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PI-based Multi-objective Load-frequency Control in a Restructured Power System

H. Bevrani¹, Y. Mitani², K. Tsuji¹

1 Department of Electrical Eng., Osaka University, 2-1 Yamada-Oka, Suita, Osaka 565-0871, Japan 2 Department of Electrical Eng., Kyushu Institute of Technology, Kyushu, Japan bevrani@polux.pwr.eng.osaka-u.ac.jp

Abstract: This paper addresses a new decentralized robust LFC design in a deregulated power system under bilateralbased policy scheme. The LFC problem is formulated to a PI based multi-objective control problem via a mixed H_2/H_{∞} control technique. The robust PI control problem is reduced to a static output feedback control synthesis, and it is solved using a developed iterative linear matrix inequalities algorithm to get a robust performance index close to specified optimal one. The proposed method is applied to a 3-control area power system with possible contract scenarios and a wide range of load changes. The results are compared with H_1/H_{∞} dynamic control design.

Keywords: Load frequency control, mixed H_1/H control, static output feedback control, bilateral LFC scheme, linear matrix inequalities (LMI).

1. Introduction

Naturally, LFC is a multi-objective control problem. LFC goals, i.e. frequency regulation and tracking the load changes, maintaining the tie-line power interchanges to specified values in presence of generation constraints and dynamical model uncertainties, determines the LFC synthesis as a multiobjective control problem. Therefore, it is expected that an appropriate multi-objective control strategy could be able to give a better solution for this problem 1). It is well known that each robust method is mainly useful to capture a set of special specifications. For instance, the H_2 tracking design is more adapted to deal with transient performance by minimizing the linear quadratic cost of tracking error and control input, but H_{∞} approach (and μ as a generalized H_{∞} approach) is more useful to hold closed-loop stability in presence of control constraints and uncertainties. While the H_{∞} norm is natural for norm-bounded perturbations, in many applications the natural norm for the input-output performance is the H_2 norm.

In this paper, the LFC synthesis problem is formulated as a mixed H_2/H_{∞} static output feedback (SOF) control problem to obtain a desired PI controller. An iterative linear matrix inequalities (ILMI) algorithm is developed to compute the PI parameters. The model uncertainty in each control area is covered by an unstructured multiplicative uncertainty block. The proposed strategy is applied to a three control area example. The designed robust PI controllers, which are ideally practical for industry, are compared with the mixed H_2/H_{∞} dynamic output feedback controllers (using general LMI technique²⁾. The results show the PI controllers guarantee the robust performance for a wide range of operating conditions as well as H_2/H_{∞} dynamic controllers.

This paper is organized as follows: The generalized LFC model in a bilateral-based power system market is given in section 2. Section 3 presents the problem formulation via mixed H_1/H_{∞} technique for a given control area. The PIbased multi-objective LFC design using a developed ILMI is given in section 4. The proposed methodology is applied to a 3-control area power system as a case study, in section 5. Finally to demonstrate the effectiveness of the proposed method and to compare with mixed H_1/H_2 dynamic output feedback control design, some simulation results are given in section 6.

2. Bilateral-based LFC scheme 1)

Based on the generalized LFC scheme $\frac{1}{1}$, overall power system structure can be considered as a collection of distribution companies (Discos) or separated control areas interconnected through high voltage transmission lines or tielines. Each control area has its own LFC and is responsible for tracking its own load and honoring tie-line power exchange contracts with its neighbors. There can be various combinations of contracts between each Disco and available generation companies (Gencos). On the other hand each Genco can contract with various Discos. The "generation participation matrix (GPM)" concept is defined to express these bilateral contracts in the generalized model. GPM shows the participation factor of each Genco in the considered control areas and each control area is determined by a Disco. The rows of a GPM correspond to Gencos and columns to control areas which contract power. For example, the GPM for a large scale power system with *m* control areas (Discos) and *n* Gencos, has the following structure. Where gpf_{ii} refers to "generation participation factor" and shows the participation factor of Genco *i* in the load following of area *j* (based on a

specified bilateral contract).

