f_{nom} - m_i P_i + \Omega_i, \text{ where } m_i = \frac{\Delta f_{max}}{P_{max}^i} \tag{1}$$

where, f_i, f_{nom}, m_i, P_{max}^i and P_i denote i^{th} DG's frequency, nominal frequency, droop coefficient, real active power, and power capacity, respectively, and $\Delta f_{max} = 0.005 f_{nom}$ is the maximum allowable steady-state frequency deviation. Let differentiate (1) and define $w_i = -m_i \dot{P}_i$, one then can write

$$\begin{cases} \dot{f}_i = u_i + w_i, \\ \dot{\Omega}_i = u_i, \end{cases} \tag{2}$$

where, u_i (resp. w_i) is input (resp. disturbance) signal. In order to bring the frequency back to f_{nom} and maintain the proportional Watt sharing established by droop control, one can use the following control input, as it is discussed in [19].

$$u_i = g_i^f (f_{nom} - f_i) + g_i^{\Omega} \sum\nolimits_{j \in N_i} a_{ij} (\Omega_j - \Omega_i), \tag{3}$$

where, g_i^f and g_i^{Ω} are nonnegative feedback gains and a_{ij} is defined in Section II-A. Note that in (2), the signal w_i is proportional to the active power variations and is treated as a disturbance signal. This systematic modeling simplifies the convergence and stability analyses, while, accounts for the impacts of the underlying control and physical layers on the performance of secondary control. The implementation algorithm for an example DG is given in Algorithm 1.

III. Stability Analysis

The system described by (1)-(3), can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{w}, \text{ where } \mathbf{A} = -\begin{bmatrix} \mathbf{g}_f & \mathbf{g}_{\Omega}\mathcal{L} \\ \mathbf{g}_f & \mathbf{g}_{\Omega}\mathcal{L} \end{bmatrix}. \tag{4}$$

$\mathcal{L}$ is the Laplacian matrix of the CN; $\mathbf{g}_f = \text{diag}\{g_i^f\}$, $\mathbf{g}_{\Omega} = \text{diag}\{g_i^{\Omega}\}$; $\mathbf{x} = [(\mathbf{f} - \mathbf{f}_{nom})^T, \mathbf{\Omega}^T]^T$, $\mathbf{w} = [\text{row}\{w_i\}, \mathbf{0}_n^T]^T$, where, $\mathbf{f}_{nom} = \text{col}\{f_{nom}\}$, $\mathbf{f} = \text{col}\{f_i\}$, $\mathbf{\Omega} = \text{col}\{\Omega_i\}$.

Theorem 1: Suppose that the disturbance $w_i = -m_i \dot{P}_i$ has bounded variations. If the matrix $\mathbf{A}$ is Hurwitz i.e., its eigenvalues are all in the left half plane (LHP), then, the system (4) is finite-gain L_2-stable from $\mathbf{w}$ to $\mathbf{x}$, with an induced gain less than $\gamma = [\tau^{-1} \lambda_{max}^{\mathbf{P}^2} / (\lambda_{min}^{\mathbf{Q}} - \tau)]^{0.5}$ [22]; $\mathbf{Q} = -(\mathbf{PA} + \mathbf{A}^T\mathbf{P})$ and $\mathbf{P}$ are available positive definite matrices, and $\tau > \lambda_{min}^{\mathbf{Q}}$.

Proof: Consider the Lyapunov function $V = \mathbf{x}^T\mathbf{P}\mathbf{x}$ where, $\mathbf{P}$ is a positive definite matrix. Differentiating this function and by using (4), one can write

$$\dot{V} = \mathbf{x}^T(\mathbf{PA} + \mathbf{A}^T\mathbf{P})\mathbf{x} + 2\mathbf{x}^T\mathbf{P}\mathbf{w}. \tag{5}$$

Now, if $\mathbf{A}$ is Hurwitz, then for any given positive definite matrix $\mathbf{Q} > \mathbf{0}_{2n}$ there exists $\mathbf{P} > \mathbf{0}_{2n}$ such that $-\mathbf{Q} = \mathbf{PA} + \mathbf{A}^T\mathbf{P}$ (see Theorem 4.6 in [25]). According to the Young's inequality one has $2\mathbf{x}^T\mathbf{P}\mathbf{w} \leq \tau\mathbf{x}^T\mathbf{x} + \tau^{-1}\mathbf{w}^T\mathbf{P}^2\mathbf{w}$, $\forall \tau > 0$. Substituting this inequality into (5), one can write

$$\dot{V} \leq -(\lambda_{min}^{\mathbf{Q}} - \tau)\mathbf{x}^T\mathbf{x} + \tau^{-1}\lambda_{max}^{\mathbf{P}^2}\mathbf{w}^T\mathbf{w}, \tag{6}$$

