

# Decentralized Robust Voltage Control of Islanded AC Microgrids: An LMI-Based $H_\infty$ Approach



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**Abstract**—In this paper, a decentralized control method is presented for the islanded inverter-interfaced microgrids with robust performance approach. The system is consisted of two Distributed Generations (DG) linked to their loads by inverter, RL filter and a step up transformer. The objective here is to guarantee the stability of the system and its desired performance under large load perturbation and Plug-and-Play (PnP) of the DGs. To achieve that aim, a Linear Matrix Inequality (LMI) state-feedback control technique with  $H_\infty$  guaranteed cost is used for each DG. Contributions of the designed control system are: 1) the control design is entirely decentralized; 2) the stability and the desired performance of the overall system is guaranteed; 3) the control system is completely robust against PnP functionality of the DGs; 4) the controller provides robustness against large load perturbation. The ability of the presented control system with regard to voltage tracking, load changes and topology changes is evaluated on several case studies based on simulation in SimPowerSystems Toolbox in MATLAB.

**Keywords**—decentralized control, islanded microgrid, plug-and-play (PnP), linear matrix inequality (LMI)

## I. INTRODUCTION

The increasing trend in electrical energy consumption, the environmental problems of using fossil fuels like effects of greenhouse gases and the limitation of these resources have led to a shift towards renewable energy sources. The best way of using renewable energies in the electricity network is by utilizing DG resources. Stable and Reliable connectivity of DGs is possible by means of microgrids [1], [2].

Microgrid systems are described as small scale power grids that are generally working in grid-connected mode of operation. In this mode, the microgrid system is linked to the main power grid at the Point of Common Coupling (PCC). In this operating mode, the output voltage and frequency of the microgrid system are mainly imposed by the main power grid. Responsibility of the designed control system in this mode is to perfectly share real and reactive power between DGs [3]. However, microgrid system may be disconnected from the main power grid and enter the islanding operation. In this occasion, a power mismatch occurs among DGs and loads. As a result, both voltage and the frequency diverge from their nominal values and system will lose its performance and may even become unstable [4]. Consequently, considering the necessity of network reliability and consistency in microgrid desired performance as a stand-alone system after islanding, a new control scheme must be designed for stability of voltage and frequency of the system alongside with a suitable power division between DG units [5].

Most common control method that is applied to adjust the voltage and frequency of the islanded microgrid system is droop control technique [6], [7], [8]. The original idea of the droop control has been established in [9]. In power systems that are based on rotational machines, the voltage is dependent on the reactive power and the frequency depends on the active power balance. This means that the amplitude/frequency of the voltage drops if the demand for reactive/active power increases. In addition, from the control aspect, droop control method is a decentralized, proportional control strategy that preserve the stability of voltage and frequency of the microgrid system [10]. Regardless of all the applications and advantages that droop control has brought, it has plenty of disadvantages that have led researches to move to non-droop-based control methods [11]-[17]. Coupled dynamic of both real and reactive power, and dependency on line dynamics that can cause poor performances are known as some drawbacks of droop control technique.

In non-droop techniques, the frequency in DG units is controlled via an oscillator placed inside every DG unit. Then by using a common time reference signal based on the Global Positioning System (GPS), all oscillators are synchronized [18]. Moreover, to control the PCC voltage of each DG unit, a proper voltage controller is designed, while a proper model of the microgrid is presented. Control scheme based on robust performance approach has played a major role among non-droop-based control methods. These studies consider the robust voltage regulation of the islanded microgrid by forming a convex optimization problem to design a state or output feedback controller [12]-[17]. For instance, a two three-degree-of-freedom (DoF) output-feedback controller was designed in [14] with robustness against load perturbations and disturbances. In addition, another output-feedback design was presented in [15] with distributed control strategy that only guaranteed robustness against slight variation in local loads. A mixed  $H_2/H_\infty$  robust controller was presented in [19] that provided robustness against load perturbations, while providing a good transient response by using a unique controller. However, the design did not provide robustness against topology changes and PnP functionality of the DGs. To address the problem of voltage regulation of the islanded microgrid system providing PnP capability, state-feedback control approach was considered in [12], [13], and [17]. A state-feedback control design was used in [17] enabling PnP functionality of the DGs and also the design provided robustness against PnP functionality of the DGs. However, local controllers had to be returned after each plug in or plug out of neighboring DGs. This disadvantage was later resolved in [13], where the PnP functionality was considered as a parametric uncertainty bounded in a polytope. However, the

three DoF design of the controller made the design extremely complex. Moreover, the state-feedback controller and two pre-filters have been designed as three independent optimization problem that led to a sub-optimal controller and as a result a sub-optimal performance.

A new robust  $H_\infty$  state-feedback controller via LMI is designed in this paper for the solution of the problem of robust voltage control of the islanded microgrid system containing the connection of two DGs with a unique controller for each DG that would resolve the problem of sub-optimal design and complexity of the previous works. First, we consider the loads to be unknown. However, their current is assumed to be measurable where we take it as a disturbance signal. Then, we show that PnP functionality of the DGs and topology changes can be considered as a polytopic uncertainty in system's model. Using this design, we preserve stability alongside with optimal performance of the system with a unique controller for each DG unit in order to reduce the computational complexity of the designed control system. In this design, local controllers do not need to retune their parameters in order to provide PnP capability or topology changes. To verify the efficiency of the designed control system, various case studies are simulated in the toolbox of SimPowerSystems in MATLAB.

