Scalable Robust Voltage Control of DC Microgrids with Uncertain Constant Power Loads

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***Abstract*—Constant power loads (CPLs) impose instability issues in DC microgrids due to their negative impedance characteristics. This paper studies the problem of voltage control design of DC microgrids with CPLs. It is assumed that the power of CPLs is uncertain and belongs to a given interval leading to an infinite number of equilibrium points of the system. We develop a polytope model for DC microgrids with uncertain CPLs. Using this model, a robust two-degree-of-freedom (2DOF) feedback-feedforward voltage control framework is then proposed. The voltage controller is obtained by a solution of a set of linear matrix inequalities. The voltage control design strategy for each distributed generation (DG) unit is scalable and independent of the other DGs. The effectiveness of the proposed control approach is evaluated through simulation studies in MATLAB/SimPowerSystems Toolbox.**

***Index Terms*—Constant power load (CPL), DC microgrids, Linear Matrix Inequality (LMI), load uncertainty, polytopic systems, robust voltage control, scalable control design.**

I. Introduction

DC microgrids have been broadly applied to various fields such as electric vehicles, data centers, aircrafts, spacecrafts, telecom systems, submarines, high efficiency households, and hybrid energy storage systems [1], [2]. The main advantages of DC microgrids compared to AC microgrids include higher efficiency and natural interface with renewable energy sources, electronic loads, and energy storage systems [2]. Moreover, the control systems of DC microgrids are less complex than those of AC microgrid systems where there exist several issues such as frequency regulation, reactive power control, and unbalanced load conditions.

For the efficient and reliable operation of DC microgrids, there exist several major challenges from a control perspective which must be addressed. One of the main challenges is stability issues caused by constant power loads (CPL) which demand constant power regardless of their input voltage. Examples of CPLs are loads interfaced through tightly regulated power converters such as electronic devices and electric drives [3], [4]. In a CPL, the input current decreases (increases) when the input voltage increases (decreases) [4]. Therefore, CPLs exhibit negative impedance characteristics corresponding to first-third quadrant hyperbolas in the voltage-current plane [5]. Therefore, CPLs provide a completely different scenario than classical impedance loads [6], [7].

The destabilizing effects of CPLs in DC microgrids have promoted a surge of research efforts to cope with this issue. The proposed approaches are categorized into passive (e.g. [8]–[11]) and active methods (e.g. [12]–[19]) [3]. Although passive approaches are simple, they introduce power losses and reduce efficiency. Active approaches employ feedback control-based methods [3]. A few solutions are based on nonlinear control strategies to overcome the instability problem of CPLs, e.g. [12], [13]. The majority of the literature has been devoted to stability of the linearized system around a fixed equilibrium point calculated according to a nominal value of CPLs [15], [17], [19]. For a comprehensive review of different active control techniques, one can refer to [3].

Most of the existing approaches are valid for DC microgrids with resistive loads (e.g. [1], [20]–[24]) and they have not considered CPLs. Literature on active control approaches for DC microgrids with CPLs is generally limited to specific cases: 1) single-bus DC microgrids with parallel components [15]–[19] 2) control design/stability analysis based on a single equilibrium point [5], [7], [15], [17], [19] 3) non-scalable control design/stability analysis [17]. In addition to these specific cases, the majority of the literature is devoted to stability analysis of DC microgrids with CPLs and the results for robust control synthesis are rather limited. The proposed approach in [14] has focused on the stability analysis of DC microgrids for a given range of CPLs; however, the uncertainty in CPLs has not been considered in the controller design procedure. The existing work in [16] is limited to DC microgrids with multiple parallel-connected DC-DC converters and it is not applicable to DC microgrids with a general structure. The proposed work in [17] has not considered robustness in relation to uncertainty in CPLs.

The main assumption of most existing works in the literature is that the power of constant power loads is accurately known which results in a fixed equilibrium point. However, this assumption is not desirable in real applications. It is difficult to obtain exact information on equilibrium points of DC microgrids because they depend on uncertain CPLs. Different power of CPLs leads to different equilibria. If it is assumed that the power of CPLs varies in a given interval, the set of equilibria of the system is typically infinite. One possible solution could be to consider stability analysis/control synthesis for a finite grid of equilibria. However, we may miss key equilibrium points which are achievable by the DC microgrids. Therefore, it is necessary to provide an accurate model and control design strategy which can cover the uncertainty in CPLs.

In this paper, we address the problem of voltage control design for islanded DC microgrids with a general topology. The DC microgrids consist of DC-DC power electronics converters, energy storage systems, and constant power loads. It is assumed that the power of each CPL takes arbitrary values in a given interval leading to an infinite set of equilibria. The uncertainties of the power of CPLs are modeled as a convex

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hull of a set of known vertices. The voltage controller has a hierarchical structure where the primary voltage controller of each DG tracks reference voltage signals generated/sent by a Power Management System (PMS). PMS is a centralized controller where as the primary voltage control has a decentralized architecture. The main focus of this paper is on primary control consisting of local voltage controllers associated with each DG. The local voltage controllers include two terms: feedback and feedforward control. The feedback controllers are designed such that all the equilibrium points of DC microgrids are stable. The feedforward controllers are responsible to provide desired performance according to IEEE standards [25]. The main feature of the proposed control strategy is that the design procedure is scalable. In other words, the primary voltage control system design of each DG unit is independent of other DGs. The control problem is formulated as a set of Linear Matrix Inequalities (LMIs) which can be easily solved by efficient solvers.

The contributions of this paper are as follows: 1) In terms of modeling, opposed to the works in [20] and [21], our proposed modeling approach captures the uncertainty in constant power loads as well as resistive loads. We develop a general modeling approach in terms of a polytopic model for DC microgrids with uncertain CPLs in state space framework. The model represents the dynamics of the complete system including DC-DC converters, uncertain CPLs, uncertain resistive loads, and lines. 2) In terms of control design, unlike the works done in [5], [7], [15], [17], [19], we have considered uncertainty in CPLs. The challenging problem of robust voltage control of DC microgrids with uncertain CPL is formulated as a set of LMIs with structural constraints on some decision variables. 3) The control design procedure is scalable as opposed to the proposed approach in [17]. As a result, each local voltage controller is designed only based on its own DG while the stability of whole DC microgrid system is guaranteed. 4) The voltage controllers guarantee robust stability and robust performance of DC microgrids with uncertain loads.