If one chooses $\tau > \lambda^{\mathbf{Q}}_{min}$, then integrating (6) yields

$$\int_0^\infty \mathbf{x}^T\mathbf{x}dt \leq \frac{\tau^{-1}\lambda^{\mathbf{P}^2}_{max}}{(\lambda^{\mathbf{Q}}_{min}-\tau)}\int_0^\infty \mathbf{w}^T\mathbf{w}dt + \frac{V_0}{(\lambda^{\mathbf{Q}}_{min}-\tau)}, \quad (7)$$

where, V_0 denotes the initial value of V. After a few mathematical operations, (7) can be written as inequality (4) in [22], which in turn implies that if $w_i, \forall i$, have bounded variations (i.e., belong to L_2 space), then, the system is finite-gain L_2-stable from $\mathbf{w}$ to $\mathbf{x}$ with an induced gain less than γ. ■

Remark 1: According to Theorem 1, the MG system is stable, if *i)* the matrix $\mathbf{A}$ is Hurwitz; *ii)* w_i has bounded varaitions. Referring to Section V-A of [19], and conducting similar procedure based on the Schur complement determinant formula, one can investigate the former. The latter condition, however, holds when the secondary control and the underlying MG system is stable. One can refer to [18], [26] for more info on the stability of such a system.

IV. Convergence Analysis & Online Kron Reduction (Smooth PnP Functionality)

Lemma 1: Any given Laplacian matrix $\mathcal{L}$ has a zero eigenvalue (not necessarily simple), corresponding to the eigenvector $\mathbf{1}_n$. In addition, the equation $\mathcal{L}\mathbf{x} = \mathbf{0}_n$ has a solution in the form $\alpha\mathbf{1}_n$, and $\text{rank}(\mathcal{L}) \leq n-1$ [23].

Lemma 2: If a digraph has a rooted spanning tree, then, $\mathcal{L}$ has a simple zero eigenvalue, i.e., $\text{rank}(\mathcal{L}) = n-1$ [27].

Lemma 3: If a digraph has a rooted spanning tree, then there exists a row vector $\boldsymbol{\mu} = \text{row}\{\mu_i\}$ such that $\boldsymbol{\mu}\mathcal{L} = \mathbf{0}_n^T$. Moreover, if node r is a root, then $\mu_r > 0$, and $\mu_r = 0$, if it is not a root [24].

Lemma 4: Elementary row operations does not chnage the rank and kernel of a matrix [28], [29].

Theorem 2: Suppose that the CN among the DGs of the MG system (4), described in Section II-A, has a rooted spanning tree. If $g_i^f \neq 0$ for at least one root node, then, for a stable MG formulated in (4), all the frequencies converge to the nominal frequency and the proportional active power sharing, established by droop control, is preserved, i.e., $f_i = f_{nom}$ and $m_iP_i = m_jP_j$, $\forall i, j$.

Proof: In steady state, by setting $\dot{\mathbf{x}} = \mathbf{0}_{2n}$ in (4), one has

$$\mathbf{g}_f(\mathbf{f} - \mathbf{f}_{nom}) + \mathbf{g}_\Omega\mathcal{L}\boldsymbol{\Omega} = \mathbf{0}_n, \quad (8)$$

$$\text{or equally: } \begin{bmatrix} g_1^\Omega r_1^\mathcal{L} \\ \vdots \\ g_n^\Omega r_n^\mathcal{L} \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \vdots \\ \Omega_n \end{bmatrix} + \begin{bmatrix} g_1^f\sigma \\ \vdots \\ g_n^f\sigma \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (9)$$

where, $r_1^\mathcal{L}$ is i^{th} row of $\mathcal{L}$ and $\sigma = f_{com} - f_{nom}$ with $f_{com} = f_i, \forall i$ being common frequency of MG in steady state. Assume that r^{th} DG is a root node with $g_r^f \neq 0$ then, from r^{th} row of (9) one has $\sigma = -(g_r^\Omega/g_r^f)r_r^\mathcal{L}\boldsymbol{\Omega}$. Substituting this equation for σ in other rows of (9), one can write