This paper is organized as follows. A mathematical model of the islanded AC microgrid and its state-space demonstration is presented in section II. In section III, The microgrid control system is developed. Section IV is devoted to topology changes and PnP capability of DGs in the islanded microgrid. The simulation results are presented in section V. Section VI provides conclusion for this paper.

Throughout this paper, the set of real numbers is denoted by  $\mathbb{R}$ . For matrix  $A$ , the transpose of  $A$ , the inverse of  $A$ , the inverse transpose of  $A$  and the trace of  $A$  are respectively denoted by  $A^T, A^{-1}, A^{-T}$  and  $tr(A)$ . The identity and zero matrix are denoted by  $I$  and  $0$ , respectively.

## II. INVERTER-INTERFACED MICROGRID SYSTEM MODELLING

The dynamical model of the islanded AC microgrid that is used in this paper is presented in this section. In the modelling process, each DG is consisted of a renewable energy source modeled as a DC source, a Voltage Source Converter (VSC), an  $RL$  filter, a  $Y - \Delta$  transformer with  $k$  as the transformation ratio, a capacitor for attenuating the high frequency harmonics and a load with unidentified parameters and. The schematic of an islanded AC inverter-interfaced microgrid consisted of two DGs is shown in Fig. 1. The state-space representation of the islanded AC microgrid system under balanced conditions in  $dq$  reference frame is as bellow:

$$DG \ i: \begin{cases} \frac{dV_{i,dq}}{dt} + j\omega_0 V_{i,dq} = \frac{k_i}{C_{t_i}} I_{t_i,dq} - \frac{1}{C_{t_i}} I_{L_i,dq} + \frac{1}{C_{t_i}} I_{ij,dq} \\ \frac{dI_{t_i,dq}}{dt} + j\omega_0 I_{t_i,dq} = -\frac{k_i}{L_{t_i}} V_{i,dq} - \frac{R_{t_i}}{L_{t_i}} I_{t_i,dq} + \frac{1}{L_{t_i}} V_{t_i,dq} \end{cases} \quad (1)$$

$$DG \ j: \begin{cases} \frac{dV_{j,dq}}{dt} + j\omega_0 V_{j,dq} = \frac{k_j}{C_{t_j}} I_{t_j,dq} - \frac{1}{C_{t_j}} I_{L_j,dq} + \frac{1}{C_{t_j}} I_{ij,dq} \\ \frac{dI_{t_j,dq}}{dt} + j\omega_0 I_{t_j,dq} = -\frac{k_j}{L_{t_j}} V_{j,dq} - \frac{R_{t_j}}{L_{t_j}} I_{t_j,dq} + \frac{1}{L_{t_j}} V_{t_j,dq} \end{cases} \quad (2)$$

$$Line \ ij: \frac{dI_{ij,dq}}{dt} + j\omega_0 I_{ij,dq} = -\frac{R_{ij}}{L_{ij}} I_{ij,dq} + \frac{1}{L_{ij}} V_{j,dq} - \frac{1}{L_{ij}} V_{i,dq} \quad (3)$$

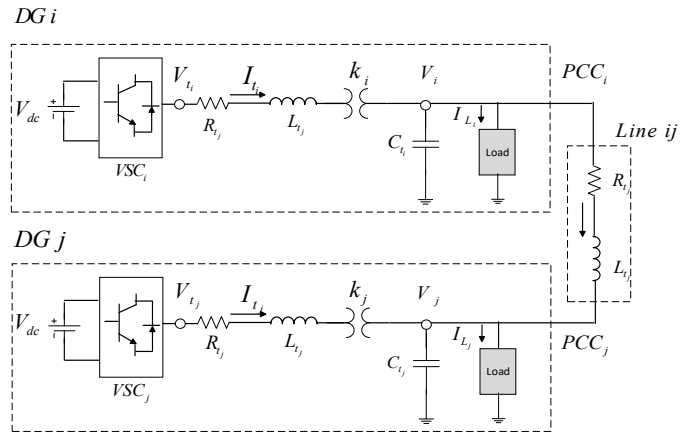


Fig. 1. Diagram of islanded AC microgrid system composed of two DGs connected through line  $ij$ .

where,  $(V_{t_i,dq}, V_{t_j,dq})$ ,  $(I_{t_i,dq}, I_{t_j,dq})$ ,  $(V_{i,dq}, V_{j,dq})$ ,  $(I_{L_i,dq}, I_{L_j,dq})$ , and  $I_{ij,dq}$  are the  $dq$  elements of the VSC voltages, filter currents, load voltages, local load currents and the current of the transmission line, respectively.