The paper is organized as follows. Section II presents a mathematical model of DC microgrids with uncertain CPLs. In Section III, a scalable robust voltage controller is designed for DC microgrids. Simulation case studies are considered in Section IV. Section V concludes the paper.

The notation used in this paper is standard. In particular, matrices I and 0 are the identity matrix and the zero matrix of appropriate dimensions, respectively. The symbols A^T and $\star$ denote the transpose of matrix A and symmetric blocks in block matrices, respectively. For symmetric matrices, $P > 0$ and $P < 0$ respectively indicate the positive-definiteness and the negative-definiteness.

II. Model of DC Microgrids

We consider an islanded DC microgrid system composed of N DGs. Each DG includes a DC voltage source, a DC-DC converter, and a local load connected at Point of Common Coupling (PCC). Loads are assumed to be either constant power or resistive. DGs are connected via distribution lines modeled by RL networks. The proposed control approach in this paper is general and can be applied to DC microgrids with different types of DC-DC converters. However, to simplify the mathematical representation of DC microgrids, we only consider the analytical model of DC microgrids with only buck converters. Similar to the work in [20], in the case that a boost converter is used, the appropriate model of the converter needs to be replaced.

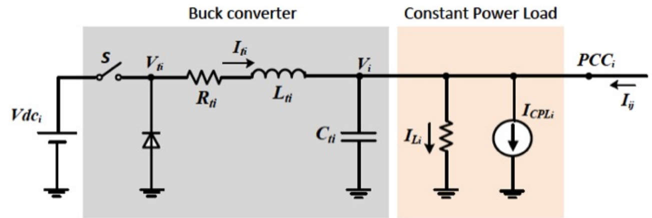

Fig. 1: A schematic diagram of a DG with a constant power load.

DC microgrids form a network represented by graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V}$ and $\mathscr{E}$ are the sets of vertices and edges, respectively. Each element in the vertex set $\mathscr{V}$ represents a DG and each element in the edge set $\mathscr{E}$ represent the distribution line between the corresponding DGs. If $(i, j) \in \mathscr{E}$, DG j is a neighbor of DG i. We assume that DG i is connected to a set of N_i different DGs.

A. Dynamical Model of a DG

This subsection is about the development of a mathematical model of a DG unit with a resistive load R_i and a constant power load as shown in Fig. 1. The dynamics of DG i connected to $N_i \subset \{1, \ldots, N\}$ units are given by:

$$
\begin{aligned}
\frac{dV_i}{dt} &= \frac{1}{C_{t_i}} I_{t_i} - \frac{1}{C_{t_i}} I_{CPL_i} - \frac{1}{C_{t_i} R_i} V_i + \sum_{j \in N_i} \frac{1}{C_{t_i}} I_{ij} \\
\frac{dI_{t_i}}{dt} &= -\frac{1}{L_{t_i}} V_i - \frac{R_{t_i}}{L_{t_i}} I_{t_i} + \frac{d_{buck_i}}{L_{t_i}} V_{dc_i}
\end{aligned} \tag{1}
$$

where $I_{CPL_i} = \frac{P_{CPL_i}}{V_i}$ and V_i, P_{CPL_i}, I_{CPL_i}, I_{ij}, d_{buck_i} are voltage at PCC i, the power of CPL i connected at PCC i, the current of CPL i, the current of the distribution line between PCC i and PCC j, and the duty cycle of the buck converter i, respectively. The distribution line ij modeled by an RL network with parameters R_{ij} and L_{ij} is mathematically described as follows:

$$
\frac{dI_{ij}}{dt} = -\frac{R_{ij}}{L_{ij}} I_{ij} + \frac{1}{L_{ij}} V_j - \frac{1}{L_{ij}} V_i \tag{2}
$$

where $j \in N_i$ and V_j is the voltage signal at PCC j. Under the assumption of quasi-stationary dynamics for the distribution lines [26], that is $\frac{dI_{ij}}{dt} = 0$), the line dynamics are described as follows:

$$
I_{ij} = \frac{V_j - V_i}{R_{ij}} \tag{3}
$$

Since the line impedance in DC systems is mainly resistive, the above assumption is reasonable. The dynamics of DG i with quasi-stationary distribution lines are given by:

$$
\begin{aligned}
\frac{dV_i}{dt} &= \frac{1}{C_{t_i}} I_{t_i} - \frac{1}{C_{t_i}} I_{CPL_i} - \frac{1}{C_{t_i} R_i} V_i + \sum_{j \in N_i} \frac{1}{C_{t_i} R_{ij}} (V_j - V_i) \\
\frac{dI_{t_i}}{dt} &= -\frac{1}{L_{t_i}} V_i - \frac{R_{t_i}}{L_{t_i}} I_{t_i} + \frac{d_{buck_i}}{L_{t_i}} V_{dc_i}
\end{aligned} \tag{4}
$$

We assume that the system (4) with CPLs admits an equilibrium, i.e. there exist constant signals $(\bar{V}_i, \bar{V}_j, \bar{I}_{t_i}, \bar{d}_{buck_i})$ such that:

$$\begin{aligned} &\frac{1}{C_{t_i}}\bar{I}_{t_i} - \frac{1}{C_{t_i}}\bar{I}_{CPL_i} - \frac{1}{C_{t_i}R_i}\bar{V}_i + \sum_{j\in N_i}\frac{1}{C_{t_i}R_{ij}}(\bar{V}_j - \bar{V}_i) = 0 \\ &-\frac{1}{L_{t_i}}\bar{V}_i - \frac{R_{t_i}}{L_{t_i}}\bar{I}_{t_i} + \frac{\bar{d}_{buck_i}}{L_{t_i}}V_{dc_i} = 0 \end{aligned} \tag{5}$$