$$
GPM = \begin{bmatrix} gpf_{11} & gpf_{12} & \cdots & gpf_{1(m-1)} & gpf_{1m} \\ gpf_{21} & gpf_{22} & \cdots & gpf_{2(m-1)} & gpf_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ gpf_{(n-1)1} & gpf_{(n-1)2} & \cdots & gpf_{(n-1)(m-1)} & gpf_{(n-1)m} \\ gpf_{n1} & gpf_{n2} & \cdots & gpf_{n(m-1)} & gpf_{nm} \end{bmatrix} (1)
$$

The generalized LFC block diagram for control area *i* in a deregulated environment is shown in Fig. 1. New information signals due to possible various contracts between Disco *i* and other Discos and Gencos are shown as dashed-line inputs, and, we can write 1):

$$
v_{Ii} = \Delta P_{Loc-i} + \Delta P_{di}
$$
 (2)

$$
v_{2i} = \sum_{\substack{j=1 \ j \neq i}}^N T_{ij} \Delta f_j \tag{3}
$$

$$
v_{3i} = \sum_{\substack{j\\j=1}} (Total\ export\ power - Total\ import\ power) = \sum_{\substack{j=1\\j\neq i}}^{N} \sum_{k=1} (S_{kj}^{n}) \Delta P_{kj} - \sum_{k=1}^{n} \sum_{\substack{j=1\\j\neq i}}^{N} gpf_{jk}) \Delta P_{ki}
$$
(4)

$$
v_{4i} = \begin{bmatrix} v_{4i-1} & v_{4i-2} & \cdots & v_{4i-n} \end{bmatrix}
$$
 (5)

$$
v_{4i-1} = \sum_{j=1}^{ } gpf_{ij}\Delta P_{Lj}
$$
\n
$$
\vdots
$$
\n(6)

$$
v_{4i\cdot n} = \sum_{j=1}^{N} g p f_{nj} \Delta P_{Lj}
$$

$$
\Delta P_{\text{tie-i, error}} = \Delta P_{\text{tie-i, actual}} - \nu_{\text{3i}} \tag{7}
$$

$$
\sum_{i=1}^{n} g p f_{ij} = I \tag{8}
$$

$$
\sum_{k=1}^{n} \alpha_{ki} = I \quad ; \quad 0 \le \alpha_{ki} \le I \tag{9}
$$

$$
\Delta P_{mi} = \sum_{j=1}^{N} g p f_{ij} \Delta P_{Lj} \tag{10}
$$

Fig. 1. Generalized LFC (bilateral-based) model in a deregulated environment.

where, Δf_i : frequency deviation, ΔP_{gi} : governor valve position, ΔP_{ci} : governor load setpoint, ΔP_{ti} : turbine power, $\Delta P_{\text{tie}-i}$: net tie-line power flow, ΔP_{di} : area load disturbance, M_i : equivalent inertia constant, D_i : equivalent damping coefficient, T_{gi} :governor time constant, T_{ii} : turbine time constant, T_{ij} : tie-line synchronizing coefficient between area *i* $\&$ *j*, *B_i* : frequency bias, *R_i* : drooping characteristic, α : ACE participation factor, *N:* number of control areas, ΔP_{Li} : contracted demand of area *i*, ΔP_{mi} : power generation of a Genco *i*, ΔP_{Loc-i} : total local demand (contracted and uncontracted) in area *i*, v_{3i} : scheduled ΔP_{tie-i} ($\Delta P_{tie-i, scheduled}$), and $\Delta P_{tie-i,actual}$: actual ΔP_{tie-i} .

3. LFC formulation via mixed H_2/H_{∞}

The main control framework in order to formulation of LFC problem via a mixed H_2/H_2 control design for a given control area (Fig. 1) is shown in Fig. 2. Δ_i models the structured uncertainty set in the form of multiplicative type and *Wi* includes the associated weighting function. It is notable that in model of power system there are several uncertainties because of parameter variations, model linearization and unmodeled dynamics due to some approximations. Usually, the uncertainties in power system can be modelled as multiplicative and/or additive uncertainties ³⁾. The output channel $z_{\infty i}$ is associated with the H_{∞} performance while the fictitious output z_{2i} is associated with LQG aspects or H_2 performance.