Based on the assumption of QSL [20],  $dI_{ij,dq}/dt = 0$ , equation (3) can be written as follow:

$$I_{ij,dq} = \frac{V_{j,dq}}{R_{ij} + j\omega_0 L_{ij}} - \frac{V_{i,dq}}{R_{ij} + j\omega_0 L_{ij}} \quad (4)$$

As a result, the state-space representation of the system can be described as follows:

$$\begin{aligned} \dot{x}_{g_i} &= A_{g_{ii}} x_{g_i} + A_{g_{ij}} x_{g_j} + B_{g_i} u_i + B_{d_i} d_i \\ y_i &= C_{g_i} x_{g_i}; \quad i, j = 1, 2 \end{aligned} \quad (5)$$

In above,  $x_{g_i} = [V_{i,d} \ V_{i,q} \ I_{t_i,d} \ I_{t_i,q}]^T$  is described the state vector,  $u_i = [V_{t_i,d} \ V_{t_i,q}]^T$  is defined as the input,  $d_i = [I_{L_i,d} \ I_{L_i,q}]^T$  is the exogenous input, and  $y_i = [V_{i,d} \ V_{i,q}]^T$  is the output vector. The state-space matrices are presented in (6) [13], [17], where  $\omega_0 = 2\pi f_0$ ,  $X_{ij} = \omega_0 L_{ij}$ ,  $Z_{ij}^2 = R_{ij}^2 + X_{ij}^2$ , and  $f_0$  is the system's nominal frequency.

$$A_{ii} = \begin{bmatrix} \frac{1}{C_u} \frac{R_{ij}}{Z_{ij}^2} & \omega_0 \frac{1}{C_u} \frac{X_{ij}}{Z_{ij}^2} & \frac{k_i}{C_u} & 0 \\ -\omega_0 + \frac{1}{C_u} \frac{X_{ij}}{Z_{ij}^2} & -\frac{1}{C_u} \frac{R_{ij}}{Z_{ij}^2} & 0 & \frac{k_i}{C_u} \\ -\frac{k_i}{L_u} & 0 & \frac{R_{ij}}{L_u} & \omega_0 \\ 0 & \frac{k_i}{L_u} & -\omega_0 & -\frac{R_{ij}}{L_u} \end{bmatrix} \quad A_{ij} = \frac{1}{C_u} \begin{bmatrix} \frac{R_{ij}}{Z_{ij}^2} & \frac{X_{ij}}{Z_{ij}^2} & 0 & 0 \\ \frac{X_{ij}}{Z_{ij}^2} & \frac{R_{ij}}{Z_{ij}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_u} & 0 \\ 0 & \frac{1}{L_u} \end{bmatrix} \quad B_{d_i} = \begin{bmatrix} -\frac{1}{c_u} & 0 \\ 0 & -\frac{1}{c_u} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (6)$$

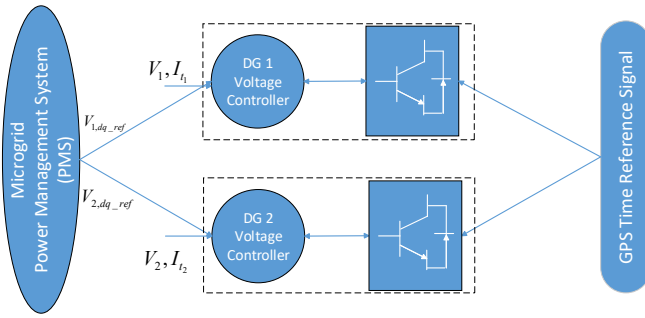


Fig. 2. Structure of non-droop technique for control of islanded AC microgrid.

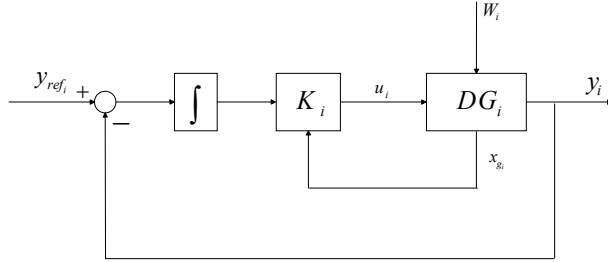


Fig. 3. Diagram of local voltage controllers for each DG unit.

### III. MICROGRID SYSTEM CONTROL SCHEME

Fig. 2 illustrates the structure of the microgrid non-droop based control scheme consisted of a PMS, DGs voltage controllers, and a frequency control technique.

#### A. Power Management System

The power management system is a crucial element for the proper and reliable operation of the microgrids. [21]. The key purpose of the PMS is to allocate optimal set-points for the system. As a result, controllers in each DG accurately share the active and the reactive power between DGs [22] and properly respond to the microgrid disturbances [23]. These set-points will be communicated to the controllers of every DG unit.

#### B. Frequency Control

In the non-droop based control method, the frequency of the system is regulated by an open loop method. For this purpose, all DGs are consisted of an internal oscillator that creates  $\theta(t) = \int_0^t \omega_0 d\tau$ . Then a global synchronization signal is transferred to the internal oscillators of each DG unit via GPS in order to synchronize them [21].

#### C. Voltage Control

The goal here is to design a control system for the microgrid specified in (5) in a decentralized manner. The focus of this paper is on designing a voltage control system that preserves stability and guarantees desired response based on a decentralized control strategy that requires no kind of communications.

1) *Design Requirements:* A voltage controller based on  $dq$  reference frame for the microgrid system represented in

(5) is desirable, in a way that the following circumstances apply.

- 1) Decentralized design of the control system.
- 2) Asymptotically stability of the overall closed-loop microgrid system.
- 3) Asymptotic tracking of all reference voltages.