Linearization of the dynamic equations in (4) around the equilibrium results in the following linear model:

$$\begin{aligned} \frac{d\tilde{V}_i}{dt} &= \frac{1}{C_{t_i}}\tilde{I}_{t_i} + \frac{P_{CPL_i}}{C_{t_i}\bar{V}_i^2}\tilde{V}_i - \frac{1}{C_{t_i}R_i}\tilde{V}_i + \sum_{j\in N_i}\frac{1}{C_{t_i}R_{ij}}(\tilde{V}_j - \tilde{V}_i) \\ \frac{d\tilde{I}_{t_i}}{dt} &= -\frac{1}{L_{t_i}}\tilde{V}_i - \frac{R_{t_i}}{L_{t_i}}\tilde{I}_{t_i} + \frac{\tilde{d}_{buck_i}}{L_{t_i}}V_{dc_i} \end{aligned} \tag{6}$$

where $\tilde{V}_i = V_i - \bar{V}_i$, $\tilde{I}_{t_i} = I_{t_i} - \bar{I}_{t_i}$, $\tilde{V}_j = V_j - \bar{V}_j$, and $\tilde{d}_{buck_i} = d_{buck_i} - \bar{d}_{buck_i}$. The linearized dynamics of DG i are presented in the state space framework as follows:

$$\begin{aligned} \dot{x}_{g_i} &= A_{g_{ii}}x_{g_i} + \sum_{j\in N_i} A_{g_{ij}}x_{g_j} + B_{g_i}u_i \\ y_i &= C_{g_i}x_{g_i} \end{aligned} \tag{7}$$

where

$$\begin{aligned} A_{g_{ii}} &= \begin{bmatrix} -\frac{1}{C_{t_i}}\left(\sum\limits_{j\in N_i}\frac{1}{R_{ij}} + \frac{1}{R_i} - \frac{P_{CPL_i}}{\bar{V}_i^2}\right) & \frac{1}{C_{t_i}} \\ -\frac{1}{L_{t_i}} & -\frac{R_{t_i}}{L_{t_i}} \end{bmatrix} \\ A_{g_{ij}} &= \begin{bmatrix} \frac{1}{R_{ij}C_{t_i}} & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{g_i} = \begin{bmatrix} 0 \\ \frac{1}{L_{t_i}} \end{bmatrix} \\ C_{g_i} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned} \tag{8}$$

where $x_{g_i} = \begin{bmatrix} \tilde{V}_i & \tilde{I}_{t_i} \end{bmatrix}^T$ is the state of DG i, $x_{g_j} = \begin{bmatrix} \tilde{V}_j & \tilde{I}_{t_j} \end{bmatrix}^T$ is the state of DG j, $u_i = \tilde{d}_{buck_i}V_{dc_i}$ is the input of DG i, and $y_i = \tilde{V}_i$ is the output of DG i. It is notable that if DG i does not contain a CPL, it is enough to consider $P_{CPL_i} = 0$ in (8).

B. A General Model of a DG with Uncertain Loads

We assume that DG i supplies an uncertain resistive load, i.e. $\underline{R}_i \le R_i \le \bar{R}_i$, and a CPL whose power is not precisely known, but it belongs to a given interval, i.e. $\underline{P}_{CPL_i} \le P_{CPL_i} \le \bar{P}_{CPL_i}$. As a result, the equilibrium of the dynamical model in (4) is not fixed and the model includes an infinite set of equilibria. In order to consider all the equilibrium points in the dynamics of DG i, a polytopic model is developed. We need to consider the following four vertices:

$$\begin{aligned} A_{i_1} &= \begin{bmatrix} -\frac{1}{C_{t_i}}\left(\sum\limits_{j\in N_i}\frac{1}{R_{ij}} + \frac{1}{\bar{R}_i} - \frac{\bar{P}_{CPL_i}}{\bar{V}_i^2}\right) & \frac{1}{C_{t_i}} \\ -\frac{1}{L_{t_i}} & -\frac{R_{t_i}}{L_{t_i}} \end{bmatrix} \\ A_{i_2} &= \begin{bmatrix} -\frac{1}{C_{t_i}}\left(\sum\limits_{j\in N_i}\frac{1}{R_{ij}} + \frac{1}{\bar{R}_i} - \frac{\underline{P}_{CPL_i}}{\bar{V}_i^2}\right) & \frac{1}{C_{t_i}} \\ -\frac{1}{L_{t_i}} & -\frac{R_{t_i}}{L_{t_i}} \end{bmatrix} \\ A_{i_3} &= \begin{bmatrix} -\frac{1}{C_{t_i}}\left(\sum\limits_{j\in N_i}\frac{1}{R_{ij}} + \frac{1}{\underline{R}_i} - \frac{\bar{P}_{CPL_i}}{\bar{V}_i^2}\right) & \frac{1}{C_{t_i}} \\ -\frac{1}{L_{t_i}} & -\frac{R_{t_i}}{L_{t_i}} \end{bmatrix} \\ A_{i_4} &= \begin{bmatrix} -\frac{1}{C_{t_i}}\left(\sum\limits_{j\in N_i}\frac{1}{R_{ij}} + \frac{1}{\underline{R}_i} - \frac{\underline{P}_{CPL_i}}{\bar{V}_i^2}\right) & \frac{1}{C_{t_i}} \\ -\frac{1}{L_{t_i}} & -\frac{R_{t_i}}{L_{t_i}} \end{bmatrix} \end{aligned} \tag{9}$$

All the equilibria lie in the following matrix which is the convex combination of the above matrices:

$$A_{g_{ii}}(\lambda) = \sum_{l=1}^{q}\lambda_l A_{i_l} \tag{10}$$

where $q = 4$ and λ_l belongs to the following set:

$$\Lambda_q = \left\{ \lambda_l \ge 0, \quad \sum_{l=1}^{q}\lambda_l = 1 \right\} \tag{11}$$

Therefore, the dynamics of a DG with uncertain loads are described by the following polytopic model:

$$\begin{aligned} \dot{x}_{g_i} &= A_{g_{ii}}(\lambda)x_{g_i} + \sum_{j\in N_i} A_{g_{ij}}x_{g_j} + B_{g_i}u_i \\ y_i &= C_{g_i}x_{g_i} \end{aligned} \tag{12}$$

where the state space matrices are given in (8)-(10).

