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ROBUST TUNING OF PI/PID CONTROLLERS USING H_∞-CONTROL TECHNIQUE

H. Bevrani
Dept. of Electrical and Computer Engineering, Kumamoto University
Kumamoto 860-8555, Japan
E-mail: bevrani@st.eecs.kumamoto-u.ac.jp

T. Hiyama
Dept. of Electrical and Computer Engineering, Kumamoto University
Kumamoto 860-8555, Japan
E-mail: hiyama@eecs.kumamoto-u.ac.jp

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In many real world control problems, it is usually desirable to meet specified control goals using controllers with simple structures like as proportional-integral (PI) and proportional-integral-derivative (PID) which are useful in industry applications. Since practically, these controllers are commonly tuned based on experiences, classical or trial-and-error approaches, they are incapable of obtaining good dynamical performance to capture design objectives. This paper addresses a systematic method to design robust PI/PID controllers based on H_∞-control technique. First the PI/PID control problem is reduced to an H_∞-static output feedback control synthesis, and then the control parameters are easily carried out using an iterative linear matrix inequalities (ILMI) algorithm. A numerical example and simulations are given to illustrate the proposed methodology.

1. Introduction

The proportional-integral (PI) and proportional-integral-derivative (PID) controllers, because of their functional simplicity, are widely used in industrial applications. However in practice, their parameters are often tuned using experiences or trial and error methods. It is clear that meeting all design specifications and control objectives by a simple PI/PID controller which is tuned based on experiences/trial-error methods is difficult.

Over the years, many different parameter tuning methods have been presented for PI and PID controllers. A survey up to 2002 is given in [1, 2]. Many of these methods are modifications of the frequency response method introduced by Ziegler and Nichols [3]. Some efforts have also been made to find analytical approaches to tune the parameters.

In this paper to calculate the controller parameters optimally, the PI/PID control problem is formulated as an H_∞ static output feedback (SOF) control problem. For this purpose an iterative linear matrix inequalities (ILMI) algorithm is developed. As a numerical example, the proposed strategy is applied to a multi-area power system for tuning of PI-based frequency regulators. The designed robust controllers, which are ideally practical for electric industry, are compared with the H_∞ dynamic output feedback controllers (using general LMI technique [4]). The results show the proposed control synthesis guarantees the robust performance for a wide range of operating conditions as well as H_∞ dynamic control design.

2. Proposed methodology

2.1. From PI/PID to SOF control design

In a given PI/PID-based control system *i*, the measured output signal (y_{oi}) performs the input signal of controller and we can write (for PID type):

$$u_i = k_{Pi} y_{oi} + k_{Ii} \int y_{oi} + k_{Di} \frac{dy_{oi}}{dt}, \tag{1}$$

where k_{Pi}, k_{Ii} and k_{Di} are constant real numbers. Therefore, by augmenting the system description to include the y_{oi}, it's integral and derivative as a new measured output vector (y_i), the PI/PID control problem becomes one of finding a static output feedback that satisfied prescribed performance requirements. In order to change (1) to a simple SOF control law as

$$u_i = K_i y_i. \tag{2}$$

We can rewrite (1) as follows

$$u_i = [k_{Pi} \quad k_{Ii} \quad k_{Di}] \begin{bmatrix} y_{oi} & \int y_{oi} & \frac{dy_{oi}}{dt} \end{bmatrix}^T. \tag{3}$$

Therefore, y_i in (2) can be expressed as follows.

$$y_i = \begin{bmatrix} y_{oi} & \int y_{oi} & \frac{dy_{oi}}{dt} \end{bmatrix}^T. \tag{4}$$

Using above transformation, the PI/PID multi-objective control problem is reduced to a static output feedback (SOF) control problem. In order to get a robust solution for the recent SOF control problem, the H_∞ technique is used. For the resulted H_∞-based SOF design we have to solve a nonconvex optimization problem, which can not be directly achieved by using general linear matrix inequalities (LMI) techniques. Here, for this purpose an iterative linear matrix inequalities (ILMI) algorithm is developed. Fig. 1 shows the proposed control strategy.