 η_{1i} , η_{2i} and η_{3i} are constant weights that must be chosen by designer to get the desired performance and considering practical constraint on control action. Experience suggests that one can fix the weights η_{1i} , η_{2i} and η_{3i} to unity and use the method with regional pole placement technique for performance tuning ⁴⁾. $G_i(s)$ and $K_i(s)$ correspond to the nominal dynamical model of the given control area and controller, respectively. Also y_i is the measured output (performed by area control error ACE), u_i is the control input and *wi* includes the perturbed and disturbance signals in control area.

Fig. 2. Mixed H_2/H_∞ -based control framework

According to Fig. 2, the LFC as a multi-objective control problem can be expressed by the following optimization problem: Design a controller that minimizes the 2-norm of the fictitious output signal z_{2i} under the constraints that the ∞ norm of the transfer function from w_{1i} to $z_{\alpha i}$ is less than one. On the other hand, the LFC design is reduced to find an

internally stabilizing controller K_i which minimizes $\left\|T_{z_{2i}w_{2i}}\right\|_{2}$ while maintaining $\left\|T_{z_{\infty i}w_{1i}}\right\|_{\infty} < 1$. This problem can be solved by convex optimization using linear matrix inequalities.

Considering Fig 1 and the proposed control framework (Fig. 2), the state space model for control area *i*, $G_i(s)$, can be obtained as

$$
\begin{aligned}\n\dot{x}_i &= A_i x_i + B_{I_i} w_i + B_{2i} u_i \\
z_{\infty i} &= C_{\infty i} x_i + D_{\infty I_i} w_i + D_{\infty 2i} u_i \\
z_{2i} &= C_{2i} x_i + D_{21i} w_i + D_{22i} u_i \\
y_i &= C_{yi} x_i + D_{yI_i} w_i\n\end{aligned} \tag{11}
$$

where

$$
x_i^T = [\Delta f_i \quad \Delta P_{tie-i} \quad \int ACE_i \quad x_{ni} \quad x_{gi}] \tag{12}
$$

$$
x_{ii} = \begin{bmatrix} \Delta P_{tli} & \Delta P_{t2i} & \cdots & \Delta P_{mi} \end{bmatrix}
$$
 (13)

$$
x_{gi} = \begin{bmatrix} \Delta P_{gli} & \Delta P_{g2i} & \cdots & \Delta P_{gni} \end{bmatrix}
$$
 (14)

$$
w_i^{\mathrm{T}} = [w_{1i} \quad w_{2i}], \ w_{2i}^{\mathrm{T}} = [v_{1i} \quad v_{2i} \quad v_{3i} \quad v_{4i}] \tag{15}
$$

$$
v_{4i}^{\ \ T} = \begin{bmatrix} v_{4i-1} & v_{4i-2} & \cdots & v_{4i-n} \end{bmatrix} \tag{16}
$$

$$
y_i = ACE_i \tag{17}
$$

$$
u_i = \Delta P_{Ci}, \quad z_{2i}^T = \begin{bmatrix} \eta_{1i} \Delta f_i & \eta_{2i} \end{bmatrix} ACE_i \quad \eta_{3i} \Delta P_{Ci} \end{bmatrix} \tag{18}
$$

and,

$$
A_{i} = \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix}, B_{Ii} = \begin{bmatrix} B_{Ii11} & B_{Ii12} \\ B_{Ii21} & B_{Ii22} \\ B_{Ii31} & B_{Ii32} \end{bmatrix}, B_{2i} = \begin{bmatrix} B_{2i1} \\ B_{2i2} \\ B_{2i3} \end{bmatrix},
$$

\n
$$
A_{i11} = \begin{bmatrix} -D_{i}/2\pi M_{i} & -1/2\pi M_{i} & 0 \\ \sum_{\substack{j=i \\ j \neq i \\ j \neq i}} T_{ij} & 0 & 0 \\ B_{i} & 1 & 0 \end{bmatrix}, A_{i12} = \begin{bmatrix} 1/2\pi M_{i} & \cdots & 1/2\pi M_{i} \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}_{3 \times n}
$$

