

Bevrani, Hassan and Mitani, Yasunori and Tsuji, Kiichiro (2004) PI-based multiobjective load-frequency control in a restructured power system. In *Proceedings SICE 2004 Annual Conference* 2, pages pp. 1745-1750, Japan.

© Copyright 2004 IEEE Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

# PI-based Multi-objective Load-frequency Control in a Restructured Power System

H. Bevrani<sup>1</sup>, Y. Mitani<sup>2</sup>, K. Tsuji<sup>1</sup>

1 Department of Electrical Eng., Osaka University, 2-1 Yamada-Oka, Suita, Osaka 565-0871, Japan 2 Department of Electrical Eng., Kyushu Institute of Technology, Kyushu, Japan bevrani@polux.pwr.eng.osaka-u.ac.jp

**Abstract:** This paper addresses a new decentralized robust LFC design in a deregulated power system under bilateralbased policy scheme. The LFC problem is formulated to a PI based multi-objective control problem via a mixed  $H_2/H_{\infty}$  control technique. The robust PI control problem is reduced to a static output feedback control synthesis, and it is solved using a developed iterative linear matrix inequalities algorithm to get a robust performance index close to specified optimal one. The proposed method is applied to a 3-control area power system with possible contract scenarios and a wide range of load changes. The results are compared with  $H_2/H_{\infty}$  dynamic control design.

**Keywords:** Load frequency control, mixed  $H_2/H_{\infty}$  control, static output feedback control, bilateral LFC scheme, linear matrix inequalities (LMI).

# 1. Introduction

Naturally, LFC is a multi-objective control problem. LFC goals, i.e. frequency regulation and tracking the load changes, maintaining the tie-line power interchanges to specified values in presence of generation constraints and dynamical model uncertainties, determines the LFC synthesis as a multiobjective control problem. Therefore, it is expected that an appropriate multi-objective control strategy could be able to give a better solution for this problem <sup>1)</sup>. It is well known that each robust method is mainly useful to capture a set of special specifications. For instance, the  $H_2$  tracking design is more adapted to deal with transient performance by minimizing the linear quadratic cost of tracking error and control input, but  $H_{\infty}$  approach (and  $\mu$  as a generalized  $H_{\infty}$  approach) is more useful to hold closed-loop stability in presence of control constraints and uncertainties. While the  $H_{\infty}$  norm is natural for norm-bounded perturbations, in many applications the natural norm for the input-output performance is the  $H_2$  norm.

In this paper, the LFC synthesis problem is formulated as a mixed  $H_2/H_{\infty}$  static output feedback (SOF) control problem to obtain a desired PI controller. An iterative linear matrix inequalities (ILMI) algorithm is developed to compute the PI parameters. The model uncertainty in each control area is covered by an unstructured multiplicative uncertainty block. The proposed strategy is applied to a three control area example. The designed robust PI controllers, which are ideally practical for industry, are compared with the mixed  $H_2/H_{\infty}$  dynamic output feedback controllers (using general LMI technique<sup>2</sup>). The results show the PI controllers guarantee the robust performance for a wide range of operating conditions as well as  $H_2/H_{\infty}$  dynamic controllers.

This paper is organized as follows: The generalized LFC model in a bilateral-based power system market is given in

section 2. Section 3 presents the problem formulation via mixed  $H_2/H_{\infty}$  technique for a given control area. The PIbased multi-objective LFC design using a developed ILMI is given in section 4. The proposed methodology is applied to a 3-control area power system as a case study, in section 5. Finally to demonstrate the effectiveness of the proposed method and to compare with mixed  $H_2/H_{\infty}$  dynamic output feedback control design, some simulation results are given in section 6.