Next, a voltage control scheme with integrator is designed to accomplish all aforesaid requirements.

2) *Voltage Controller:* as mentioned, a important control desire is the necessity of DG units to track the reference signal of voltage  $y_{ref}$ . To that aim, every DG unit is augmented using an integrator [24]:

$$\dot{v}_i = y_{ref_i} - y_i = y_{ref_i} - C_{g_i} x_{g_i} \quad (7)$$

hence, augmented microgrid is defined as:

$$\begin{aligned} \dot{\hat{x}}_{g_i} &= \hat{A}_{g_{ii}} \hat{x}_{g_i} + \hat{A}_{g_{ij}} \hat{x}_{g_j} + \hat{B}_{g_i} u_i + \hat{B}_{d_i} \hat{d}_i \\ \hat{y}_i &= \hat{C}_{g_i} \hat{x}_{g_i}; \\ z_i &= \hat{C}_{g_i} \hat{x}_{g_i} + D_{g_i} u_i + D_{d_i} \hat{d}_i \end{aligned} \quad (8)$$

in above,  $\hat{x}_{g_i} = [V_{i,d} \ V_{i,q} \ I_{t,i,d} \ I_{t,i,q} \ v_{i,d} \ v_{i,q}]^T$ ,  $u_i = [V_{t,i,d} \ V_{t,i,q}]^T$ ,  $\hat{d}_i = [I_{L,i,d} \ I_{L,i,q} \ y_{ref,i,d} \ y_{ref,i,q}]^T$ ,  $\hat{y}_i = [V_{i,d} \ V_{i,q} \ v_{i,d} \ v_{i,q}]^T$ , and  $z(t) \in \mathbb{R}^{n_w}$  is the controlled outputs including errors and control signal.

$$\begin{aligned} \hat{A}_{g_{ii}} &= \begin{bmatrix} A_{g_{ii}} & 0 \\ -C_{g_i} & 0 \end{bmatrix}, \quad \hat{A}_{g_{ij}} = \begin{bmatrix} A_{g_{ij}} & 0 \\ 0 & 0 \end{bmatrix} \\ \hat{B}_{g_i} &= \begin{bmatrix} B_{g_i} \\ 0 \end{bmatrix}, \quad \hat{B}_{d_i} = \begin{bmatrix} B_{d_i} & 0 \\ 0 & I \end{bmatrix} \\ \hat{C}_{g_i} &= \begin{bmatrix} C_{g_i} & 0 \\ 0 & I \end{bmatrix} \end{aligned} \quad (9)$$

The aim is to design a decentralized controllers  $K_i$  that are described by the following control laws:

$$u_i(t) = K_i \hat{x}_{g_i}; \quad i = 1, 2 \quad (10)$$

The closed-loop representation of the  $i^{th}$  augmented subsystem using state-feedback controller  $K_i$  are as bellow:

$$\begin{aligned} \dot{\hat{x}}_{g_i}(t) &= (\hat{A}_{g_{ii}} + \hat{B}_{g_i} K_i) \hat{x}_{g_i}(t) + \hat{A}_{g_{ij}} \hat{x}_{g_j}(t) + \hat{B}_{d_i} \hat{d}_i(t) \\ \hat{y}_i(t) &= \hat{C}_{g_i} \hat{x}_{g_i}(t) \\ z_i(t) &= (\hat{C}_{g_i} + D_{g_i}) \hat{x}_{g_i}(t) + D_{d_i} \hat{d}_i(t) \end{aligned} \quad (11)$$

Now by the means of the following theorem [25], we consider the development of the robust  $H_\infty$  state-feedback controller for islanded microgrid system (11).

**Theorem 1:** let  $\varepsilon \neq 0, \xi \in (-1,1)$ , and  $\gamma > 0$ . if there exist Lyapunov matrices  $W_i = W_i^T \in \mathbb{R}^{n_x \times n_x}$ , for  $i = 1,2, X_i \in \mathbb{R}^{n_x \times n_x}$  and  $Z_i \in \mathbb{R}^{n_u \times n_x}$  such that:

$$Y_i := \begin{bmatrix} W_i + \tilde{A}_{g_i} X_i + \bar{B}_{g_i} Z_i + X_i^T \tilde{A}_{g_i}^T + Z_i^T \bar{B}_{g_i}^T & * & * & * \\ \xi X_i^T \tilde{A}_{g_i}^T + \xi Z_i^T \bar{B}_{g_i}^T - \bar{A}_{g_i} X_i - \bar{B}_{g_i} Z_i & Y_{i(2,2)} & * & * \\ \bar{B}_{g_i}^T & -\bar{B}_{g_i}^T & -I & * \\ C_{g_i} X_i + D_{g_i} Z_i & \xi C_{g_i} X_i + \xi D_{g_i} Z_i & D_{g_i} & -\gamma I \end{bmatrix} < 0$$

$$Y_{i(2,2)} := -W_i - \xi \hat{A}_{g_i} X_i - \xi \bar{B}_{g_i} Z_i - \xi X_i^T \hat{A}_{g_i}^T - \xi Z_i^T \bar{B}_{g_i}^T$$

$$i=1,2 \quad (12)$$

with  $\tilde{A}_{g_{ii}} := \varepsilon A_{g_{ii}} - I/(2\varepsilon)$ ,  $\hat{A}_{g_{ii}} := \varepsilon A_{g_{ii}} + I/(2\varepsilon)$ ,  $\bar{B}_d := \varepsilon B_d$ , and  $\bar{B}_{g_i} := \varepsilon B_{g_i}$ . Subsequently, the controller gains  $K_i = Z_i X_i^{-1}$ ;  $i = 1,2$  ensures the closed-loop system (11) is asymptotically stable by an  $H_\infty$  guaranteed cost given by  $\sqrt{\gamma}$ .