III. Robust Voltage Control Synthesis of DC Microgrids with Uncertain CPLs

This section is devoted to the design of decentralized voltage controllers for N-bus DC microgrids with uncertain loads. The main duty of the controllers is to provide stability and desired performance of DC microgrids with CPLs, where the power of CPLs is assumed to be uncertain, but physically bounded in given intervals.

A. Structure of Proposed Voltage Controllers

The proposed voltage control strategy in this paper is based on a hierarchical control framework which consists of two main layers with different time-scales and architectures. The first layer referred to as Power Management System (PMS) is a centralized control. PMS centrally solves a power flow problem and broadcasts the power set points for each DG to properly share the power among DGs based on either a cost function associated with each DG or a market signal [27]. The set points are then transmitted to the second layer called primary voltage control. Primary control is a decentralized voltage control composed of local voltage controllers associated with each DG. The duty of the primary control is to stabilize the voltage of DC microgrids and compensate for the voltage deviations in steady state conditions.

B. Stabilizing voltage controllers

Suppose that PMS sends the voltage reference V_{ref_i} to the lower layer. The primary control includes an integrator to track the reference signals V_{ref_i}. The dynamics of the integrator are given by:

$$\begin{aligned} \dot{v}_i &= V_{ref_i} - V_i \\ &= -C_{g_i}x_{g_i} \end{aligned} \tag{13}$$

The augmented model of DG i with the integrator dynamics in (13) is given by:

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_{g_i} \\ \dot{v}_i \end{bmatrix} &= \hat{A}_{g_{ii}} \begin{bmatrix} x_{g_i} \\ v_i \end{bmatrix} + \sum_{j \in N_i} \hat{A}_{g_{ij}} \begin{bmatrix} x_{g_j} \\ v_j \end{bmatrix} + \hat{B}_{g_i} u_i \\
y_i &= \hat{C}_{g_i} \begin{bmatrix} x_{g_i} \\ v_i \end{bmatrix}
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
\hat{A}_{g_{ii}} &= \begin{bmatrix} A_{g_{ii}}(\lambda) & 0 \\ -C_{g_i} & 0 \end{bmatrix}, \quad \hat{A}_{g_{ij}} = \begin{bmatrix} A_{g_{ij}} & 0 \\ 0 & 0 \end{bmatrix} \\
\hat{B}_{g_i} &= \begin{bmatrix} B_{g_i} \\ 0 \end{bmatrix}, \quad \hat{C}_{g_i} = \begin{bmatrix} C_{g_i} & 0 \end{bmatrix}
\end{aligned} \tag{15}
$$

The main objective is to design a voltage control gain K_i for each DG such that the stability of the DC microgrids with uncertain loads is guaranteed. The controller K_i receives a feedback from x_{g_i} and v_i. The control law is expressed as follows:

$$
u_i = K_i \begin{bmatrix} x_{g_i} \\ v_i \end{bmatrix} \tag{16}
$$

The structure of the controller is fully decentralized which means that the controller K_i only receives information from DG i.

C. Closed-loop Dynamics of an N-bus DC Microgrid with a Decentralized Voltage Control

An N-bus DC microgrid composed of uncertain CPLs connected at PCCs with the local voltage controllers in (16) is described as follows:

$$
\begin{aligned}
\dot{\hat{x}}(t) &= (\hat{A}(\lambda) + \hat{B}K)\hat{x} \\
\hat{y}(t) &= \hat{C}\hat{x}(t)
\end{aligned} \tag{17}
$$

where $\hat{x} = [\hat{x}_{g_1}^T \dots \hat{x}_{g_N}^T]^T$, $\hat{x}_{g_i} = \begin{bmatrix} x_{g_i}^T & v_i \end{bmatrix}^T$, $\hat{y} = [y_1 \dots y_N]^T$, and

$$
\begin{aligned}
\hat{A}(\lambda) &= \begin{bmatrix} \hat{A}_{g_{11}}(\lambda) & \hat{A}_{g_{12}} & \cdots & \hat{A}_{g_{1N}} \\ \hat{A}_{g_{21}} & \hat{A}_{g_{22}}(\lambda) & \cdots & \hat{A}_{g_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{A}_{g_{N1}} & \hat{A}_{g_{N2}} & \cdots & \hat{A}_{g_{NN}}(\lambda) \end{bmatrix} \\
\hat{B} &= diag\left(\hat{B}_{g_1}, \dots, \hat{B}_{g_N}\right) \\
\hat{C} &= diag\left(\hat{C}_{g_1}, \dots, \hat{C}_{g_N}\right) \\
K &= diag\left(K_1, \dots, K_N\right)
\end{aligned} \tag{18}
$$

Note that $\hat{A}_{g_{ij}} = 0$ iff $j \notin N_i$. The decentralized controller K stabilizes the DC microgrid systems with uncertain resistive loads $\underline{R}_i \le R_i \le \bar{R}_i$ and constant power loads $\underline{P}_{CPL_i} \le P_{CPL_i} \le \bar{P}_{CPL_i}$ if and only if the polytopic closed-loop state matrix $\hat{A}(\lambda) + \hat{B}K$ is stable for all values of $\lambda \in \Lambda_q$. In the next subsection, we explain how to design the decentralized stabilizing voltage control gains K_i.

D. Scalable Voltage Controller Design of DC Microgrids with Uncertain CPLs

If the coupling terms $A_{g_{ij}}$ are zero, the stability of the overall microgrid system in (17) and (18) can be ensured by the stability of each DG. The main challenge is how to design the local voltage control gains K_i in a scalable approach such that the closed-loop stability of the overall closed-loop system is guaranteed. In the following theorem, we present the conditions for the design of stabilizing local primary voltage controllers K_i which allow us to guarantee the stability of the closed-loop system in (17) and (18).The feedback gains K_i are designed via the following theorem.