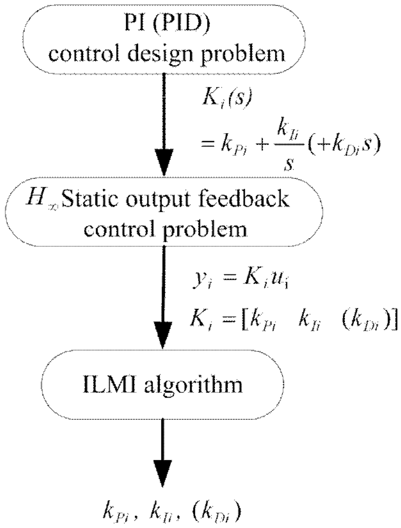

Fig. 1. Design framework.

2.2. H_∞ based SOF control design

This section gives a brief overview of H_∞ based SOF control. Consider a linear time invariant system *G(s)* with the following state-space realization.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x \end{aligned} \qquad (5)$$

where *x* is the state variable vector, *w* is the disturbance and other external input vector, *z* is the controlled output vector and *y* is the measured output vector.

The H_∞ based SOF control problem is to find a static output feedback law $u = Ky$, as shown in Fig. 2, such that the resulted closed-loop system is internally stable, and the H_∞ norm from *w* to *z* is smaller than γ, a specified positive number, i.e.

$$\|T_{zw}(s)\|_\infty < \gamma. \qquad (6)$$

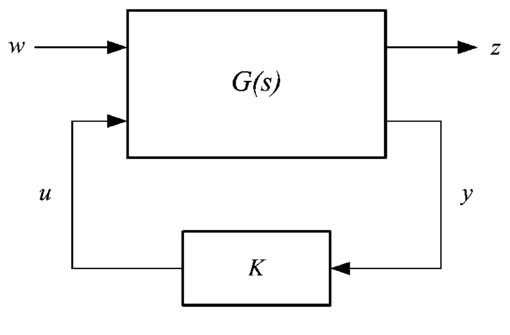

Fig. 2. Closed-loop system via H_∞ control.

Lemma 1. It is assumed that (A, B_2, C_2) is stabilizable and detectable. The matrix *K* is an H_∞ controller, if and only if there exists a symmetric matrix $X > 0$ such that

(7)
$$\begin{bmatrix} A_{cl}^T X + X A_{cl} & X B_{cl} & C_{cl}^T \\ B_{cl}^T X & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0$$

where

$$A_{cl} = A + B_2 K C_2\,,\ B_{cl} = B_1$$
$$C_{cl} = C_1 + D_{12} K C_2\,,\ D_{cl} = 0$$

The proof is given in [5]. We can rewrite (7) as the following matrix inequality [6],

(8)
$$\overline{XB}K\overline{C} + (\overline{XB}K\overline{C})^T + \overline{A}^T\overline{X} + \overline{XA} < 0\,,$$

where

$$\overline{A} = \begin{bmatrix} A & B_1 & 0 \\ 0 & -\gamma I/2 & 0 \\ C_1 & 0 & -\gamma I/2 \end{bmatrix},\ \overline{B} = \begin{bmatrix} B_2 \\ 0 \\ D_{12} \end{bmatrix},$$

(9)
$$\overline{C} = \begin{bmatrix} C_2 & 0 & 0 \end{bmatrix},\ \overline{X} = \begin{bmatrix} X & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Hence, the H_∞ based SOF control problem is reduced to find $X > 0$ and K such that matrix inequality (8) holds. It is a generalized static output feedback stabilization problem of the system $(\overline{A},\overline{B},\overline{C})$ which can be solved via lemma 2.

Lemma 2. The system (A, B, C) that may also be identified by the following representation:

(10)
$$\dot{x} = Ax + Bu$$
$$y = Cx$$

is stabilizable via static output feedback if and only if there exist $P>0$, $X>0$ and K satisfying the following quadratic matrix inequality.

(11)
$$\begin{bmatrix} A^T X + XA - PBB^T X - XBB^T P + PBB^T P & (B^T X + KC)^T \\ B^T X + KC & -I \end{bmatrix} < 0$$

Proof. According to the Schur complement, the quadratic matrix inequality (11) is equivalent to the following matrix inequality

(12)
$$A^T X + XA - PBB^T X - XBB^T P + PBB^T P$$
$$+ (B^T X + KC)^T (B^T X + KC) < 0\,.$$

For this new inequality notation, the sufficiency and necessity of theorem are already proven [6].

2.3. Developed ILMI algorithm

The solution way given by lemma 2 can not be directly achieved by using general LMI technique to solve the consequent nonconvex optimization problem. On the other hand, the matrix inequality (11) points to an iterative approach to solve the matrix K and X, namely, if P is fixed, then it reduces to an LMI problem in the unknowns K and X. For this purpose, we introduce an iterative LMI algorithm that is mainly based on the approach given in [6]. The key point is to formulate the H_∞ problem via a generalized static output stabilization feedback such that all eigenvalues of (A-BKC) shift towards the left half-plane through the reduction of *a*, a real number, to close to feasibility of (11).