$$g_r^\Omega r_r^\mathcal{L}\boldsymbol{\Omega} + g_r^f\sigma = 0, \quad (10)$$

$$\mathcal{L}_3\boldsymbol{\Omega} = \begin{bmatrix} \vdots \\ g_i^\Omega r_i^\mathcal{L} - \frac{g_i^f}{g_r^f}g_r^\Omega r_r^\mathcal{L} \\ \vdots \end{bmatrix}\boldsymbol{\Omega} = \mathbf{0}_{n-1}, \forall i \neq r. \quad (11)$$

For r^{th} DG which is a root, from Lemma 3 one can write

$$\sum_{i=1}^n \mu_i r_i^\mathcal{L} = \mathbf{0}_n^T \Rightarrow r_r^\mathcal{L} = -\sum_{i=1\&i\neq r}^n \frac{\mu_i}{\mu_r}r_i^\mathcal{L}, \quad (12)$$

where, from Lemma 3, $\mu_i > 0$ if i^{th} DG is root node, and $\mu_i = 0$ if it is not. Equation (12) implies that r^{th} row of $\mathcal{L}$ can be expressed as a linear combination of its other rows. Hence, applying elementary row operations to $\mathcal{L}$ and considering Lemma 4, one can obtain the matrix $\mathcal{L}_1 = [\mathbf{0}_n, \mathcal{L}_2^T]^T$ with the following properties.

$$\begin{cases} \text{rank}(\mathcal{L}_1) = \text{rank}(\mathcal{L}), \\ \ker(\mathcal{L}_1) = \ker(\mathcal{L}), \end{cases} \quad \mathcal{L}_2 = \begin{bmatrix} \vdots \\ r_i^\mathcal{L} \\ \vdots \end{bmatrix}, \forall i \neq r, \quad (13)$$

In addition, since $\mathcal{L}_1 = [\mathbf{0}_n, \mathcal{L}_2^T]^T$, one definitely has

$$\begin{cases} \text{rank}(\mathcal{L}_2) = \text{rank}(\mathcal{L}_1), \\ \ker(\mathcal{L}_2) = \ker(\mathcal{L}_1). \end{cases} \quad (14)$$

On the other hand, substituting (12) for $r_r^\mathcal{L}$ in (11) yields

$$\mathcal{L}_3 = \begin{bmatrix} \vdots \\ g_i^\Omega r_i^\mathcal{L} + \frac{g_i^f}{g_r^f}g_r^\Omega \sum_{i=1\&i\neq r}^n \frac{\mu_i}{\mu_r}r_i^\mathcal{L} \\ \vdots \end{bmatrix}, \forall i \neq r. \quad (15)$$

Looking at (13) and (15), one can see that $\mathcal{L}_3$ can be obtained from $\mathcal{L}_2$ after applying a series of elementary row operations. Therefore, according to (13), (14), and Lemma 4 one has

$$\begin{cases} \text{rank}(\mathcal{L}_3) = \text{rank}(\mathcal{L}_2) = \text{rank}(\mathcal{L}_1) = \text{rank}(\mathcal{L}), \\ \ker(\mathcal{L}_3) = \ker(\mathcal{L}_2) = \ker(\mathcal{L}_1) = \ker(\mathcal{L}). \end{cases} \quad (16)$$

According to (16), one can say that the equations (11) and $\mathcal{L}\boldsymbol{\Omega} = \mathbf{0}_n$ have the same general solution. On the other hand, from Lemma 1 and Lemma 2, the solution of $\mathcal{L}\boldsymbol{\Omega} = \mathbf{0}_n$ is in the form $\boldsymbol{\Omega} = \alpha\mathbf{1}_n$, and $\mathcal{L}$ has a simple eigenvalue i.e., $\text{rank}(\mathcal{L}) = n-1$. Accordingly, (11) has a solution in the form $\boldsymbol{\Omega} = \alpha\mathbf{1}_n$ and $\text{rank}(\mathcal{L}_3) = n-1$, as well. This, in turn, implies that $\Omega_i = \Omega_j, \forall i, j$, and thus $r_r^\mathcal{L}\boldsymbol{\Omega} = 0$. Since g_r^Ω, g_r^f both are positive, substituting $r_r^\mathcal{L}\boldsymbol{\Omega} = 0$ into (10) yields $\sigma = f_i - f_{nom} = 0$. Considering this equation and the result $\Omega_i = \Omega_j, \forall i, j$ one can infer from (1) that $f_i = f_{nom}$ and $m_iP_i = m_jP_j$, $\forall i, j$. ■