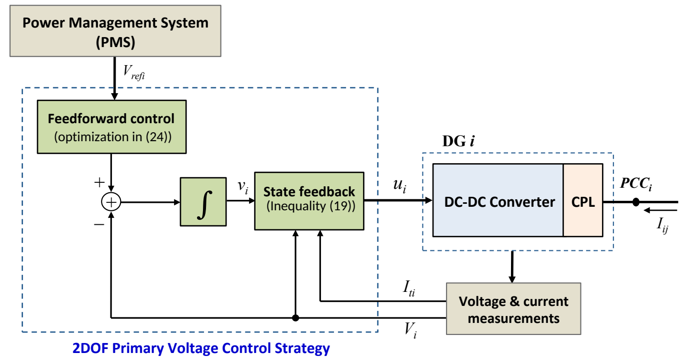

Fig. 2: Block diagram of the proposed voltage control framework for DG i.

Theorem 1. *The local voltage controllers K_i stabilize the closed-loop DC microgrid system with uncertain loads described in (14)-(15) if there exist Lyapunov matrices $P_i^l > 0$, slack matrices (Y_i, G_i), and a small positive scalar ε such that*

$$
\begin{bmatrix} \hat{A}_{g_{ii}}^l G_i + G_i^T (\hat{A}_{g_{ii}}^l)^T + \hat{B}_{g_i} Y_i + Y_i^T \hat{B}_{g_i}^T & \star \\ P_i^l - G_i + \varepsilon(\hat{A}_{g_{ii}}^l G_i + \hat{B}_{g_i} Y_i)^T & -\varepsilon(G_i + G_i^T) \end{bmatrix} < 0 \tag{19}
$$

where $l = 1, \dots, q$ and the slack matrix G_i has the following fixed structure:

$$
G_i = \left[\begin{array}{c|c} \eta_i & 0_{1\times 2} \\ \hline G_{21_i} & G_{22_i} \end{array}\right] \tag{20}
$$

where matrices G_{21_i} and G_{22_i} are of appropriate dimensions. The positive scalar parameter $\eta_i > 0$ is chosen such that $\frac{\eta_i}{R_{ij}C_{t_i}} \approx 0$ for $j \in N_i$. The robust local controller is parametrized as $K_i = Y_i G_i^{-1}$.

Proof. Check Appendix. ☐

The design of robust voltage controllers K_i using Theorem 1 is independent of the global model of the DC microgrid systems and only needs parameters of each DG $(\hat{A}_{g_{ii}}, \hat{B}_{g_i})$. Therefore, the voltage control design using Theorem 1 is scalable.

Remarks.

- The proposed conditions given in (19) are Linear Matrix Inequalities (LMIs) in terms of unknown matrices P_i^l, G_i, and Y_i. The LMI-based conditions can be efficiently solved by a semidefinite programming solver [28].
- To limit the two norm of the controller K_i, the following constraints on Y_i and G_i are added:

$$
\begin{aligned}
\begin{bmatrix} -\beta I & Y_i^T \\ Y_i & -I \end{bmatrix} &< 0 \\
\begin{bmatrix} G_i + G_i^T & I \\ I & \frac{1}{2}\delta_i \end{bmatrix} &> 0
\end{aligned} \tag{21}
$$

They imply that $\|K_i\|_2 < \sqrt{\beta_i}\delta_i$.
- The proposed LMIs given in (19) guarantee the robust stability of the DC microgrids with uncertain resistive

and constant power loads whose values belong to given intervals.

- The voltage controller design approach proposed in (19) has many important features: 1) It provides the robust stability of the DC microgrid systems with parameter uncertainties in resistive and constant power loads. 2) The structure of the local primary voltage controllers is fully decentralized. 3) The closed loop system asymptotically tracks all the reference voltage signals. 4) The design of the local voltage controller is scalable.

E. Feedforward Voltage Controllers

The voltage controllers K_i are stabilizing controllers. In order to speed up the response and improve the transient behavior of the microgrid system according to the IEEE standards [25], a local feedforward controller K_i^r for each DG is developed. The feedforward term is operated as a pre-filter whose duty is to filter the reference voltage V_{ref_i}. The closed-loop system of each DG is described as follows:

$$V_i = (T_i(s)K_i^r(s))\,V_{ref_i} \tag{22}$$

where

$$T_i(s) = \hat{C}_i\left(sI - (\hat{A}_{g_{ii}} + \hat{B}_{g_i}K_i)\right)^{-1}\begin{bmatrix} 0 \\ I \end{bmatrix} \tag{23}$$

To design the feedforward controllers $K_i^r(s)$, a reference tracking model (reference model) $T_{d_i}(s)$ is designed according to the desired performance of DG i. Then, the controllers $K_i^r(s)$ are designed by solving the following H_∞ optimization problem:

$$\begin{aligned} &\min_{K_i^r(s)} \quad \gamma_i \\ &\text{s.t.} \quad \|T_i(s)K_i^r(s) - T_{d_i}(s)\|_\infty < \gamma_i \end{aligned} \tag{24}$$

The schematic diagram of the overall two-degree-of-freedom (2DOF) primary voltage control framework for DG i is depicted in Fig. 2.

F. Proposed Algorithm for Design of 2DOF Primary Voltage Controllers

The overall 2DOF voltage control design is based on the following steps.

Input: Model of each DG with uncertainty (the four vertices given in (9)).

Output: Robust state feedback controllers K_i and feedforward controllers K_i^r.

1) The model of each DG is augmented with an integrator. The dynamics of the augmented model are presented as (14)-(15).
2) We solve a set of LMIs given in (19) with the structural constraint on the slack matrices G_i in (20). The decision variables of the LMIs are Y_i, G_{21_i}, G_{22_i}, and P_i^l. The feedback control term is obtained as $K_i = Y_iG_i^{-1}$. The LMI conditions can be solved using YALMIP [28] as an interface and SDP solvers such as MOSEK [29].
3) A reference model is chosen based on the desired performance of DC microgrids. Then, the feedforward controller is designed via the optimization algorithm in (24). The optimization problem can be solved via some MATLAB commands such as hinfstruct, looptune, and systune.