# 2. Bilateral-based LFC scheme <sup>1)</sup>

Based on the generalized LFC scheme <sup>1)</sup>, overall power system structure can be considered as a collection of distribution companies (Discos) or separated control areas interconnected through high voltage transmission lines or tielines. Each control area has its own LFC and is responsible for tracking its own load and honoring tie-line power exchange contracts with its neighbors. There can be various combinations of contracts between each Disco and available generation companies (Gencos). On the other hand each Genco can contract with various Discos. The "generation participation matrix (GPM)" concept is defined to express these bilateral contracts in the generalized model. GPM shows the participation factor of each Genco in the considered control areas and each control area is determined by a Disco. The rows of a GPM correspond to Gencos and columns to control areas which contract power. For example, the GPM for a large scale power system with m control areas (Discos) and nGencos, has the following structure. Where  $gpf_{ij}$  refers to

"generation participation factor" and shows the participation factor of Genco i in the load following of area j (based on a specified bilateral contract).

$$GPM = \begin{bmatrix} gpf_{11} & gpf_{12} & \cdots & gpf_{1(m-1)} & gpf_{1m} \\ gpf_{21} & gpf_{22} & \cdots & gpf_{2(m-1)} & gpf_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ gpf_{(n-1)1} & gpf_{(n-1)2} & \cdots & gpf_{(n-1)(m-1)} & gpf_{(n-1)m} \\ gpf_{n1} & gpf_{n2} & \cdots & gpf_{n(m-1)} & gpf_{nm} \end{bmatrix}$$
(1)

The generalized LFC block diagram for control area *i* in a deregulated environment is shown in Fig. 1. New information signals due to possible various contracts between Disco *i* and other Discos and Gencos are shown as dashed-line inputs, and, we can write <sup>1</sup>):

$$v_{li} = \Delta P_{Loc-i} + \Delta P_{di} \tag{2}$$

$$v_{2i} = \sum_{\substack{j=l\\j\neq i}}^{N} T_{ij} \Delta f_j \tag{3}$$

$$v_{3i} = \sum_{j \neq i} (Total \ export \ power - Total \ import \ power)$$
$$= \sum_{\substack{j=1\\j\neq i}}^{N} (\sum_{k=1}^{n} gpf_{kj}) \Delta P_{Lj} - \sum_{k=1}^{n} (\sum_{\substack{j=1\\j\neq i}}^{N} gpf_{jk}) \Delta P_{Li}$$
(4)

$$v_{4i} = \begin{bmatrix} v_{4i-1} & v_{4i-2} & \cdots & v_{4i-n} \end{bmatrix}$$
(5)

$$v_{4i-1} = \sum_{j=1}^{n} gpf_{1j} \Delta P_{Lj}$$
  

$$\vdots$$
(6)

$$v_{4i-n} = \sum_{j=l}^{N} gpf_{nj} \Delta P_{Lj}$$

$$\Delta P_{tie-i,error} = \Delta P_{tie-i,actual} - v_{3i} \tag{7}$$

$$\sum_{i=1}^{n} gpf_{ij} = 1 \tag{8}$$

$$\sum_{k=l}^{n} \alpha_{ki} = l \quad ; \quad 0 \le \alpha_{ki} \le l$$
<sup>(9)</sup>

$$\Delta P_{mi} = \sum_{j=1}^{N} gp f_{ij} \Delta P_{Lj} \tag{10}$$



Fig. 1. Generalized LFC (bilateral-based) model in a deregulated environment.

where,  $\Delta f_i$ : frequency deviation,  $\Delta P_{gi}$ : governor valve position,  $\Delta P_{ci}$ : governor load setpoint,  $\Delta P_{ii}$ : turbine power,  $\Delta P_{tie-i}$ : net tie-line power flow,  $\Delta P_{di}$ : area load disturbance,  $M_i$ : equivalent inertia constant,  $D_i$ : equivalent damping coefficient,  $T_{gi}$ : governor time constant,  $T_{ii}$ : turbine time constant,  $T_{ij}$ : tie-line synchronizing coefficient between area *i* & *j*,  $B_i$ : frequency bias,  $R_i$ : drooping characteristic,  $\alpha$ : ACE participation factor, *N*: number of control areas,  $\Delta P_{Li}$ : contracted demand of area *i*,  $\Delta P_{mi}$ : power generation of a Genco *i*,  $\Delta P_{Loc-i}$ : total local demand (contracted and uncontracted) in area *i*,  $v_{3i}$ : scheduled  $\Delta P_{tie-i}$  ( $\Delta P_{tie-i, scheduled}$ ), and  $\Delta P_{tie-i, actual}$ : actual  $\Delta P_{tie-i}$ .