In the conventional PnP scenarios (see e.g., [12]–[16], [19]), it is assumed that when a DG gets disconnected, its communication links are all interrupted. This may hamper the CN's connectivity. To avoid probable disconnection in the CN when a DG gets unplugged, an algorithm is introduced in what follows. Once k^{th} DG gets disconnected, *i)* does not need to cooperate with other DGs to reach consensus, i.e., $g_i^\Omega = 0$; *ii)* does not send the local data Ω_k to its out-neighbors; instead, it sends the average of incoming data, received from in-neighbors, i.e., $\bar{\Omega}_k = \frac{1}{d_k}\sum_{j\in N_k} a_{kj}\Omega_j$. See Algorithm 1.

Theorem 3: Suppose that the CN among the DGs of the MG system (4), described in Section II-A, has a rooted spanning

Algorithm 1 Secondary Control Implementation of k^{th} DG, Augmented with Online Kron Reduction Algorithm.

initialization:
1: Activate the secondary control ▷ Activation time
2: Send the local data Ω_k to out-neighbors (receivers)
3: Receive the in-neighbors (senders) data $\Omega_j, \forall j \in N_k$
4: Compute u_k & Ω_k in (2) & (3)
5: **while** secondary control is active **do**
6: Receive the in-neighbors (senders) data $\Omega_j, \forall j \in N_k$
7: **if** DG is connected **then** ▷ Plugged-in
8: Compute u_k & Ω_k in (2) & (3)
9: Send the local data Ω_k to out-neighbors (receivers)
10: **end if**
11: **if** DG is disconnected **then** ▷ Plugged-out
12: Set $g_k^{\Omega} = 0$ and compute u_k & Ω_k in (2) & (3)
13: Compute the average data $\bar{\Omega}_k = \frac{1}{d_k}\sum_{j\in N_k}\Omega_j$
14: Send the data $\bar{\Omega}_k$ to out-neighbors (receivers)
15: **end if**
16: **end while**

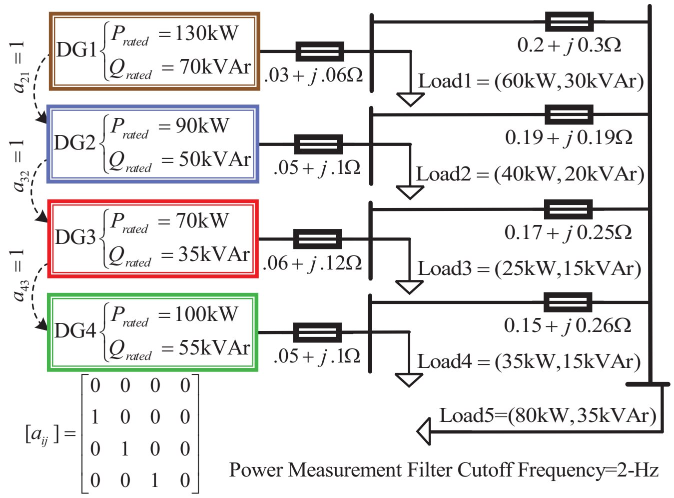

Fig. 1. Test MG system and its specifications. $r + jx$ denotes the related line impedence and $[a_{ij}]$ is the CN Adjacency matrix. Communication links and electrical connections are denoted by dashed and solid lines, respectively.

tree. If the CN topology is *invariant and locally known*, then the proposed online Kron reduction in Algorithm 1 (lines 11-15), results in a new CN with a reduced Laplacian matrix, and the existing spanning tree is preserved.

