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Bilateral based robust load frequency control

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Abstract

Load frequency control (LFC) has been one of the major subjects in electric power system design/operation and is becoming much more significant today in accordance with increasing size and the changing structure and complexity of interconnected power systems. In practice, power systems use simple proportional-integral (PI) controllers for frequency regulation and load tracking. However, since the PI controller parameters are usually tuned based on classical or trial and error approaches, they are incapable of obtaining good dynamical performance for a wide range of operating conditions and various load changes scenarios in a restructured power system.

This paper addresses a new decentralized robust LFC design in a deregulated power system under a bilateral based policy scheme. In each control area, the effect of bilateral contracts is taken into account as a set of new input signals in a modified traditional dynamical model. The LFC problem is formulated as a multi-objective control problem via a mixed H_2/H_∞ control technique. In order to design a robust PI controller, the control problem is reduced to a static output feedback control synthesis, and then, it is solved using a developed iterative linear matrix inequalities algorithm to get a robust performance index close to a specified optimal one. The proposed method is applied to a 3 control area power system with possible contract scenarios and a wide range of load changes. The results of the proposed multi-objective PI controllers are compared with H_2/H_∞ dynamic controllers.

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1. Introduction

In a deregulated environment, load frequency control (LFC) as an ancillary service acquires a fundamental role to maintain the electrical system reliability at an adequate level. That is why there has been increasing interest for designing load frequency controllers with better performance during recent years, and several optimal and robust control strategies have been developed for LFC synthesis according to the changing environment of power system operation under deregulation. Some of them suggest complex state feedback or high order dynamic controllers, which are not practical for industry practices. Usually, the existing LFC in practical power systems uses proportional-integral (PI) type controllers that are tuned online based on classical and trial and error approaches. Furthermore, in most published reports, only one single norm is used to capture design specifications, while meeting all LFC design objectives by a single norm-based control approach with regard to the increasing complexity and changing power system structure is difficult.

Naturally, LFC is a multi-objective control problem. LFC goals, i.e. frequency regulation and tracking load changes and maintaining tie line power interchanges to specified values in the presence of generation constraints and dynamical model uncertainties, determines the LFC synthesis as a multi-objective control problem. Therefore, it is expected that an appropriate multi-objective control strategy could be able to give a better solution for this problem [1]. It is well known that each robust method is mainly useful to capture a set of special specifications. For instance, the H_2 tracking design is more adapted to deal with transient performance by minimizing the linear quadratic cost of tracking error and control input, but the H_∞ approach (and μ as a generalized H_∞ approach) is more useful in holding closed loop stability in the presence of control constraints and uncertainties. While the H_∞ norm is natural for norm bounded perturbations, in many applications the natural norm for the input–output performance is the H_2 norm.

In this paper, the LFC synthesis problem is formulated as a mixed H_2/H_∞ static output feedback (SOF) control problem to obtain a desired PI controller. An iterative linear matrix inequalities (ILMI) algorithm is developed to compute the PI parameters. The model uncertainty in each control area is covered by an unstructured multiplicative uncertainty block. The proposed strategy is applied to a three control area example. The designed robust PI controllers, which are ideally practical for industry, are compared with the mixed H_2/H_∞ dynamic output feedback controllers (using the general LMI technique [2]). The results show the PI controllers guarantee the robust performance for a wide range of operating conditions as well as H_2/H_∞ dynamic controllers. The preliminary steps of this work are given in Refs. [1,3].

This paper is organized as follows: the generalized LFC model in a bilateral based power system market is given in Section 2. Section 3 presents the problem formulation via a mixed H_2/H_∞ technique for a given control area. The PI based multi-objective LFC design using a developed iterative LMI (ILMI) is given in Section 4. The proposed methodology is applied to a 3 control area power system as a case study in Section 5. Finally, to demonstrate the effectiveness of the proposed method and to compare with mixed H_2/H_∞ dynamic output feedback control design, some simulation results are given in Section 6.