In summary, the H_∞ based SOF controller design based on ILMI approach for a given system consists of the following steps:

Step 1. Compute the new system $(\bar{A},\bar{B},\bar{C})$, according to (9). Set $i=1$ and $\Delta\gamma = \Delta\gamma_0$. Let $\gamma_i = \gamma_0$ a positive real number.

Step 2. Select $Q > 0$, and solve $\bar{X}$ from the following algebraic Riccati equation:

(13) $$\bar{A}^T \bar{X} + \bar{X}\bar{A} - \bar{X}\bar{B}\bar{B}^T \bar{X} + Q = 0$$

Set $P_1 = \bar{X}$.

Step 3. Solve the following optimization problem for $\bar{X}_i$, K_i and a_i. Minimize a_i subject to the LMI constraints:

(14) $$\begin{bmatrix} \bar{A}^T\bar{X}_i + \bar{X}_i\bar{A} - P_i\bar{B}\bar{B}^T\bar{X}_i - \bar{X}_i\bar{B}\bar{B}^T P_i + P_i\bar{B}\bar{B}^T P_i - a_i\bar{X}_i & (\bar{B}^T\bar{X}_i + K_i\bar{C})^T \\ \bar{B}^T\bar{X}_i + K_i\bar{C} & -I \end{bmatrix}$$

(15) $$\bar{X}_i = \bar{X}_i^T > 0$$

Denote a_i^* as the minimized value of a_i.

Step 4. If $a_i^* \le 0$, go to step 8.

Step 5. For $i > 1$ if $a_{i-1}^* \le 0$, K_{i-1} is desired H_∞ controller and $\gamma^* = \gamma_i + \Delta\gamma$ indicates a lower bound such that the above system is H_∞ stabilizable via static output feedback.

Step 6. Solve the following optimization problem for $\bar{X}_i$ and K_i:

Minimize trace($\bar{X}_i$) subject to the above LMI constraints (14) and (15) with $a_i = a_i^*$. Denote $\bar{X}_i^*$ as the $\bar{X}_i$ that minimized trace($\bar{X}_i$).

Step 7. Set $i=i+1$ and $P_i = \bar{X}_{i-1}^*$, then go to step 3.

Step 8. Set $\gamma_i = \gamma_i - \Delta\gamma$, $i=i+1$. Then do steps 2 to 4.

The matrix inequalities (14) and (15) give a sufficient condition for the existence of the static output feedback controller.

3. Numerical Example

To illustrate the effectiveness of the proposed design strategy, it is applied to a three control area power system example which is the same as given in [7], to design decentralized PI-based frequency regulators. According to (5), we can calculate the state-space model for each area system as follows:

(16) $$\begin{aligned} \dot{x}_i &= A_i x_i + B_{1_i} w_i + B_{2_i} u_i \\ z_i &= C_{1i} x_i + D_{12_i} u_i \qquad , i = 1, 2, 3, \\ y_i &= C_{2i} x_i \end{aligned}$$

where x_i is the state variable vector, w_i is the disturbance and area interface vector, z_i is the controlled output vector and y_i is the measured output vector which is performed by area control error (ACE) signal. For the current example, the system order is 9. The mentioned variables and vectors are considered as follows:

(17) $$z_i^T = \begin{bmatrix} \eta_{1i}\Delta f & \eta_{2i}\int ACE_i & \eta_{3i}\Delta P_{tie\text{-}i} & \eta_{4i}\Delta P_{Ci} \end{bmatrix},$$

(18) $$x_i^T = [\Delta f_i \quad \Delta P_{tie-i} \quad \int ACE_i \quad x_{ti} \quad x_{gi}],$$

(19) $$x_{ti} = \begin{bmatrix} \Delta P_{t1i} & \Delta P_{t2i} & \cdots & \Delta P_{tni} \end{bmatrix},\ x_{gi} = \begin{bmatrix} \Delta P_{g1i} & \Delta P_{g2i} & \cdots & \Delta P_{gni} \end{bmatrix},$$