Now the conditions have to be created in such a way that the design of the controller is decentralized. In other words, the interaction terms  $\hat{A}_{g_{ij}} X_j + X_j^T \hat{A}_{g_{ij}}^T$  should be neglected. Next, we indicate that by certain assumptions, fulfilling (12) result in the overall system asymptotic stability.

3) *Control scheme Based on the Idea of Neutral Interactions:* neutral interaction in [26] indicates that the interaction terms are negligible if and only if the interaction matrix  $\hat{A}_c = \hat{A} - \hat{A}_d$ , where we define  $\hat{A}_d = \text{diag}(\hat{A}_{g_{11}}, \dots, \hat{A}_{g_{11}})$ , is factorized as follows:

$$\hat{A}_c = X^T S \quad (13)$$

where  $X$  is the slack matrix defined in (12). Also  $S$  is defined as a skew-symmetric matrix, i.e.,  $S^T = -S$ .

Based on three assumptions bellow, the interactions are neutral.

1)  $C_{t_i} = C_s$ , for  $i = 1,2$ .

2) Decentralized controllers are designed such that (12) holds with:

$$X_i = \begin{bmatrix} \eta I_{2 \times 2} & 0 \\ 0 & X_{22_i} \end{bmatrix}; \quad i=1,2 \quad (14)$$

where  $\eta > 0$  is mutual for all matrices.

3) It holds  $\eta R_{ij}/(C_s Z_{ij}^2) \approx 0$  for  $i, j = 1,2$ , where  $Z_{ij} = [R_{ij} + j\omega_0 L_{ij}]$ .

if the above-mentioned conditions are true, the interaction terms  $\hat{A}_{g_{ij}} X_j + X_j^T \hat{A}_{g_{ij}}^T \approx 0$  [17].

#### IV. PLUG-AND-PLAY CAPABILITY

Here, we aim to guarantee the closed-loop microgrid system stability in the event that a DG unit is plugged in or out.

##### A. Robust Strategy Against PnP Functionality of DGs

Studying equation (6) easily indicate that the plugging in or plugging out of a DG  $i$  to or from DG  $j$  can simply effect matrix  $\hat{A}_{g_{ii}}$  of both DGs. Consequently, two possibilities for each of the two DGs are considered:

- 1) Minimum value of  $(R_{ij}/Z_{ij}^2)$  and  $(X_{ij}/Z_{ij}^2)$
- 2) Maximum value of  $(R_{ij}/Z_{ij}^2)$  and  $(X_{ij}/Z_{ij}^2)$

in two cases above, minimum of  $(R_{ij}/Z_{ij}^2)$  and  $(X_{ij}/Z_{ij}^2)$  are defined for the situation where there is no connection between two DGs, and the maximum values are for the time that they are connected to each other.

Accordingly, matrices  $\hat{A}_{g_{ii}}$  of the DGs have a polytopic type uncertainty like bellow:

$$\hat{A}_{g_{ii}}(\lambda) = \lambda_1 \hat{A}_{g_{ii}}^1 + \lambda_2 \hat{A}_{g_{ii}}^2, \quad \sum_{i=1}^2 \lambda_i = 1, \quad 0 < \lambda_i < 1 \quad (15)$$

Eventually, we develop a state-feedback control system for the augmented uncertain system  $(\hat{A}_{g_{ii}}(\lambda), \hat{B}_{g_i}, \hat{C}_{g_i}, 0)$  in a decentralize manner using following convex optimization problem:

$$\min_{W_{21}, Z_i, X_i, \eta} \eta$$

$$Y_i^j := \begin{bmatrix} W_i^j + \tilde{A}_{g_i}^j X_i + \bar{B}_{g_i} Z_i + X_i^T \tilde{A}_{g_i}^{jT} + Z_i^T \bar{B}_{g_i}^{jT} & * & * & * \\ \xi X_i^T \tilde{A}_{g_i}^{jT} + \xi Z_i^T \bar{B}_{g_i}^{jT} - \bar{A}_{g_i}^j X_i - \bar{B}_{g_i} Z_i & Y_{i(2,2)}^j & * & * \\ \bar{B}_{g_i}^{jT} & -\bar{B}_{g_i}^{jT} & -I & * \\ C_{g_i} X_i + D_{g_i} Z_i & \xi C_{g_i} X_i + \xi D_{g_i} Z_i & D_{g_i} & -\gamma I \end{bmatrix} < 0$$

$$W_i^j = W_i^{jT} > 0, \quad \eta > 0$$

$$Y_{i(2,2)}^j := -W_i^j - \xi \hat{A}_{g_i}^j X_i - \xi \bar{B}_{g_i} Z_i - \xi X_i^T \hat{A}_{g_i}^{jT} - \xi Z_i^T \bar{B}_{g_i}^{jT}$$