2. Bilateral based LFC scheme [1]

In a deregulated environment, the common LFC objectives, i.e. restoring the frequency and the net interchanges to their desired values for each control area, remain. In Ref. [1], a traditional based dynamical model is generalized for a given control area in the deregulated environment under a bilateral LFC scheme, following the idea presented in Ref. [4]. This section gives a brief overview on the generalized LFC model that uses all the information required in a vertically operated utility industry plus the contract data information.

Based on the mentioned model, the overall power system structure can be considered as a collection of distribution companies (Discos) or separate control areas interconnected through high voltage transmission lines or tie lines. Each control area has its own LFC and is responsible for tracking its own load and honoring tie line power exchange contracts with its neighbors. There can be various combinations of contracts between each Disco and the available generation companies (Gencos). On the other hand, each Genco can contract with various Discos. The "generation participation matrix (GPM)" concept is defined to express these bilateral contracts in the generalized model. The GPM shows the participation factor of each Genco in the considered control areas, and each control area is determined by a Disco. The rows of a GPM correspond to Gencos and the columns to the control areas that contract power. For example, the GPM for a large scale power system with m control areas (Discos) and n Gencos, has the following structure, in which gpf_{ij} refers to "generation participation factor" and shows the participation factor of Genco i in the load following of area j (based on a specified bilateral contract):

$$\mathrm{GPM} = \begin{bmatrix} gpf_{11} & gpf_{12} & \cdots & gpf_{1(m-1)} & gpf_{1m} \\ gpf_{21} & gpf_{22} & \cdots & gpf_{2(m-1)} & gpf_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ gpf_{(n-1)1} & gpf_{(n-1)2} & \cdots & gpf_{(n-1)(m-1)} & gpf_{(n-1)m} \\ gpf_{n1} & gpf_{n2} & \cdots & gpf_{n(m-1)} & gpf_{nm} \end{bmatrix} \tag{1}$$

The generalized LFC block diagram for control area i in a deregulated environment is shown in Fig. 1. New information signals due to possible various contracts between Disco i and other Discos and Gencos are shown as dashed line inputs, and, we can write [1]:

$$v_{1i} = \Delta P_{\mathrm{Loc}-i} + \Delta P_{di} \tag{2}$$

$$v_{2i} = \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij}\Delta f_j \tag{3}$$

$$\begin{aligned} v_{3i} &= \sum(\text{Total export power} - \text{Total import power}) \\ &= \sum_{\substack{j=1 \\ j\neq i}}^{N}\left(\sum_{k=1}^{n} gpf_{kj}\right)\Delta P_{Lj} - \sum_{k=1}^{n}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N} gpf_{jk}\right)\Delta P_{Li} \end{aligned} \tag{4}$$

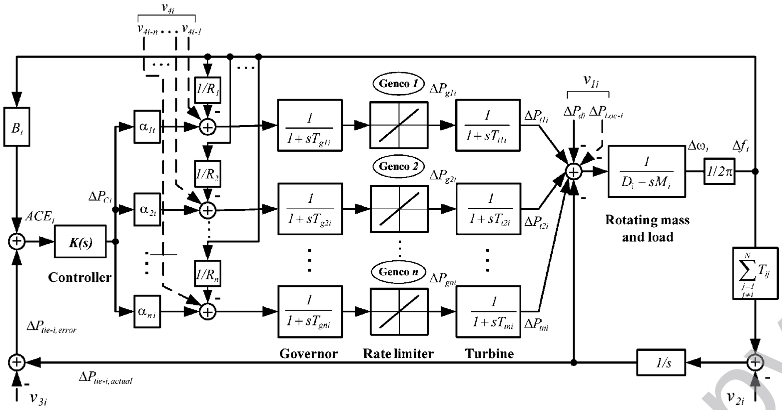

Fig. 1. Generalized LFC (bilateral based) model in a deregulated environment.