# **3. LFC formulation via mixed** $H_2/H_{\infty}$

The main control framework in order to formulation of LFC problem via a mixed  $H_2/H_{\infty}$  control design for a given control area (Fig. 1) is shown in Fig. 2.  $\Delta_i$  models the structured uncertainty set in the form of multiplicative type and  $W_i$  includes the associated weighting function. It is notable that in model of power system there are several uncertainties because of parameter variations, model linearization and unmodeled dynamics due to some approximations. Usually, the uncertainties in power system can be modelled as multiplicative and/or additive uncertainties <sup>3)</sup>. The output channel  $z_{\infty i}$  is associated with the  $H_{\infty}$  performance while the fictitious output  $z_{2i}$  is associated with LQG aspects or  $H_2$  performance.

 $\eta_{1i}$ ,  $\eta_{2i}$  and  $\eta_{3i}$  are constant weights that must be chosen by designer to get the desired performance and considering practical constraint on control action. Experience suggests that one can fix the weights  $\eta_{1i}$ ,  $\eta_{2i}$  and  $\eta_{3i}$  to unity and use the method with regional pole placement technique for performance tuning <sup>4</sup>).  $G_i(s)$  and  $K_i(s)$  correspond to the nominal dynamical model of the given control area and controller, respectively. Also  $y_i$  is the measured output (performed by area control error ACE),  $u_i$  is the control input and  $w_i$  includes the perturbed and disturbance signals in control area.



**Fig. 2.** Mixed  $H_2/H_{\infty}$ -based control framework

According to Fig. 2, the LFC as a multi-objective control problem can be expressed by the following optimization problem: Design a controller that minimizes the 2-norm of the fictitious output signal  $z_{2i}$  under the constraints that the  $\infty$ -norm of the transfer function from  $w_{1i}$  to  $z_{\infty i}$  is less than one. On the other hand, the LFC design is reduced to find an

internally stabilizing controller  $K_i$  which minimizes  $\|T_{z_{2i} w_{2i}}\|_2$  while maintaining  $\|T_{z_{\infty i} w_{Ii}}\|_{\infty} < 1$ . This problem can be solved by convex optimization using linear matrix inequalities.

Considering Fig 1 and the proposed control framework (Fig. 2), the state space model for control area *i*,  $G_i(s)$ , can be obtained as

$$\dot{x}_{i} = A_{i}x_{i} + B_{I_{i}}w_{i} + B_{2i}u_{i} z_{\infty i} = C_{\infty i}x_{i} + D_{\infty I_{i}}w_{i} + D_{\infty 2i}u_{i} z_{2i} = C_{2i}x_{i} + D_{2I_{i}}w_{i} + D_{22i}u_{i} y_{i} = C_{yi}x_{i} + D_{yI_{i}}w_{i}$$

$$(11)$$

where

$$x_i^T = [\Delta f_i \quad \Delta P_{tie-i} \quad \int ACE_i \quad x_{ti} \quad x_{gi}]$$
(12)

$$x_{ti} = \begin{bmatrix} \Delta P_{t1i} & \Delta P_{t2i} & \cdots & \Delta P_{tni} \end{bmatrix}$$
(13)

$$x_{gi} = \begin{bmatrix} \Delta P_{g1i} & \Delta P_{g2i} & \cdots & \Delta P_{gni} \end{bmatrix}$$
(14)

$$w_i^{T} = \begin{bmatrix} w_{1i} & w_{2i} \end{bmatrix}, \ w_{2i}^{T} = \begin{bmatrix} v_{1i} & v_{2i} & v_{3i} \end{bmatrix}$$
(15)

$$v_{4i}^{T} = \begin{bmatrix} v_{4i-1} & v_{4i-2} & \cdots & v_{4i-n} \end{bmatrix}$$
(16)