Proof: To investigate the effects of this algorithm on CN topology and its Laplacian matrix, let us define the vector of neighborhood errors $\mathbf{z} = \text{col}\{z_i\} = -\mathcal{L}\boldsymbol{\Omega}$, employed in (3)-(4), where, i^{th} DG's local neighborhood errors is

$$z_i = \sum_{j\in N_i} a_{ij}(\Omega_j - \Omega_i) = -d_i\Omega_i + a_{ik}\Omega_k + \sum_{j\neq k} a_{ij}\Omega_j. \quad (17)$$

Consider that k^{th} DG is plugged-out. If the CN is *invariant and locally known* by k^{th} DG, one can compute $\bar{\Omega}_k = \frac{1}{d_k}\sum_{j\in N_k} a_{kj}\Omega_j$ and substitute it for Ω_k in (17). Then, the neighborhood error of other DGs can be written as

$$z_i = -d_i\Omega_i + \sum_{j\neq k}(a_{ij} + \frac{a_{ik}a_{kj}}{d_k})\Omega_j, \forall i \& j \neq k. \quad (18)$$

One can write (18) in the compact form $\mathbf{z}_{n-1} = -\mathcal{L}_{red}\boldsymbol{\Omega}_{n-1}$. Consider the reduced matrix $\mathcal{L}_{red} = [l_{ij}] \in \mathbb{R}^{(n-1)\times(n-1)}$, obtained from the foregoing procedure. From (18) one has

$$\begin{cases} l_{ij} = (\sum_{j\neq k} a_{ij}) + a_{ik}, & i = j, \\ l_{ij} = -(a_{ij}|_{j\neq k}) - (a_{ik}a_{kj}/d_k), & i \neq j. \end{cases} \quad (19)$$

Note that for the DGs, $a_{ij}|_{j\neq k}$ in both pre-outage and post-outage CN topology is the same, while, a_{ik} and a_{kj} only correspond with the pre-outage CN topology. According to (19), one can write $l_{ii} + \sum_{j=1}^{n-1} l_{ij} = 0$, where, the in-degree of the disconnected DG i.e., $d_k = \sum_j a_{kj}$ is used. Hence, the reduced matrix $\mathcal{L}_{red}$ is a Laplacian matrix (by definition) for the post-outage communication network (k^{th} DG's outage), with the Adjacency matrix $\mathcal{A}_{red} = [a_{ij}^{red}]$, where, $a_{ij}^{red} = a_{ij} + \frac{a_{ik}}{d_k}a_{kj}$. Moreover, under the proposed Kron reduction method, the communication medium of the unplugged DG, e.g., k^{th} DG here, interconnects the neighboring DGs (in-neighbors to out-neighbors); therefore, the data definitely finds a path to cross the disconnected DG. In other words, under the proposed algorithm, any direct path crossing the disconnected DG is preserved, so are the former available spanning trees.■

Remark 2: To compute $\bar{\Omega}_k$ and apply the CN reconfiguration in Algoritm 1, k^{th} DG only requires the information of its in-nieghbors, which are *locally known* to it. Thus, the online Kron reduction strategy is easily *distributed and scalable*.

Remark 3: Theorem 3 investigates the existence of the spanning tree, and does not discuss the variation of the garph algebraic connectivity (reflected by the second smallest eigenvalue of the Laplacian matrix [23]). However, interested reader can refer to [30] for more info about the impacts of Kron reduction on Laplacian matrices.

V. Case Studies & Simulation Results

In order to verify the effectiveness of the proposed controller and the online Kron reduction, a 50-Hz, 220-Volt islanded MG with four voltage-controlled DGs (with nonidentical capacities) is simulated in MATLAB/Simscape Electrical™ software environment. Fig. 1 depicts the test MG, where the specifications of different parts are also given. Two studies are presented in this section as follows. *Study 1)* Controller performance assessment over activation and load change stages and validation of Theorem 2, and *Study 2)* assessment of the PnP functionality with and without the online Kron reduction strategy, presented in Theorem 3.

A. Study 1: Controller Performance & Theorem 2 Verification

In this study, the gains g_i^{Ω} are selected as $[g_1^{\Omega}, g_2^{\Omega}, g_3^{\Omega}, g_4^{\Omega}] = [20, 18, 16, 14]$. Fig. 2 indicates the simulation results. Prior to $t = 5s$, the MG is engaged with droop control, the secondary Hz-Watt controller is activated at $t = 5s$, load 5 becomes 2 times grater at $t = 10s$ and restores to its initial value at $t = 15s$, and load 1 gets plugged-out & plugged-in at $t = 15s$ &

$t = 20s$, respectively. From Fig. 1, the CN has a spanning tree, rooted at 1st DG. Accordingly, in what follows, the described scenario is simulated under two different cases.