TABLE I: Parameters of DC microgrid in Fig. 3.

Electrical parameters				
DGs	DC-DC converter parameters $R_t(\Omega)$	$L_t(mH)$	Shunt capacitance $C_t(mF)$	Reference voltage V_{ref} (V)
DG 1	0.2	1.8	2.2	47.9
DG 2	0.3	2.0	1.9	48
DG 3	0.1	2.2	1.7	47.7
DG 4	0.5	3.0	2.5	48
DG 5	0.4	1.2	2.0	47.8
DG 6	0.6	2.5	3.0	48.1

Distribution network parameters		
Line impedance	Symbol	Value
Line impedance between DG 1 and DG 2	Z_{12}	$R_{12} = 0.05\Omega$, $L_{12} = 2.1\mu H$
Line impedance between DG 1 and DG 3	Z_{13}	$R_{13} = 0.07\Omega$, $L_{13} = 1.8\mu H$
Line impedance between DG 3 and DG 4	Z_{34}	$R_{34} = 0.06\Omega$, $L_{34} = 1.0\mu H$
Line impedance between DG 2 and DG 4	Z_{24}	$R_{24} = 0.04\Omega$, $L_{24} = 2.3\mu H$
Line impedance between DG 4 and DG 5	Z_{45}	$R_{45} = 0.08\Omega$, $L_{45} = 1.8\mu H$
Line impedance between DG 1 and DG 6	Z_{16}	$R_{16} = 0.1\Omega$, $L_{16} = 2.5\mu H$
Line impedance between DG 5 and DG 6	Z_{56}	$R_{56} = 0.08\Omega$, $L_{56} = 3.0\mu H$

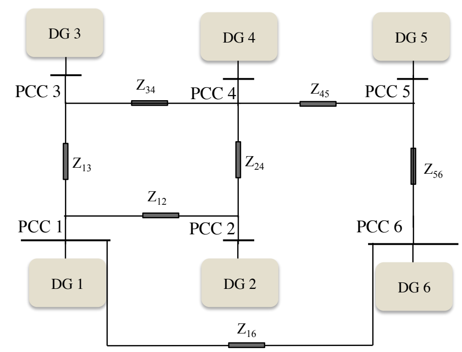

Fig. 3: Layout of an islanded DC microgrid consisting of 6 DGs.

IV. SIMULATION RESULTS

We consider an islanded DC microgrid consisting of 6 DGs with buck converters as graphically shown in Fig. 3. The parameters of each DG and the distribution network are given in Table I. The input DC voltage of the buck converters is $V_{dc_i} = 100V$, for $i = 1,\ldots,6$. It is assumed that DG1, DG2, and DG5 supply a constant power load with power demand $P_{CPL_1} = 300 \pm 100W$ (33% uncertainty), $P_{CPL_2} = 350 \pm 100W$ (28% uncertainty), and $P_{CPL_5} = 550 \pm 100W$ (18% uncertainty), respectively. The resistive loads at each PCC are subject to uncertainty as $5\Omega \leq R_1 \leq 15\Omega$, $2\Omega \leq R_2 \leq 10\Omega$, $10\Omega \leq R_3 \leq 30\Omega$, $1\Omega \leq R_4 \leq 5\Omega$, $2\Omega \leq R_5 \leq 10\Omega$, and $2\Omega \leq R_6 \leq 10\Omega$.

The proposed algorithm in Subsection III-F is used and implemented in MATLAB to design the voltage controllers for all 6 DGs. The parameters of the designed controllers K_i, $i = 1,\ldots,6$ are as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.003 & -0.0031 & 44.475 \end{bmatrix} \\ K_2 &= \begin{bmatrix} 0.0013 & -0.0032 & 57.443 \end{bmatrix} \\ K_3 &= \begin{bmatrix} -0.0012 & -0.0089 & 18.102 \end{bmatrix} \\ K_4 &= \begin{bmatrix} -0.0012 & -0.0179 & 128.682 \end{bmatrix} \\ K_5 &= \begin{bmatrix} 0.0025 & -0.007 & 80.93 \end{bmatrix} \\ K_6 &= \begin{bmatrix} 0.0051 & -0.0287 & 185.276 \end{bmatrix} \end{aligned} \tag{25}$$

A. Case Study 1: Uncertain Constant Power Loads

The first case study assesses the performance of the proposed control strategy with respect to uncertain power demand

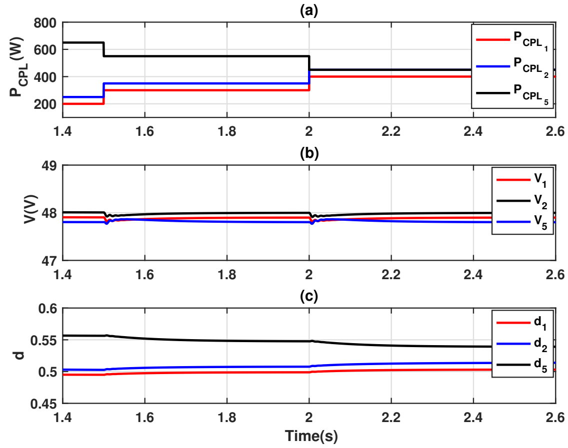

Fig. 4: Dynamical response of DG1, DG2, and DG5 with uncertain CPLs: (a) uncertain power of CPLs, (b) voltage signal at PCC1, PCC2, and PCC5, and (c) control signals of DG1, DG2, and DG5.