(20) $$y_i^T = ACE_i,\ u_i = \Delta P_{Ci},$$

(21) $$w_i^T = \begin{bmatrix} w_{1i} & w_{2i} \end{bmatrix},\ w_{1i} = \Delta P_{di},\ w_{2i} = \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}\Delta f_j,$$

where Δf_i – frequency deviation, ΔP_{gi} – overnor valve position, ΔP_{ci} – governor load setpoint, ΔP_{ti} – turbine power, ΔP_{tie-i} net tie-line power flow, ΔP_{di} area load disturbance, M_i – equivalent inertia constant, D_i – equivalent damping coefficient, T_{gi} – governor time constant, T_{ti} – turbine time constant, T_{ij} – tie-line synchronizing coefficient between area *i* & *j*, B_i – frequency bias, R_i – drooping characteristic, α – ACE participation factor, $\Delta P_{tie\text{-}i}$ – tie-line power changes, η_i – constant weight.

And here,

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix},\ B_{1i} = \begin{bmatrix} B_{1i1} \\ B_{1i2} \\ B_{1i3} \end{bmatrix},\ B_{2i} = \begin{bmatrix} B_{2i1} \\ B_{2i2} \\ B_{2i3} \end{bmatrix},$$

$$C_{1i} = \begin{bmatrix} c_{1i} & 0_{4\times n} & 0_{4\times n} \end{bmatrix},\ c_{1i} = \begin{bmatrix} \eta_{1i} & 0 & 0 \\ 0 & 0 & \eta_{2i} \\ 0 & 0 & 0 \\ 0 & \eta_{3i} & 0 \end{bmatrix},$$

$$D_{12i} = \begin{bmatrix} 0 \\ 0 \\ \eta_{4i} \\ 0 \end{bmatrix},\ C_{2i} = \begin{bmatrix} B_i & 1 & 0 & 0_{1\times n} & 0_{1\times n} \end{bmatrix},$$

$$A_{i11} = \begin{bmatrix} -D_i/M_i & -1/M_i & 0 \\ 2\pi\sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} & 0 & 0 \\ B_i & 1 & 0 \end{bmatrix},\ A_{i12} = \begin{bmatrix} 1/M_i & \cdots & 1/M_i \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}_{3\times n}$$

$$A_{i22} = -A_{i23} = diag\begin{bmatrix} -1/T_{t1i} & -1/T_{t2i} & \cdots & -1/T_{tni} \end{bmatrix},$$

$$A_{i33} = diag\begin{bmatrix} -1/T_{g1i} & -1/T_{g2i} & \cdots & -1/T_{gni} \end{bmatrix}$$

$$A_{i31}=\begin{bmatrix}-1/(T_{g1i}R_{1i}) & 0 & 0\\ \vdots & \vdots & \vdots\\ -1/(T_{gni}R_{ni}) & 0 & 0\end{bmatrix},\ A_{i13}=A_{i21}{}^{T}=0_{3\times n},\ A_{i32}=0_{n\times n}$$

$$B_{1i1}=\begin{bmatrix}-1/M_i & 0\\ 0 & -2\pi\\ 0 & 0\end{bmatrix},\ B_{1i2}=B_{1i3}=0_{n\times 3},$$

$$B_{2i1}=0_{3\times 1}, B_{2i2}=0_{n\times 1},\ B_{2i3}{}^{T}=\begin{bmatrix}\alpha_{1i}/T_{g1i} & \alpha_{2i}/T_{g2i} & \cdots & \alpha_{ni}/T_{gni}\end{bmatrix},$$

$\eta_{1i}=0.4,\ \eta_{2i}=1.075,\ \eta_{3i}=0.39,\ \eta_{4i}=333.$

The system data and parameters values are assumed the same as in [7]. For each control area, in addition to the proposed control strategy to obtain the robust PI controller, an H_∞ dynamic output feedback controller using LMI control toolbox [1] is designed. The function *hinflmi* gives an optimal H∞ controller through the minimizing the guaranteed robust performance index subject to the constraint given by the matrix inequalities and returns the controller *K(s)* with optimal robust performance index. The resulted controllers using the *hinflmi* function are dynamic type and have the following state-space form, whose orders are the same as size of plant model (here 9).

$$\dot{x}_{ki}=A_{ki}x_{ki}+B_{ki}y_i \qquad (22)$$
$$u_i=C_{ki}x_{ki}+D_{ki}y_i$$

At the next step, according to the synthesis methodology described in section 2, a set of three decentralised robust PI controllers are designed. Using ILMI approach, the controllers are obtained following several iterations. For example, for area control 3, the final result is obtained after 29 iterations. Some iterations are listed in Table 1.The control parameters for three areas are shown in Table 2.