$$\tilde{A}_{g_i}^j = \varepsilon_i A_{g_i}^j - I/(2\varepsilon_i), \quad \hat{A}_{g_i}^j = \varepsilon_i A_{g_i}^j + I/(2\varepsilon_i), \quad \bar{B}_{g_i}^j := \varepsilon_i B_{g_i}^j, \quad \text{and } \bar{B}_{g_i} := \varepsilon_i B_{g_i}$$

$$i=1,2; \quad j=1,2 \quad (16)$$

##### B. Algorithm 1: Decentralized Control Algorithm of The Inverter-Interfaced Microgrids in Islanding Mode

Here, a procedure for designing local controllers  $K_i$  for DG  $i$  is given.

*Step 1:* Form vertices  $\tilde{A}_{g_{ii}}^1$  and  $\tilde{A}_{g_{ii}}^2$ .

*Step 2:* Execute the given structure in (14) for the slack matrix  $X_i$ .

*Step 3:* Fix the scalars  $\varepsilon \neq 0, \xi \in (-1,1)$  and  $\gamma > 0$  and solve the convex optimization problem (16) for each DG.

*Step 4:* Set  $K_i = Z_i X_i^{-1}$  as state-feedback gains for each DG of augmented microgrid system (11).

TABLE I

MICROGRID ELECTRICAL PARAMETERS

DGs	Filter parameters		Shunt capacitance	Load parameters		Reference voltages	
	$R_f (m\Omega)$	$L_f (m\Omega)$	$C_f (\mu F)$	KW	Kvar	$V_{d,ref} (pu)$	$V_{q,ref} (pu)$
DG 1	1.2	93.7	62.86	20	10	0.9	0.436
DG 2	1.6	94.8	62.86	20	10	0.8	0.6

DC bus voltage	$V_{dc} = 2000V$
Power base value	$S_{base} = 8KVA$
Voltage base value	$V_{base,low} = 0.5KV, V_{base,high} = 11.5KV$
VSC voltage	$V_{VSC} = 600V$
VSC rated power	$S_{VSC} = 3MVA$
Transformer voltage ratio	$K_i = 0.6/13.8KV (\Delta/Y)$
System nominal frequency	$f_0 = 60Hz$
Line impedance	$R = 1.1 \Omega, L = 600 mH$

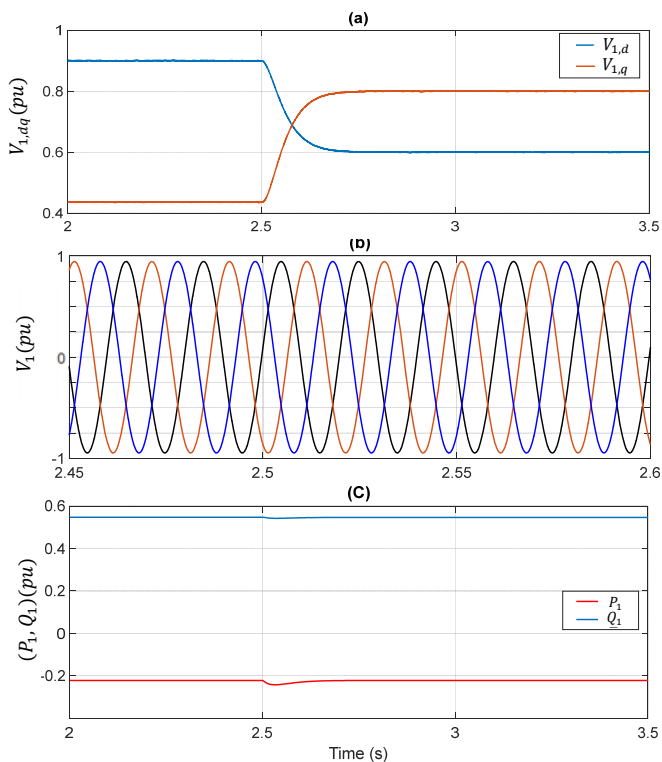


Fig. 4. Performance of DG1 against reference changing (a)  $dq$  components of PCC voltages of DG 1, (b) Instant PCC voltages of DG 1, (c) DG1 real and reactive power.

## V. SIMULATION RESULTS

In this section, the effectiveness of the presented controller is evaluated by simulation of the system of Fig. 2 with MATLAB/SimPowerSystems toolbox. Parameters of the DGs and line is given is Table I.

### A. Voltage Tracking

Proper reference voltage tracking for the two DGs are shown in Fig. 2. References are set according to the values represented in Table I. At  $t = 2.5 s$ ,  $d$  and  $q$  elements of the reference voltages of DG 1 shifts from 0.9 and 0.436 pu to 0.6 and 0.8 pu, respectively. Response of DG 1 to the new values of reference voltages are shown in Fig. 4. Fig. 4(a) illustrates the  $d$  and  $q$  elements of voltage in PCC 1 and shows that the proposed controller successfully regulates PCC 1 voltages without any steady state error. Fig. 4(b) and (c) illustrates the instant PCC 1 voltages and real and reactive output power of DG 1.