1) Case 1: Theorem 2 Assumption Holds: In this case the gains g_i^f are selected as $[g_1^f, g_2^f, g_3^f, g_4^f] = [12, 0, 0, 6]$, which means that for the rooted DG the frequency feedback gain is positive, i.e., $g_1^f \neq 0$. According to Fig. 2(a)-(b), the proposed controller well restores the frequencies back to 50-Hz and shares the active powers properly (equal actual/maximum powers), even in the presence of multiple severe load changes. This implies that Theorem 2 holds.

2) Case 2: Theorem 2 Assumption does not Hold: In this case the gains g_i^f are selected as $[g_1^f, g_2^f, g_3^f, g_4^f] = [0, 10, 8, 6]$, which means that for the rooted DG the frequency feedback gain is zero, i.e., $g_1^f = 0$, although the other non-root DGs' frequency gains are positive. According to Fig. 2(c)-(d), in this case, the proposed controller neither restores the frequencies back to 50-Hz nor shares the active powers properly (non-equal actual/maximum powers), although the MG system is stable and converges to an operating point. This implies that Theorem 2 does not hold.

B. Study 2: Plug-and-Play Functionality with & without the Online Kron Reduction Strategy i.e., Theorem 3 Verification

In this study, the gains are selected as $[g_1^\Omega, g_2^\Omega, g_3^\Omega, g_4^\Omega] = [20, 18, 16, 14]$ & $[g_1^f, g_2^f, g_3^f, g_4^f] = [12, 10, 8, 6]$. It is assumed that the MG is subjected to the proposed secondary control. The results are given in Fig. 3. At $t = 5s$, 3rd DG gets disconnected and returns back to the MG at $t = 15s$. In order to highlight the effectiveness of the proposed online Kron reduction strategy, it is assumed that the load 1 is switched off at $t = 10s$, when 3rd DG does not inject power to MG. From Fig. 1, the CN has a spanning tree, rooted at 1st DG. Therefore, interruption of 3rd DG's communication links, because of its outage, cuts this tree such that 4th becomes isolated. This PnP scenario is simulated for two different cases.

1) Case 1: Disabled Online Kron Reduction: In this case, following the conventional approach in the literature, it is assumed that once 3rd DG gets disconnected, its corresponding communication links all are interrupted. According to Fig. 3(a)-(b), once the DG leaves the secondary controlled MG, although the frequencies all are well-controlled, 4th DG does not participate in proportional Watt sharing (unlike 1st and 2nd DGs), until 3rd DG gets plugged-in. This happens as a result of the spanning tree interruption.

2) Case 2: Enabled Online Kron Reduction: Herein, the former case scenario is repeated, but in the presence of the Kron reduction strategy. From Fig. 3(c)-(d), once 3rd DG leaves the MG, subject to the proposed strategy in Algorithm 1, all the remaining DGs participate in both Hz & Watt control tasks. This validates Theorem 3, i.e., the spanning tree is preserved and the requirements of Theorem 2 all are satisfied.

VI. Conclusion

The secondary frequency control of autonomous microgrids is investigated, from a cybernetic topological point-of-view. Firstly, by concentrating on the droop equation and communication layer, stability of the system with L_2-stability approach is analyzed. Secondly, a comprehensive convergence analysis is conducted and minimum cyber requirements for achieving proper Hz restoration and Watt sharing are extracted. On top of the mentioned analyses, an online Kron reduction strategy is proposed which preserves the existing spanning trees from interruptions induced by DGs outages. A theoretical analysis, supporting this online Kron reduction strategy is presented, as well. The effectiveness of the proposed control strategy is validated by simulating a test microgrid system in MATLAB/Simscape Electrical™ software environment.