$$y_i = ACE_i \tag{17}$$

$$u_i = \Delta P_{C_i}, \quad z_{2i}^T = \left[ \eta_{1i} \Delta f_i \quad \eta_{2i} \int ACE_i \quad \eta_{3i} \Delta P_{C_i} \right]$$
(18)

and,

$$\begin{split} A_{i} &= \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix}, B_{1i} = \begin{bmatrix} B_{111} & B_{112} \\ B_{121} & B_{1i22} \\ B_{1i31} & B_{1i32} \end{bmatrix}, B_{2i} = \begin{bmatrix} B_{2i1} \\ B_{2i2} \\ B_{2i3} \end{bmatrix}, \\ A_{i11} &= \begin{bmatrix} -D_{i}/2\pi M_{i} & -1/2\pi M_{i} & 0 \\ \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} & 0 & 0 \\ B_{i} & 1 & 0 \end{bmatrix}, A_{i12} = \begin{bmatrix} 1/2\pi M_{i} & \cdots & 1/2\pi M_{i} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}_{3\times n} \\ A_{i22} &= -A_{i23} = diag \begin{bmatrix} -1/T_{i1i} & -1/T_{i2i} & \cdots & -1/T_{ini} \end{bmatrix}, \\ A_{i33} &= diag \begin{bmatrix} -1/T_{g1i} & -1/T_{g2i} & \cdots & -1/T_{gni} \end{bmatrix}, \\ A_{i33} &= diag \begin{bmatrix} -1/T_{g1i} & 0 & 0 \\ \vdots & \vdots & \vdots \\ -1/(T_{gni}R_{ni}) & 0 & 0 \end{bmatrix}, A_{i13} = A_{i21}^{T} = 0_{3\times n}, A_{i32} = 0_{m\times n}, \\ B_{1i12} &= \begin{bmatrix} -1/2\pi M & 0 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 0 & \cdots & 0 \end{bmatrix}_{3\times (3+n)}, \\ B_{1i22} &= 0_{n\times (3+n)}, B_{1i32} = \begin{bmatrix} 0_{n\times 3} & b \end{bmatrix}, \\ b &= diag \begin{bmatrix} 1/T_{g1i} & 1/T_{g2i} & \cdots & 1/T_{gni} \end{bmatrix}, \end{split}$$

 $B_{2i1} = 0_{3\times 1}, B_{2i2} = 0_{n\times 1}, B_{2i3}^{T} = \left[\alpha_{1i}/T_{g_{1i}} \quad \alpha_{2i}/T_{g_{2i}} \quad \cdots \quad \alpha_{ni}/T_{g_{ni}}\right],$   $B_{1i11} = 0_{3\times 1}, B_{1i21} = 0_{n\times 1}, B_{1i31}^{T} = \left[\alpha_{1i}/T_{g_{1i}} \quad \alpha_{2i}/T_{g_{2i}} \quad \cdots \quad \alpha_{ni}/T_{g_{ni}}\right],$   $C_{\infty i} = 0_{1\times(2n+3)}, D_{\infty 1i} = \left[-1 \quad 0_{1\times(3+n)}\right], D_{\infty 2i} = 1,$  $C_{2i} = \left[c_{2i1} \quad c_{2i2}\right], c_{2i1} = \left[\begin{array}{c}\eta_{1i} \quad 0 \quad 0\\ 0 \quad 0 \quad \eta_{2i}\\ 0 \quad 0 \quad 0\end{array}\right], c_{2i2} = 0_{3\times 2n},$ 

$$D_{21i} = \theta_{3 \times (4+n)}, \ D_{22i} = \begin{bmatrix} 0\\ 0\\ \eta_{3i} \end{bmatrix}.$$

# 4. PI-based multi-objective LFC design

Assume K(s) in Fig. 1 is a PI controller. We can formulate the PI in the following SOF control law  $^{1)}$ ,

$$u_{i} = \begin{bmatrix} k_{Pi} & k_{Ii} \end{bmatrix} \begin{bmatrix} ACE_{i} \\ \int ACE_{i} \end{bmatrix}, \quad (u_{i} = K_{i}y_{i})$$
(19)