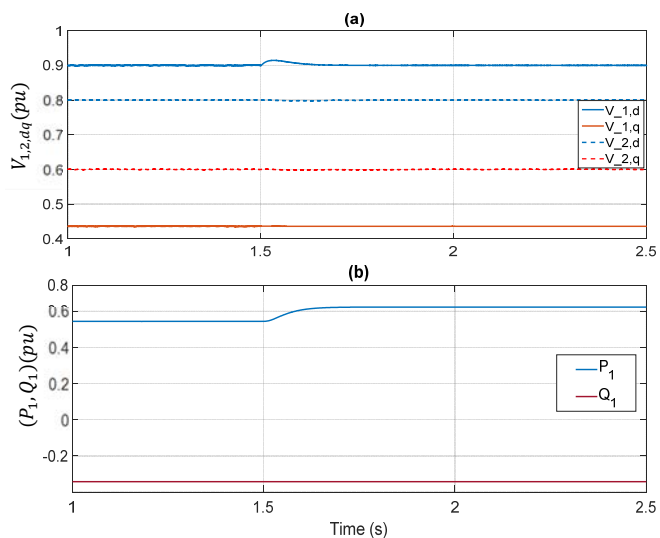


Fig. 5. Performance of DG1 and DG2 against load changes (a)  $dq$  components of PCC voltages of both DG1 and DG2, (b) DG1 and DG2 active and reactive power.

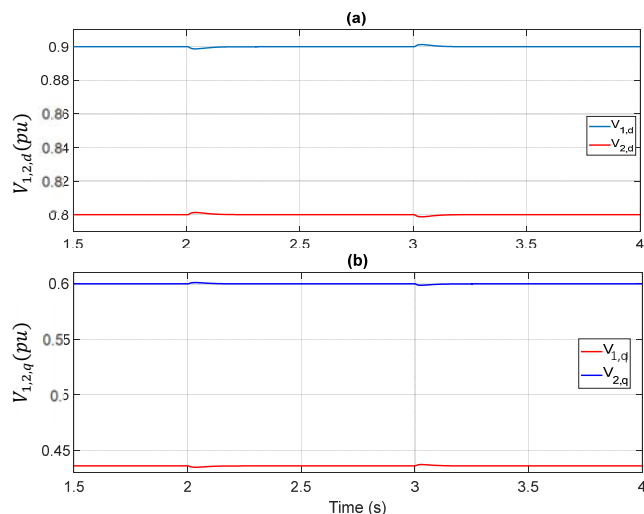


Fig. 6. Performance of DGs against PnP (a)  $d$  component of PCC voltages of both DGs, (b)  $q$  component of PCC voltages of both DGs.

### B. Robustness Against Load Changes

In this case, the proposed controller is evaluated against changes in local load. At  $t = 1.5 s$ , active load for DG 1 is changed from 20 to 25 KW. The results shown in in Fig. 5 illustrate the robustness of designed control system against deviation in local load parameters.

### C. Performance in Case of PnP functionality

The objective in this scenario is to validate the robustness of the designed controller against PnP functionality of DGs shown in Fig. 2. To conduct this scenario, we assume that at  $t = 2 s$ , DG2 is plugged out. Therefore, this disconnection, affects the dynamics of DG1. Then, at  $t = 3 s$ , DG2 is pugged back in to the microgrid system. The dynamic behavior of DGs is shown in Fig. 6. The results proves the capability of the designed control system to be robust against PnP functionality of DGs. Results confirm that the proposed controller guarantees stability and the desired performance of the closed-loop system in this scenario.

## VI. CONCLUSION

A robust  $H_{\infty}$  control strategy for voltage control of an islanded AC microgrid was developed in this paper. The configuration of the control system was entirely decentralized and the proposed control system is the result of a convex LMI-based optimization problem with a linearly parameter dependent Lyapunov function. Robustness of the proposed controller with regard to PnP capability and load uncertainties was achieved by a unique controller. Consequently, the stability and the desired performance of the microgrid is guaranteed in the case of the plug in or plug out of DGs and load perturbations. Several case studies were investigated in SimPowerSystem Toolbox of MATLAB that verified fast tracking response.