\n
$$
A_{i22} = -A_{i23} = diag[-1/T_{11i} & -1/T_{12i} & \cdots & -1/T_{ini}],
$$

\n
$$
A_{i33} = diag[-1/T_{g1i} & -1/T_{g2i} & \cdots & -1/T_{gm}]\,,
$$

\n
$$
A_{i31} = \begin{bmatrix} -1/(T_{g1i}R_{1i}) & 0 & 0 \\ \vdots & \vdots & \vdots \\ -1/(T_{gm}R_{ni}) & 0 & 0 \end{bmatrix}, A_{i13} = A_{i21}^T = 0_{3 \times n}, A_{i32} = 0_{m \times n},
$$

\n
$$
B_{1i12} = \begin{bmatrix} -1/2\pi M & 0 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 0 & \cdots & 0 \end{bmatrix}_{3 \times (3+n)}
$$

\n
$$
B_{1i22} = 0_{m \times (3+n)}, B_{1i32} = \begin{bmatrix} 0_{m \times 3} & b \end{bmatrix},
$$

\n
$$
b = diag[I/T_{g1i} &
$$

*B2i1 03*u*¹ B2i2 0n*u*¹* , , > @ *1i g1i 2i g2i ni gni T 2i3 B Į /T Į /T Į /T* , $B_{1i11} = 0_{3\times1}, B_{1i21} = 0_{n\times1}, B_{1i31}^T = \left[\alpha_{1i}/T_{gli} \quad \alpha_{2i}/T_{g2i} \quad \cdots \quad \alpha_{ni}/T_{gni} \right],$ $C_{\infty i} = 0_{\{x(2n+3)}}, D_{\infty I i} = \begin{bmatrix} -1 & 0_{\{x(3+n)\}}, D_{\infty 2i} = 1, \end{bmatrix}$ $\mathcal{L}_{2i} = \begin{bmatrix} c_{2i1} & c_{2i2} \end{bmatrix}, c_{2iI} = \begin{bmatrix} \eta_{Ii} & 0 & 0 \\ 0 & 0 & \eta_{2i} \end{bmatrix}, c_{2i2} = 0_{3\times 2i}$ *0 0 0 0 0 Ș* η_{Ii} 0 0 $C_{2i} = \begin{bmatrix} c_{2i1} & c_{2i2} \end{bmatrix}$, $c_{2i1} = \begin{bmatrix} 0 & 0 & \eta_{2i} \\ 0 & 0 & 0 \end{bmatrix}$, $c_{2i2} = 0_{3\times 2}$ » ¼ º « \mathbf{r} $\overline{\mathsf{L}}$ ª $=[c_{2i1} \quad c_{2i2}], c_{2i1} = [0 \quad 0 \quad \eta_{2i}], c_{2i2} = 0_{3\times 2n},$

$$
D_{21i} = 0_{3 \times (4+n)}, \ D_{22i} = \begin{bmatrix} 0 \\ 0 \\ \eta_{3i} \end{bmatrix}.
$$

4. PI-based multi-objective LFC design

Assume $K(s)$ in Fig. 1 is a PI controller. We can formulate the PI in the following SOF control law $\frac{1}{1}$,

$$
u_i = [k_{Pi} \quad k_{H}] \left[\begin{array}{c} ACE_i \\ \int ACE_i \end{array} \right], \quad (u_i = K_i y_i)
$$
 (19)

Therefore, y_i in (17) can be augmented to following form

$$
y_i^T = [ACE_i \quad \int ACE_i]
$$
 (20)

and for the corresponded coefficients in (11), we can write

$$
C_{yi} = [c_{yi} \quad 0_{2 \times 2n}], c_{yi} = \begin{bmatrix} \beta_i & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D_{yli} = 0_{2 \times (4+n)}
$$

Consider a linear time invariant system $G_i(s)$ with the state-space realization of (11). A mixed H_2/H_{∞} SOF control design can be expressed in following optimization problem.