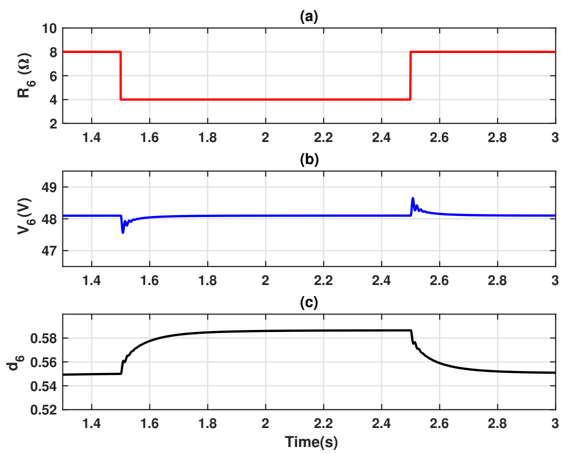

Fig. 5: Dynamical response of DG6 with frequent load changes: (a) resistive load changes at PCC6, (b) voltage signal at PCC6, and (c) control signal of DG6.

of CPLs. To this end, we assume that the power of CPL connected at PCC1, PCC2, and PCC5 is changing in the given range at different transition times as shown in Fig. 4(a). The voltage signals at PCC1, PCC2, and PCC5 are depicted in Fig. 4(b). The control signal of all three DGs are shown in Fig. 4(c). Note that, since the control signal is $u = V_{dc}d$, only the duty cycle d is shown in Fig. 4(c). The results illustrate that the uncertainty of the output power of CPLs does not influence the stability of the microgrid system. In other words, the DC microgrid system is robustly stable with respect to uncertain CPLs.

B. Case Study 2: Uncertain Resistive Loads

Case study 2 evaluates the performance of the proposed voltage controller in resistive load uncertainty. The voltage references are set according to values given in Table I. The load resistance at PCC6 is changed from its nominal value 8Ω to 4Ω at $t = 1.5s$. Then, it is stepped up to its nominal value at $t = 2.5s$. The voltage signal at PCC6 and control signal of DG6 are shown in Fig. 5.

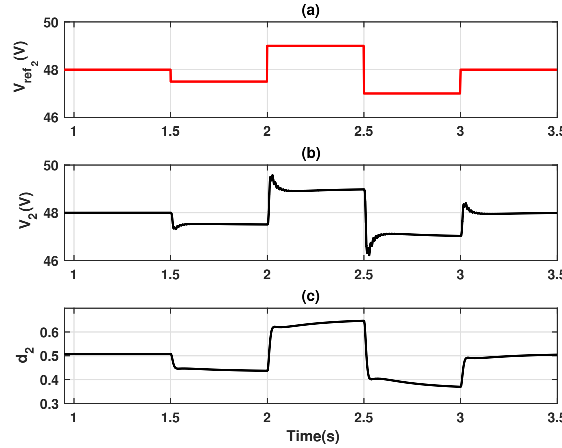

Fig. 6: Dynamical response of DG2 due to different reference voltage changes: (a) reference voltage for DG2, (b) voltage signal at PCC2, and (c) control signal of DG2.

C. Case Study 3: Voltage Tracking

We study the performance and transient behavior of DG2 in voltage tracking scenario. DG1, DG2, and DG5 respectively supply a constant power load with power demand $P_{CPL_1} = 230$W, $P_{CPL_2} = 350$W, and $P_{CPL_5} = 570$W. The voltage references for all DGs are initially set according to reference values given in Table I. Then, the voltage reference for DG2 is stepped down/up to different values at different times. Fig. 6 shows the dynamic responses of DG2. The results show that the proposed control technique is able to regulate the load voltage at PCCs with zero steady state error and small transient time.

D. Case Study 4: Disconnection/Connection of CPLs

In this case study, we assume that the CPL with $P_{CPL_1} = 250W$ at PCC1 is totally disconnected at $t = 1.5s$ and reconnected at $t = 2.5s$. The voltage signals of DG1 at PCC1 and its neighboring DGs are depicted in Fig. 7(a) and Fig. 7(c), respectively. The control signal of DG1 (d_1) is shown in Fig. 7(b). The results indicate that the transient response of the neighboring DGs of DG1 due to the disconnection/connection of CPL1 is negligible.

E. Case Study 5: Plug-and-Play Functionality of DGs

In this case study, we evaluate the capability of the proposed controllers in plug-and-play (PnP) functionality of DGs. To this end, it is assumed that DG1 leaves the microgrid system in Fig.3 at $t = 1.5s$ and plugged in at $t = 2.5s$. Due to this plug-and-play operation, all the connections attached to DG1, i.e. DG2, DG3, and DG6, are affected. The dynamic response of all these DGs are depicted in Fig. 8. The results reveal the robust performance of the voltage controllers to plug-and-play functionality of DGs.

F. Case Study 6: Microgrid Topology Change

This case study assesses the robustness of the proposed control strategy against a change in the topology and architecture of the DC microgrid system in Fig.3. We assume that the line

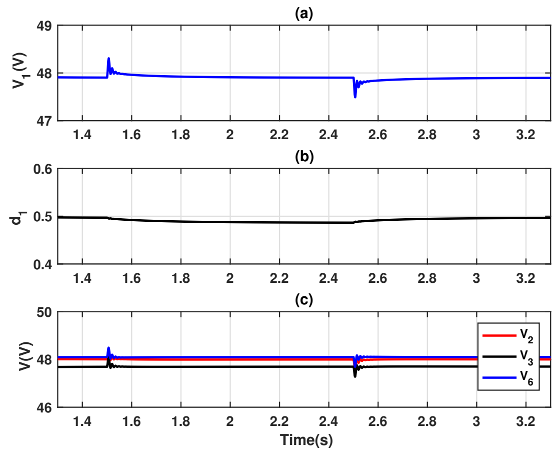

Fig. 7: Dynamical response of DG1 and its neighbors due to disconnection of the CPL at PCC1 at $t = 1.5s$ and its reconnection at $t = 2.5s$: (a) voltage signal at PCC1, (b) control signal of DG1 (d_1), and (c) voltage signals at PCC2, PCC3, and PCC6.