Therefore,  $y_i$  in (17) can be augmented to following form

$$y_i^T = [ACE_i \quad \int ACE_i] \tag{20}$$

and for the corresponded coefficients in (11), we can write

$$C_{yi} = \begin{bmatrix} c_{yi} & \theta_{2\times 2n} \end{bmatrix}, c_{yi} = \begin{bmatrix} \beta_i & 1 & 0\\ 0 & 0 & 1 \end{bmatrix}, D_{y1i} = \theta_{2\times (4+n)}$$

Consider a linear time invariant system  $G_i(s)$  with the state-space realization of (11). A mixed  $H_2/H_{\infty}$  SOF control design can be expressed in following optimization problem.

<u>Optimization problem</u>: Determine an admissible SOF law  $K_i$ , belong to family of internally stabilizing SOF gains  $K_{sof}$ ,

$$u_i = K_i y_i , \ K_i \in K_{sof}$$

$$\tag{21}$$

such that

$$\inf_{K_i \in K_{sof}} \left\| T_{z_{2i} w_{2i}} \right\|_2 \text{ subject to } \left\| T_{z_{\infty i} w_{1i}} \right\|_\infty < 1$$
(22)

This problem defines a robust performance synthesis problem where the  $H_2$  norm is chosen as the performance measure. Recently, several methods are proposed to obtain the suboptimal solution for the  $H_2$ ,  $H_{\infty}$  and  $H_2/H_{\infty}$  SOF control problems <sup>5,6)</sup>.

On substitution of (21) into (11), it is easy to find that for each control area the state space realization of closed-loop system will be given as

$$\begin{aligned} \dot{x}_i &= A_{ic} x_i + B_{lic} w_i \\ z_{\infty i} &= C_{\infty ic} x_i \\ z_{2i} &= C_{2ic} x_i \\ y_i &= C_{yic} x_i \end{aligned}$$
(23)

and we can write,

$$\left\|T_{z_{2i}\,w_{2i}}\right\|_{2}^{2} = trace(C_{2ic}L_{C}C_{2ic}^{T})$$
(24)

where  $L_C$  denotes the controllability Gramian of the pair  $(A_{ic}, B_{Iic})$ . Lemma 2 in Ref. [6] gives a solution for  $H_2$  suboptimal SOF control problem. Following, a new ILMI algorithm is introduced to get a suboptimal solution for the above optimization problem. Specifically, the proposed algorithm formulates the  $H_2/H_{\infty}$  SOF control as a general SOF stabilization problem (see theorem 2 in Ref. [7]) to get a

family of  $H_2$  stabilizing controllers  $K_{sof}$ . Then the designed controller  $K_i \in K_{sof}$  will be chosen such that

$$\left|\gamma_{2}^{*}-\gamma_{2}\right|<\varepsilon, \quad \gamma_{\infty}=\left\|T_{z_{\infty i} w_{Ii}}\right\|_{\infty}<1$$
(25)

where  $\varepsilon$  is a small real positive number,  $\gamma_2^*$  is resulted  $H_2$  performance by  $K_i$  subject to given constraint in (25) and  $\gamma_2$  is resulted  $H_2$  optimal performance index from applied standard  $H_2/H_{\infty}$  dynamic output feedback control to the control area *i* as shown in Fig. 3.

Using lemma 2 in Ref. [6], a family of  $H_2$  stabilizing SOF gains  $K_{sof}$  can be obtained. But we are looking for the solution of such controller within this family which satisfy the given constraint in (25). Developed algorithm, gives an iterative LMI suboptimal solution to obtain a  $H_2/H_{\infty}$  SOF controller for a given power system control area:

*Step 1.* Compute the state-space model (11) for the given control area.

**Step 2.** Compute the optimal guaranteed  $H_2$  performance index  $\gamma_2$  using function *hinfmix* in MATLAB based LMI control toolbox <sup>2)</sup> to design standard  $H_2/H_{\infty}$  output dynamic controller as descript in section 3, for the performed system in step 1.