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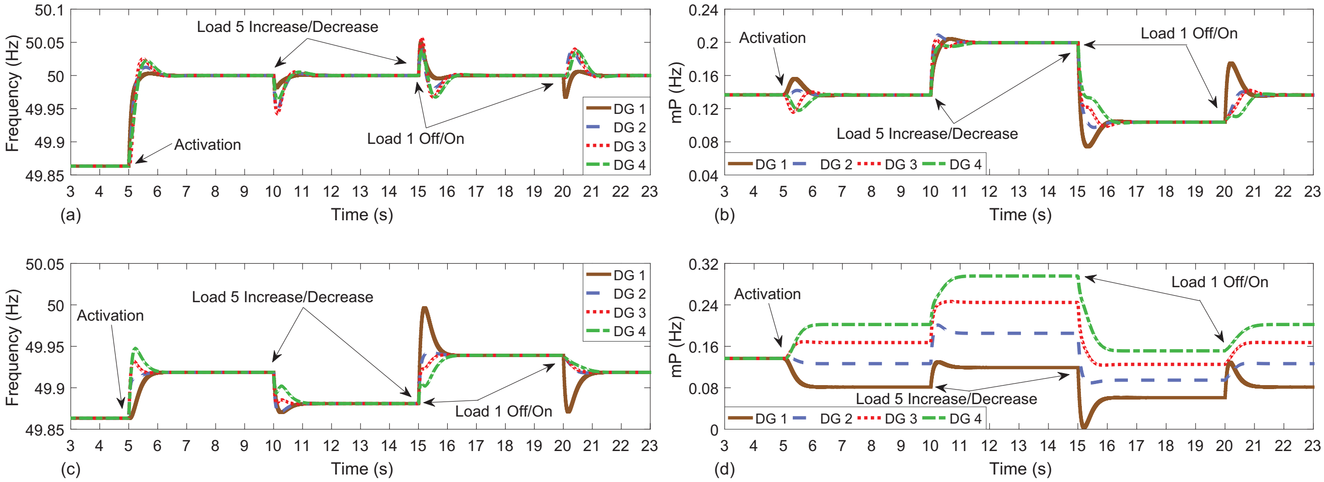

Fig. 2. Performance of the controller; (a)-(b) Case 1 ($g_1^f \neq 0$), and (c)-(d) Case 2 ($g_1^f = 0$). (a)&(c) frequencies and (b)&(d) $m_i P_i, \forall i$.

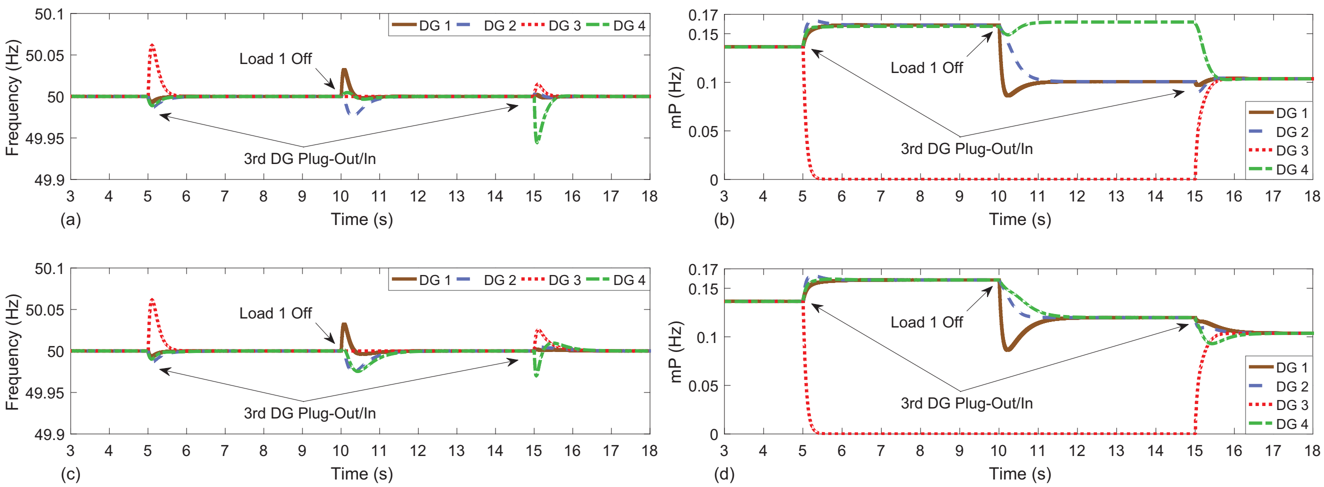

Fig. 3. Plug and Play functionality of 3rd DG; (a)-(b) Case 1 (disabled online Kron reduction strategy), and (c)-(d) Case 2 (enabled online Kron reduction strategy). (a)&(c) frequencies and (b)&(d) $m_i P_i, \forall i$.

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