Optimization problem: Determine an admissible SOF law K_i , belong to family of internally stabilizing SOF gains K_{soft} ,

$$
u_i = K_i y_i , K_i \in K_{\text{soft}}
$$
 (21)

such that

,

$$
\inf_{K_i \in K_{SOf}} \|T_{z_{2i} w_{2i}}\|_2 \ \ subject \ to \ \|T_{z_{\infty i} w_{1i}}\|_{\infty} < 1 \tag{22}
$$

This problem defines a robust performance synthesis problem where the H_2 norm is chosen as the performance measure. Recently, several methods are proposed to obtain the suboptimal solution for the H_2 , H_{∞} and H_2/H_{∞} SOF control problems $5, 6$.

On substitution of (21) into (11), it is easy to find that for each control area the state space realization of closed-loop system will be given as

$$
\begin{aligned}\n\dot{x}_i &= A_{ic} x_i + B_{lic} w_i \\
z_{\infty i} &= C_{\infty ic} x_i \\
z_{2i} &= C_{2ic} x_i \\
y_i &= C_{yic} x_i\n\end{aligned} \tag{23}
$$

and we can write,

$$
\left\|T_{z_{2i}w_{2i}}\right\|_{2}^{2} = trace(C_{2ic}L_{C}C_{2ic}^{T})
$$
\n(24)

where L_c denotes the controllability Gramian of the pair (A_{ic}, B_{lic}) . Lemma 2 in Ref. [6] gives a solution for H_2 suboptimal SOF control problem. Following, a new ILMI algorithm is introduced to get a suboptimal solution for the above optimization problem. Specifically, the proposed algorithm formulates the H_2/H_{∞} SOF control as a general SOF stabilization problem (see theorem 2 in Ref. [7]) to get a

family of H_2 stabilizing controllers K_{soft} . Then the designed controller $K_i \in K_{\text{soft}}$ will be chosen such that

$$
\left| \gamma_2^* - \gamma_2 \right| < \varepsilon \,, \quad \gamma_\infty = \left\| T_{z_{\infty i} w_{Ii}} \right\|_{\infty} < I \tag{25}
$$

where ε is a small real positive number, γ_2^* is resulted H_2 performance by K_i subject to given constraint in (25) and γ ² is resulted H_2 optimal performance index from applied standard H_2/H_{∞} dynamic output feedback control to the control area *i* as shown in Fig. 3.

Using lemma 2 in Ref. [6], a family of H_2 stabilizing SOF gains K_{soft} can be obtained. But we are looking for the solution of such controller within this family which satisfy the given constraint in (25). Developed algorithm, gives an iterative LMI suboptimal solution to obtain a H_1/H_{∞} SOF controller for a given power system control area:

Step 1. Compute the state-space model (11) for the given control area.

Step 2. Compute the optimal guaranteed H_2 performance index γ ₂ using function *hinfmix* in MATLAB based LMI control toolbox² to design standard H_2/H_{∞} output dynamic controller as descript in section 3, for the performed system in step 1.

Step 3. Set $i =1$, $\Delta \gamma_2 = \Delta \gamma_0$ and let $\gamma_{2i} = \gamma_0 > \gamma_2$. $\Delta \gamma_0$ and γ ⁰ are positive real numbers.

Step 4. Select $Q = Q_0 > 0$, and solve *X* from the following algebraic Riccati equation

 $A_i X + X A_i^T - X C_{yi}^T C_{yi} X + Q = 0, \quad X > 0$ (26) Set $P_1 = X$.

Step 5. Solve the following optimization problem for X_i , K_i and a_i :

Minimize a_i subject to the bellow LMI constraints:

$$
\begin{bmatrix} A_i X_i + X_i A_i^T + B_{1i} B_{1i}^T - P_i C_{yi}^T C_{yi} X_i - X_i C_{yi}^T C_{yi} P_i + P_i C_{yi}^T C_{yi} P_i - a_i X_i \\ (B_{2i} K_i + X_i C_{yi}^T)^T \\ B_{2i} K_i + X_i C_{yi}^T \end{bmatrix} < 0
$$

$$
\begin{array}{cc} -I & \mathbf{J} \\ (27) & \end{array}
$$

 $trace(C_{2ic}L_{c}C_{2ic}^{T}) < \gamma_{2i}$ (28)

 $X_i = X_i^T > 0$ (29)

Denote a_i^* as the minimized value of a_i .