**Step 3.** Set i = l,  $\Delta \gamma_2 = \Delta \gamma_0$  and let  $\gamma_{2i} = \gamma_0 > \gamma_2$ .  $\Delta \gamma_0$  and  $\gamma_0$  are positive real numbers.

**Step 4.** Select  $Q = Q_0 > 0$ , and solve X from the following algebraic Riccati equation

 $A_{i}X + XA_{i}^{T} - XC_{yi}^{T}C_{yi}X + Q = 0, \quad X > 0$ (26) Set  $P_{i} = X$ .

*Step 5.* Solve the following optimization problem for  $X_i$ ,  $K_i$  and  $a_i$ :

Minimize  $a_i$  subject to the bellow LMI constraints:

$$\begin{bmatrix} A_{i}X_{i} + X_{i}A_{i}^{T} + B_{1i}B_{1i}^{T} - P_{i}C_{yi}^{T}C_{yi}X_{i} - X_{i}C_{yi}^{T}C_{yi}P_{i} + P_{i}C_{yi}^{T}C_{yi}P_{i} - a_{i}X_{i} \\ (B_{2i}K_{i} + X_{i}C_{yi}^{T})^{T} \\ B_{2i}K_{i} + X_{i}C_{yi}^{T} \end{bmatrix} < 0$$

 $trace(C_{2ic}L_C C_{2ic}^{T}) < \gamma_{2i}$   $\tag{28}$ 

 $X_i = X_i^T > 0 \tag{29}$ 

Denote  $a_i^*$  as the minimized value of  $a_i$ .

Step 6. If  $a_i^* \leq 0$ , go to step 10.

**Step 7.** For i > 1 if  $a_{i-1}^* \le 0$ ,  $K_{i-1} \in K_{sof}$  is an  $H_2$  controller and go to step 10. Otherwise go to step 8.

*Step 8.* Solve the following optimization problem for  $X_i$  and  $K_i$ :

Minimize  $trace(X_i)$  subject to LMI constraints (27-29) with  $a_i = a_i^*$ . Denote  $X_i^*$  as the  $X_i$  that minimized  $trace(X_i)$ .

*Step 9.* Set i = i+1 and  $P_i = X_{i-1}^*$ , then go to step 5. *Step 10.* Set  $\gamma_{2i} = \gamma_{2i} - \Delta \gamma_2$ , i = i+1. Then do steps 4 to 6. *Step 11.* If

$$\gamma_{\infty,i-l} = \left\| W_i K_{i-l} G_i (I + K_{i-l} G_i)^{-l} \right\|_{\infty} \le 1$$
(30)

the  $K_{i-1}$  is an  $H_2/H_{\infty}$  SOF controller and  $\gamma_2^* = \gamma_{2i} - \Delta \gamma_2$ indicates a lower  $H_2$  bound such that the obtained controller satisfies (25). Otherwise set  $\gamma_{2i} = \gamma_{2i} - \Delta \gamma_2$ , i=i+1, then do steps 4-6.

# 5. Application to a 3-control area power system

To illustrate the effectiveness of proposed control strategy, a three control area power system, shown in Fig. 3, is considered as a test system. It is assumed that each control area includes two Gencos, which use the same ACE participation factor. The power system parameters are considered the same as in Ref. [1].



Fig. 3. Three control area power system

#### 5.1. Uncertainty and performance weights selection

In this example with regards to uncertainties, it is assumed that the rotating mass and load pattern parameters have uncertain values in each control area. The variation range for  $D_i$  and  $M_i$  parameters in each control area is assumed  $\pm \%20$ . Following, these uncertainties is modelled as an unstructured multiplicative uncertainty block that contains all the information available about  $D_i$  and  $M_i$  variations.