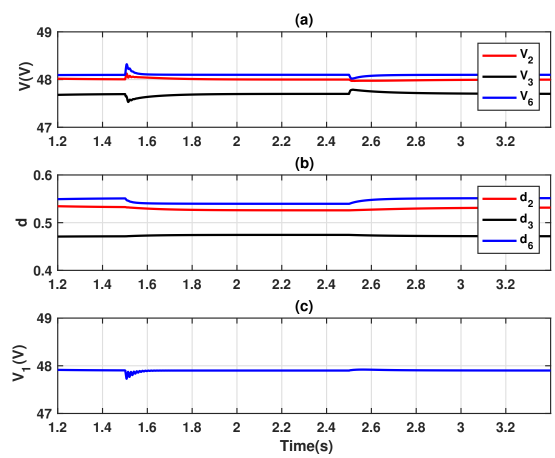

Fig. 8: Dynamical response of DG1, DG2, DG3, and DG6 due to plug-out of DG1 at $t = 1.5s$ and its plug-in at $t = 2.5s$: (a) voltage signals at PCC2, PCC3, and PCC6, (b) control signals of DG2, DG3, and DG6, and (c) voltage signal at PCC1.

between DG5 and DG6 is disconnected at $t = 2s$ due to a fault. As a result, the topology of the DC microrgrid system is changed. The dynamical response of DG5 and DG6 due to this microrgid topology change is plotted in Fig. 9. The results show that the voltage controllers are robust to uncertainties affected the microgrid topology.

V. Conclusions

This paper focuses on scalable robust voltage controller synthesis of DC microgrids with constant power loads (CPLs). It is assumed that the power of CPLs is uncertain and may be varied in a given interval. As a result, the set of equilibria of the DC microgrid system is infinite. To deal with this problem and consider all feasible set of the equilibria, a polytopic model is developed. Then, we design a robust voltage controller for each DG based on a set of linear matrix inequalities. The control design procedure is scalable that means the voltage controller design for each DG is

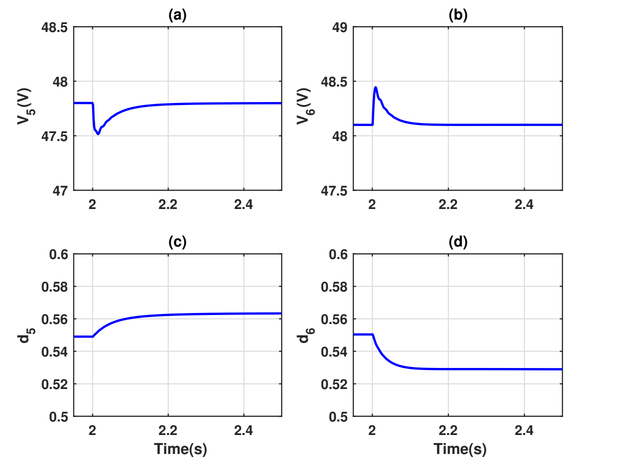

Fig. 9: Dynamical response of DG5 and DG6 due to a fault at $t = 2s$: (a) voltage signal at PCC5, (b) voltage signal at PCC6, (c) control signal of DG5, and (d) control signal of DG6.

independent of other DGs. Different case studies conducted in MATLAB/SimPowerSystems Toolbox illustrate the effectiveness of the proposed voltage control technique for DC microgrids with uncertain CPLs.

VI. Appendix

Proof of Theorem 1

Proof. We need to show that the conditions in (19) guarantee the robust stability of the closed-loop state matrix $\hat{A}(\lambda)+\hat{B}K$. The convex combination of the conditions given in Theorem 1 leads to the following condition:

$$\begin{bmatrix} \hat{A}_{g_{ii}}(\lambda)G_i + G_i^T\hat{A}_{g_{ii}}^T(\lambda) + \hat{B}_{g_i}Y_i + Y_i^T\hat{B}_{g_i}^T & \star \\ P_i(\lambda) - G_i + \varepsilon(\hat{A}_{g_{ii}}(\lambda)G_i + \hat{B}_{g_i}Y_i)^T & -\varepsilon(G_i+G_i^T) \end{bmatrix} < 0 \tag{26}$$

where $P_i(\lambda) = \sum_{l=1}^{q} \lambda_l P_i^l$. According to the Schur complement lemma [30], a set of the above conditions for $i = 1,\ldots,N$ is equivalent to

$$\begin{bmatrix} \hat{A}_D(\lambda)G + G^T\hat{A}_D^T(\lambda) + \hat{B}Y + Y^T\hat{B}^T & \star \\ P(\lambda) - G + \varepsilon(G^T\hat{A}_D^T(\lambda) + Y^T\hat{B}^T) & -\varepsilon(G+G^T) \end{bmatrix} < 0 \tag{27}$$

where $\hat{A}_D(\lambda) = \operatorname{diag}\left(\hat{A}_{g_{11}}(\lambda),\ldots,\hat{A}_{g_{NN}}(\lambda)\right)$, $P(\lambda) = \operatorname{diag}\left(P_1(\lambda),\ldots,P_N(\lambda)\right)$, $G = \operatorname{diag}\left(G_1,\ldots,G_N\right)$, and $Y = \operatorname{diag}\left(Y_1,\ldots,Y_N\right)$. By setting the coefficient η_i in (20) very small, the following term is almost zero

$$\begin{bmatrix} \hat{A}_C G + G^T\hat{A}_C^T & \varepsilon\hat{A}_C G \\ \varepsilon G^T\hat{A}_C^T & 0 \end{bmatrix} \approx 0 \tag{28}$$

where $\hat{A}_C = \hat{A}(\lambda) - \hat{A}_D(\lambda)$, because

$$\hat{A}_{g_{ij}}G_j = G_j^T(\hat{A}_{g_{ij}})^T = \begin{bmatrix} \frac{\eta_i}{R_{ij}C_{t_i}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{29}$$

for $i = 1,\ldots,N$ and $j \in N_i$. Since η_i is chosen to be very small, $\hat{A}_{g_{ij}}G_j \approx 0$. As a result, the following condition holds

$$\begin{bmatrix} \hat{A}(\lambda)G + G^T\hat{A}^T(\lambda) + \hat{B}Y + Y^T\hat{B}^T & \star \\ P(\lambda) - G + \varepsilon(G^T\hat{A}^T(\lambda) + Y^T\hat{B}^T) & -\varepsilon(G+G^T) \end{bmatrix} < 0 \tag{30}$$

The inequality above guarantees the robust stability of the closed-loop state matrix $\hat{A}(\lambda)+\hat{B}K$ for all $\lambda \in \Lambda_q$ [31]. $\square$

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