Step 6. If $a_i^* \leq 0$, go to step 10.

Step 7. For $i > 1$ if $a_{i-1}^* \le 0$, $K_{i-1} \in K_{\text{soft}}$ is an H_2 controller and go to step 10. Otherwise go to step 8.

Step 8. Solve the following optimization problem for *X_i* and K_i :

Minimize $trace(X_i)$ subject to LMI constraints (27-29) with $a_i = a_i^*$. Denote X_i^* as the X_i that minimized $trace(X_i)$.

Step 9. Set $i = i+1$ and $P_i = X_{i-1}^*$, then go to step 5. *Step 10.* Set $\gamma_{2i} = \gamma_{2i} - \Delta \gamma_2$, $i = i+1$. Then do steps 4 to 6. *Step 11.* If

$$
\gamma_{\infty,i\text{-}I} = \left\| W_i K_{i\text{-}I} G_i (I + K_{i\text{-}I} G_i)^{-1} \right\|_{\infty} \le I \tag{30}
$$

the K_{i-1} is an H_2/H_∞ SOF controller and $\gamma_2^* = \gamma_{2i} - \Delta \gamma_2$ indicates a lower H_2 bound such that the obtained controller satisfies (25). Otherwise set $\gamma_{2i} = \gamma_{2i} - \Delta \gamma_2$, $i=i+1$, then do steps 4-6.

5. Application to a 3-control area power system

To illustrate the effectiveness of proposed control strategy, a three control area power system, shown in Fig. 3, is considered as a test system. It is assumed that each control area includes two Gencos, which use the same ACE participation factor. The power system parameters are considered the same as in Ref. [1].

Fig. 3. Three control area power system

5.1. Uncertainty and performance weights selection

In this example with regards to uncertainties, it is assumed that the rotating mass and load pattern parameters have uncertain values in each control area. The variation range for D_i and M_i parameters in each control area is assumed \pm %20. Following, these uncertainties is modelled as an unstructured multiplicative uncertainty block that contains all the information available about D_i and M_i variations.

Let $\hat{G}_i(s)$ denotes the transfer function from the control input u_i to control output y_i at operating points other than nominal point. Following a practice common in robust control, we will represent this transfer function as

$$
\left| \Delta_i(s) W_i(s) \right| = \left| \hat{G}_i(s) - G_{0i}(s) J G_{0i}(s)^{-1} \right| ; \quad G_{0i}(s) \neq 0 \quad (31)
$$
\nwhere,

\n
$$
\left\| \Delta_i(s) \right\|_{\infty} = \sup_{\omega} \left| \Delta_i(s) \right| \leq I \quad (32)
$$

 Δ_i (s) shows the uncertainty block corresponding to uncertain parameters and $G_{\theta i}(s)$ is the nominal transfer function model. Thus, $W_i(s)$ is such that its respective magnitude bode plot covers the bode plots of all possible plants. For example, using (31) some sample uncertainties corresponding to different values of D_i and M_i for area 1 can be obtained as shown in Fig. 4. The uncertainties due to both set of parameters variation can be modeled by using a norm bonded multiplicative uncertainty to cover all possible plants as follows.

$$
W_1(s) = \frac{0.3986s + 0.0786}{s + 0.6888}
$$
\n(33)

Fig. 4. Uncertainty plots due to parameters changes in area 1; *Di* (dotted), M_i (dash-dotted) and W_i (solid).

Fig. 4 clearly shows that the weight (33) used in our design provides a conservative design at low and high frequencies but it gives a good trade-off between robustness and controller complexity. Using the same method, the uncertainty weighting functions for areas 2 and 3 are computed as follows.

$$
W_2(s) = \frac{0.3088s + 0.0487}{s + 0.6351}, \ W_3(s) = \frac{0.3483s + 0.0751}{s + 0.7826}
$$
 (34)