Let  $\hat{G}_i(s)$  denotes the transfer function from the control input  $u_i$  to control output  $y_i$  at operating points other than nominal point. Following a practice common in robust control, we will represent this transfer function as

$$\begin{aligned} \left| \Delta_{i}(s) W_{i}(s) \right| &= \left| [\hat{G}_{i}(s) - G_{0i}(s)] G_{0i}(s)^{-1} \right| ; \quad G_{0i}(s) \neq 0 \quad (31) \end{aligned}$$
  
where,  
$$\left\| \Delta_{i}(s) \right\|_{\infty} = \sup_{\omega} \left| \Delta_{i}(s) \right| \leq 1 \quad (32)$$

 $\Delta_i(s)$  shows the uncertainty block corresponding to uncertain parameters and  $G_{0i}(s)$  is the nominal transfer function model. Thus,  $W_i(s)$  is such that its respective magnitude bode plot covers the bode plots of all possible plants. For example, using (31) some sample uncertainties corresponding to different values of  $D_i$  and  $M_i$  for area 1 can be obtained as shown in Fig. 4. The uncertainties due to both set of parameters variation can be modeled by using a norm bonded multiplicative uncertainty to cover all possible plants as follows.

$$W_1(s) = \frac{0.3986s + 0.0786}{s + 0.6888} \tag{33}$$



Fig. 4. Uncertainty plots due to parameters changes in area 1;  $D_i$  (dotted),  $M_i$  (dash-dotted) and  $W_i$  (solid).

Fig. 4 clearly shows that the weight (33) used in our design provides a conservative design at low and high frequencies but it gives a good trade-off between robustness and controller complexity. Using the same method, the uncertainty weighting functions for areas 2 and 3 are computed as follows.

$$W_2(s) = \frac{0.3088s + 0.0487}{s + 0.6351}, \quad W_3(s) = \frac{0.3483s + 0.0751}{s + 0.7826}$$
(34)

The selection of performance constant weights  $\eta_{1i}$ ,  $\eta_{2i}$ and  $\eta_{3i}$  is dependent on specified performance objectives and must be chosen by designer. In fact an important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives <sup>1)</sup>. Here, a set of suitable values for constant weights is chosen as follows:

$$\eta_{1i} = 1.25, \ \eta_{2i} = 0.001, \ \eta_{3i} = 1.5$$
 (35)

## 5.2. Mixed $H_2/H_{\infty}$ dynamic and SOF control design

For the sake of comparison, in addition to proposed control strategy to synthesis the robust PI controller, a mixed  $H_2/H_{\infty}$  dynamic output feedback controller is designed for each area, using *hinfmix* function in LMI control toolbox <sup>2</sup>). This function gives an optimal  $H_2/H_{\infty}$  controller through the mentioned optimization problem (22) and returns the controller *K*(*s*) with optimal  $H_2$  performance index  $\gamma_2$ . The resulted controllers are dynamic type whose orders are the same as size of generalized plant model (8<sup>th</sup> order in the present paper).

At the next step, according to synthesis methodology described in section 4, a set of three decentralized robust PI controllers are designed. The control parameters are shown in table 1. The optimal performance indices for dynamic and PI controllers are listed in table 2.

The resulted robust performance indices of both synthesis methods ( $\gamma_{2i}$  and  $\gamma_{2i}^*$ ) are close to each other. It shows that although the proposed ILMI approach gives a set of much simpler controllers (PI) than the dynamic  $H_2/H_{\infty}$  design, however they holds robust performance as well as dynamic  $H_2/H_{\infty}$  controllers.

Table 1. PI control parameters from ILMI design

Parameters	Areal	Area 2	Area 3
$k_{Pi}$	-0.1250	-0.0015	-0.4278
$k_{Ii}$	-5.00E-04	-5.14E-04	-5.30E-04

Table 2. Robust performance indices				
Performance index	Areal	Area 2	Area 3	
$\gamma_{2i}$ (Dynamic)	2.1835	1.7319	2.1402	
$\gamma_{2i}^{*}$ (PI)	2.2900	1.8321	2.2370	
$\gamma_{\infty i}$ (Dynamic)	0.4177	0.3339	0.3536	
$\gamma_{\infty i}^{*}$ (PI)	0.3986	0.3088	0.3483	

## 6. Simulation results

In order to demonstrate the effectiveness of the proposed strategy, some simulations were carried out. In these simulations, the proposed PI controllers were applied to the three control area power system described in Fig. 3. The performance of the closed-loop system using the designed PI controllers in comparison of full-order  $H_2/H_{\infty}$  dynamic controllers is tested in presence of load demands, disturbances and uncertainties.