Table 1. ILMI algorithm result for design of K_3.

Iteration	γ	k_{P3}	k_{I3}
1	449.3934	-0.0043	-0.0036
5	419.1064	-0.0009	-0.0042
11	352.6694	0.1022	-0.2812
14	340.2224	-0.0006	-0.0154
19	333.0816	-0.0071	-0.1459
22	333.0332	0.0847	-0.2285
24	333.0306	0.0879	-0.2382
26	333.0270	0.0956	-0.2537
28	333.0265	0.0958	-0.2560
29	333.0238	-0.0038	-0.2700

Table 2. Control parameters (ILMI design).

Parameter	*Area 1*	*Area 2*	*Area 3*
a^*	-0.0246	-0.3909	-0.2615
k_{Pi}	-9.8e-03	-2.6e-03	-3.8e-03
k_{Ii}	-0.5945	-0.3432	-0.2700

The resulted robust performance indices of both synthesis methods are too close to each other and are shown in Table 3. It shows that although the proposed ILMI approach gives a set of much simpler controllers than the dynamic H_∞ design, however they hold robust performance as well as dynamic H_∞ controllers.

Table 3. Robust performance index.

Control design	*Control structure*	*Performance index*	*Area1*	*Area 2*	*Area 3*
H_∞	9^{th} order	γ	333.0084	333.0083	333.0080
ILMI	PI	γ^*	333.0261	333.0147	333.0238

In order to demonstrate the effectiveness of the proposed control design, some simulations were carried out. The performance of the closed-loop system using the robust PI controllers in comparison of dynamic H_∞ controllers and is tested for serious load disturbances.

Here, we will present the responses of closed loop system using both proposed method (ILMI) and designed dynamic H_∞ controllers in case of applying a large step load disturbance to each control area:

$$\Delta P_{di} = 0.105\ pu,\quad i = 1, 2, 3\ .$$

Frequency deviation (Δf) and area control error (ACE) signals of the closed-loop system are shown in Fig. 3. Using the proposed method (ILMI), the area control error and frequency deviation of all control areas are quickly driven back to zero as well as full dynamic H_∞ control.

4. Conclusion

A new methodology is developed to design of PI/PID controllers, which are common in the real-world control systems. The PI/PID control problem is transferred to an H_∞ based static output feedback control problem and then it is solved using an iterative linear matrix inequalities algorithm. The proposed method was successfully applied to a 3-control area power system example for regulating the area frequency.

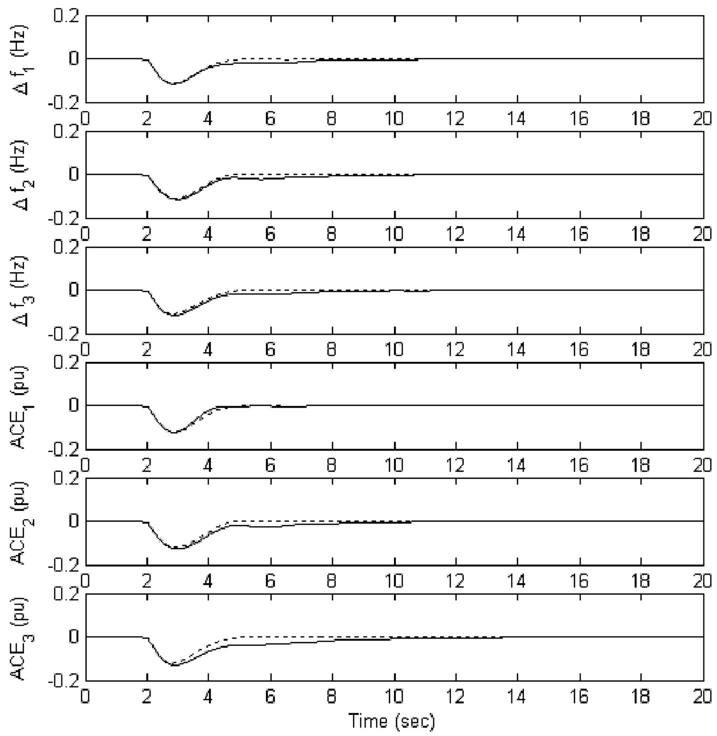

Fig. 3. Closed-loop system response following large step load demands. Solid (ILMI-based PI controllers), Dotted (dynamic H_∞ controllers).

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