The selection of performance constant weights η_{1i} , η_{2i} and η_{3i} is dependent on specified performance objectives and must be chosen by designer. In fact an important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives 1). Here, a set of suitable values for constant weights is chosen as follows:

$$
\eta_{1i} = 1.25, \ \eta_{2i} = 0.001, \ \eta_{3i} = 1.5 \tag{35}
$$

5.2. Mixed H_2/H_∞ **dynamic and SOF control design**

For the sake of comparison, in addition to proposed control strategy to synthesis the robust PI controller, a mixed H_1/H_{∞} dynamic output feedback controller is designed for each area, using *hinfmix* function in LMI control toolbox². This function gives an optimal H_2/H_{∞} controller through the mentioned optimization problem (22) and returns the controller $K(s)$ with optimal H_2 performance index γ_2 . The resulted controllers are dynamic type whose orders are the same as size of generalized plant model $(8th$ order in the present paper).

At the next step, according to synthesis methodology described in section 4, a set of three decentralized robust PI controllers are designed. The control parameters are shown in table 1. The optimal performance indices for dynamic and PI controllers are listed in table 2.

The resulted robust performance indices of both synthesis methods (γ_{2i} and γ_{2i}^*) are close to each other. It shows that although the proposed ILMI approach gives a set of much simpler controllers (PI) than the dynamic H_1/H_{∞} design, however they holds robust performance as well as dynamic H_2/H_{∞} controllers.

Table 1. PI control parameters from ILMI design

Parameters	Areal	Area 2	Area 3
K_{pi}	-0.1250	-0.0015	-0.4278
k_{E}	$-5.00E-04$ $-5.14E-04$		$-5.30E-04$

6. Simulation results

In order to demonstrate the effectiveness of the proposed strategy, some simulations were carried out. In these simulations, the proposed PI controllers were applied to the three control area power system described in Fig. 3. The performance of the closed-loop system using the designed PI controllers in comparison of full-order H_1/H_2 dynamic controllers is tested in presence of load demands, disturbances and uncertainties.

Case 1:

In this case, the closed-loop performance is tested in face of both step contracted load demand and uncertainties. It is assumed a large load demand 100 MW (0.1 pu) is requested by each Disco, following %20 decrease in uncertain parameters D_i and M_i . Furthermore, assume Discos contract with the available Gencos according to the following GPM,

$$
GPM^T = \begin{bmatrix} 0.25 & 0.5 & 0 & 0.25 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 \\ 0 & 0 & 0.75 & 0 & 0 & 0.25 \end{bmatrix}
$$

Frequency deviation, area control error (ACE1 and ACE2) and tie-line power changes are shown in Fig. 5. Using the proposed method, the area control error and frequency deviation of all areas are quickly driven back to zero. The tieline power flows are properly convergence to specified values.

Case 2:

Consider the case 1 again. Assume in addition to specified contracted load demands (0.1 pu) and %20 decrease in D_i and M_i , a bounded random step load change as a large uncontracted demand (shown in Fig. 6a) is appeared in each control area, where

$$
-50\,MW \leq \Delta P_{di} \leq +50\,MW
$$

The purpose of this scenario is testing the robustness of the proposed controllers against uncertainties and random large load disturbances. The closed-loop response for areas 1 and 3 are shown in Figs. 6b and 6c.

Fig. 5. a) Frequency deviation; b) area control error and tie-line powers; solid (ILMI-based PI), dotted (dynamic H_2/H_{∞}).

Fig. 6. Power system response for case 2. (a) random load patterns b) area-1,c) area-3; solid (ILMI-based PI), dotted (dynamic H_1/H_2).

The simulation results demonstrate that the proposed ILMI-based PI controllers track the load fluctuations and meet robustness for a wide range of load disturbances and possible bilateral contract scenarios as well as H_2/H_{∞} dynamic controllers.

7. Conclusion

The LFC problem in a multi-area power system is formulated as a decentralized multi-objective optimization control problem via mixed H_2/H_∞ technique. An iterative LMI approach has been proposed for a bilateral-based LFC scheme. Design strategy includes enough flexibility to set the desired level of performance and gives a set of simple PI controllers, which commonly useful in the real-world power systems. The proposed method was applied to a three control area power system. It was shown that the proposed simple ILMI-based PI controllers are capable to guarantee the robust performance as well as H_2/H_{∞} dynamic controllers.

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(b)