<u>Case 1:</u>

In this case, the closed-loop performance is tested in face of both step contracted load demand and uncertainties. It is assumed a large load demand 100 MW (0.1 pu) is requested by each Disco, following %20 decrease in uncertain parameters  $D_i$  and  $M_i$ . Furthermore, assume Discos contract with the available Gencos according to the following GPM,

$$GPM^{T} = \begin{bmatrix} 0.25 & 0.5 & 0 & 0.25 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 \\ 0 & 0 & 0.75 & 0 & 0 & 0.25 \end{bmatrix}$$

Frequency deviation, area control error (ACE1 and ACE2) and tie-line power changes are shown in Fig. 5. Using the proposed method, the area control error and frequency deviation of all areas are quickly driven back to zero. The tie-line power flows are properly convergence to specified values.

<u>Case 2:</u>

Consider the case 1 again. Assume in addition to specified contracted load demands (0.1 pu) and %20 decrease in  $D_i$  and  $M_i$ , a bounded random step load change as a large uncontracted demand (shown in Fig. 6a) is appeared in each control area, where

$$-50 MW \le \Delta P_{di} \le +50 MW$$

The purpose of this scenario is testing the robustness of the proposed controllers against uncertainties and random large load disturbances. The closed-loop response for areas 1 and 3 are shown in Figs. 6b and 6c.



**Fig. 5.** a) Frequency deviation; b) area control error and tie-line powers; solid (ILMI-based PI), dotted (dynamic  $H_2/H_{\infty}$ ).





**Fig. 6.** Power system response for case 2. (a) random load patterns b) area-1,c) area-3; solid (ILMI-based PI), dotted (dynamic  $H_2/H_{\infty}$ ).

The simulation results demonstrate that the proposed ILMI-based PI controllers track the load fluctuations and meet robustness for a wide range of load disturbances and possible bilateral contract scenarios as well as  $H_2/H_{\infty}$  dynamic controllers.

# 7. Conclusion

The LFC problem in a multi-area power system is formulated as a decentralized multi-objective optimization control problem via mixed  $H_2/H_{\infty}$  technique. An iterative LMI approach has been proposed for a bilateral-based LFC scheme. Design strategy includes enough flexibility to set the desired level of performance and gives a set of simple PI controllers, which commonly useful in the real-world power systems. The proposed method was applied to a three control area power system. It was shown that the proposed simple ILMI-based PI controllers are capable to guarantee the robust performance as well as  $H_2/H_{\infty}$  dynamic controllers.

### References

- H. Bevrani, Y. Mitani, K. Tsuji, Robust AGC: traditional structure versus restructured scheme, IEEJ Trans. on Power and Energy, vol. 124, no. 5, 2004.
- [2] P. Gahinet, A. Nemirovski, A. J. Laub, M. Chilali, LMI Control Toolbox, The MathWorks, Inc., 1995.
- [3] H. Bevrani, Y. Mitani, K. Tsuji, On robust load-frequency regulation in a restructured power system, IEEJ Trans. on Power and Energy, vol. 124, no. 2, pp. 190-198, 2004.
- [4] P. Gahinet, M. Chilali,  $H_{\infty}$  -design with pole placement constraints, IEEE Trans. Automatic Control, vol. 41, no. 3, pp. 358-367, 1996.
- [5] F. Leibfritz, An LMI-based algorithm for designing suboptimal static H<sub>2</sub>/H<sub>∞</sub> output feedback controllers, SIAM J. Control Optim., vol. 39, no. 6, pp. 1711-1735, 2001.
- [6] F. Zheng, Q. G. Wang, H. T. Lee, On the design of multivariable PID controllers via LMI approach, Automatica, vol. 38, pp. 517-526, 2002.
- [7] Y. Y. Cao, X. Suny, W. J. Mao, Static output feedback stabilization: an ILMI approach, Automatica, vol. 34, no. 12, pp. 1641-1645, 1998.