## REFERENCES

- [1] D. E. Olivares, A. Mehrizi-sani, A. H. Etemadi, C. A. Canizares, R. Iravani, M. Kazerani, A. H. Hajimiragha, O. Gomis-Bellmunt, M. Saadifard, R. Palma-Benke, G. A. Jimenez-Estevez, and N. D. Hatziargyrios, "Trends in microgrid control," *IEEE Transaction on Smart Grid*, vol. 5, no. 4, pp. 1905-1919, 2014.
- [2] J. M. Guerrero, M. Chandorkar, T. Lee, and P. C. Loh. "Advanced control architectures for intelligent microgrids—Part I: Decentralized and hierarchical control." *IEEE Transactions on Industrial Electronics*, vol. 60, no. 4, pp. 1254-1262, 2013.
- [3] A. Parisio, E. Rikos, and L. Glielmo, "A model predictive control approach to microgrid operation optimization," *IEEE Transaction on Control System Technology*, vol. 22, no. 5, pp. 1813-1827, 2014.
- [4] M. Yazdani and A. Mehrizi-Sani, "Distributed control techniques in microgrids," *IEEE Transaction on Smart Grids*, vol. 5, no. 6, pp. 2901-2909, 2014.
- [5] A. Bidram, F. L. Lewis, and A. Davoudi. "Distributed control systems for small-scale power networks: Using multiagent cooperative control theory" *IEEE Control systems magazine*, vol. 34, no. 6, pp. 56-77, 2014.
- [6] J. M. Guerrero, M. Chandorkar, T. Lee, and P. C. Loh. "Advanced control architectures for intelligent microgrids—Part I: Decentralized and hierarchical control." *IEEE Transactions on Industrial Electronics*, vol. 60, no. 4, pp. 1254-1262, 2013.
- [7] J. M. Guerrero, J. C. Vasquez, J. Matas, L. G. de Vicuna, and M. Castilla, "Hierarchical control of droop-controlled AC and DC microgrids—A general approach towards standardization," *IEEE Transaction on Industrial Electronics*, Vol. 58, no. 1, pp. 158-172, 2011.
- [8] J. Schiffer, R. Ortega, A. Astolfi, J. Raisch, and T. Sezi, "Conditions for Stability of droop-controlled inverter-based microgrids," *Automatica*, vol. 50, no. 10, pp. 2457-2469, 2014.
- [9] M. C. Chandorkar, D. M. Divan, and R. Adapa, "Control of parallel connected inverters in standalone AC supply systems," *IEEE Transaction on Industry Applications*, vol. 29, no. 1, pp. 136-143, 1993.
- [10] P. Piagi, R. H. Lasseter. "Autonomous control of microgrids." *IEEE Power Engineering Society General Meeting*, pp. 8, 2006.
- [11] J. M. Guerrero, M. Josep, J. Matas, L. G. De Vicuna, M. Castilla, and J. Miret. "Decentralized control for parallel operation of distributed generation inverters using resistive output impedance." *IEEE Transactions on industrial electronics*, vol. 54, no. 2, pp. 994-1004, 2007.
- [12] M. S. Sadabadi, A. Haddadi, H. Karimi, and A. Karimi. "A robust active damping control strategy for an LCL-based grid-connected DG unit." *IEEE Transactions on Industrial Electronics*, vol. 64, no. 10, pp. 8055-8065, 2017.
- [13] M. S. Sadabadi, Q. Shafiee, A. Karimi. "Plug-and-play voltage stabilization in inverter-interfaced microgrids via a robust control strategy." *IEEE Transactions on Control Systems Technology*, vol. 25, no. 3, pp. 781-791, 2017.
- [14] M. Babazadeh and H. Karimi, "A robust two-degree-of-freedom control strategy for an islanded microgrid," *IEEE Transaction on Power Delivery*, vol. 28, no. 3, pp. 1339-1347, 2013.
- [15] M. S. Sadabadi, A. Karimi, H. Karimi, "Fixed-order decentralized/distributed control of islanded inverter-interfaced microgrids", *Control Engineering practice*, vol. 45, pp. 174-193, 2015.
- [16] A. H. Etemadi, E. J. Davison, and R. Iravani. "A generalized decentralized robust control of islanded microgrids." *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 3102-3113, 2014.
- [17] S. Rivero, F. Sarzo, and G. Ferrari-Trecate. "Plug-and-play voltage and frequency control of islanded microgrids with meshed topology." *IEEE Transactions on Smart Grid*, vol. 6, no. 3, pp. 1176-1184, 2014.
- [18] C.-T. Lee, C.-C. Chu, and P.-T. Cheng, "A new droop control method for the autonomous operation of distributed energy resource interface converters," *IEEE Transaction on Power Electronics*, vol. 28, no. 4, pp. 1980-1993, 2013.
- [19] L. Sedghi, and A. Fakharian, "Robust voltage regulation in islanded microgrids: A LMI based mixed  $H_2/H_{\infty}$  control approach." *24th Mediterranean Conference on Control and Automation (MED)*, pp. 431-436, 2016.
- [20] V. Venkatasubramanian, H. Schattler, and J. Zaborszky, "Fast time-varying phasor analysis in the balanced three-phase large electric power system," *IEEE Trans. Autom. Control*, vol. 40, no. 11, pp. 1975-1982, 1995.
- [21] F. Katiraei and M. R. Iravani, "Power management strategies for a microgrid with multiple distributed generation units," *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp. 1821-1831, 2006.
- [22] A. H. Etemadi, E. J. Davison, and R. Iravani. "A decentralized robust control strategy for multi-DER microgrid-part I: Fundamental concepts." *IEEE Transactions on Power Delivery*, vol. 27, no. 4, pp. 1843-1853, 2012.
- [23] F. Katiraei, R. Iravani, N. Hatziargyriou. And A. Dimeas, "Microgrids management," *IEEE Power Energy Mag.*, vol. 6, no. 3, pp. 54-65, 2008.
- [24] S. Skogestad, and I. Postlethwaite, "Multivariable feedback control analysis and design," Hoboken, NJ, USA: Wiley, 1996.
- [25] L. A. Rodrigues, R. C. L. F. Oliveira, and J. F. Camino, "Parameterized LMIs for robust and state feedback control of continuous-time polytopic systems." *International Journal of Robust and Nonlinear Control*, vol. 28, no. 3, pp. 940-952, 2018.