$$v_{4i} = \begin{bmatrix} v_{4i-1} & v_{4i-2} & \ldots & v_{4i-n} \end{bmatrix} \tag{5}$$

$$\begin{aligned} v_{4i-1} &= \sum_{j=1}^{N} gpf_{1j}\Delta P_{Lj} \\ &\vdots \\ v_{4i-n} &= \sum_{j=1}^{N} gpf_{nj}\Delta P_{Lj} \end{aligned} \tag{6}$$

$$\Delta P_{\text{tie}-i,\text{error}} = \Delta P_{\text{tie}-i,\text{actual}} - v_{3i} \tag{7}$$

$$\sum_{i=1}^{n} gpf_{ij} = 1 \tag{8}$$

$$\sum_{k=1}^{n} \alpha_{ki} = 1; \quad 0 \leqslant \alpha_{ki} \leqslant 1 \tag{9}$$

$$\Delta P_{mi} = \sum_{j=1}^{N} gpf_{ij}\Delta P_{Lj} \tag{10}$$

where

Δf_i frequency deviation
ΔP_{gi} governor valve position
ΔP_{ci} governor load setpoint
ΔP_{ti} turbine power
$\Delta P_{\text{tie}-i}$ net tie line power flow
ΔP_{di} area load disturbance

M_i equivalent inertia constant
D_i equivalent damping coefficient
T_{gi} governor time constant
T_{ti} turbine time constant
T_{ij} tie line synchronizing coefficient between areas i and j
B_i frequency bias
R_k drooping characteristic
α area control error (ACE) participation factor
N number of control areas
ΔP_{Li} contracted demand of area i
ΔP_{mi} power generation of Genco i
$\Delta P_{\mathrm{Loc}-i}$ total local demand (contracted and uncontracted) in area i
v_{3i} scheduled $\Delta P_{\mathrm{tie}-i}$ ($\Delta P_{\mathrm{tie}-i,\mathrm{scheduled}}$)
$\Delta P_{\mathrm{tie}-i,\mathrm{actual}}$ actual $\Delta P_{\mathrm{tie}-i}$

Interested readers can find details on the above LFC modeling and simulation for a given restructured power system in Refs. [1,4].

3. LFC formulation via mixed H_2/H_∞

The main control framework in order to formulate the LFC problem via a mixed H_2/H_∞ control design for a given control area (Fig. 1) is shown in Fig. 2. The removed part of the block diagram (right hand) is the same as Fig. 1. In Fig. 2, the Δ_i model the structured uncertainty set in the form of a multiplicative type, and W_i includes the associated weighting function. It is notable that in the model of a power system, there are several uncertainties because of parameter variations,

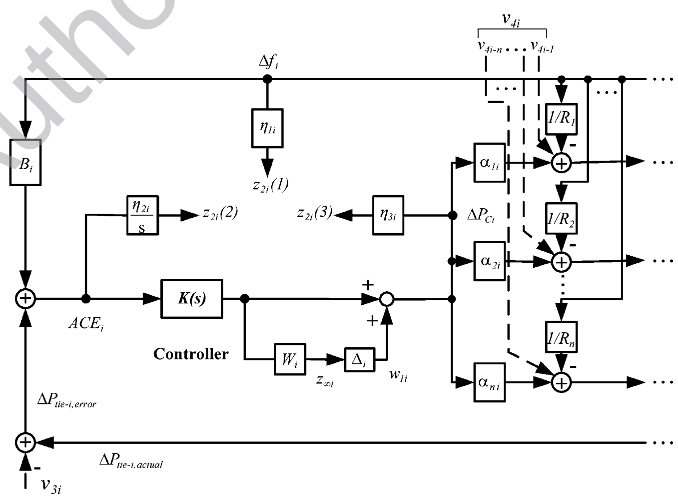

Fig. 2. Proposed control strategy.

model linearization and unmodeled dynamics due to some approximations. Usually, the uncertainties in the power system can be modeled as multiplicative and/or additive uncertainties [5]. The output channel $z_{\infty i}$ is associated with the H_∞ performance, while the fictitious output z_{2i}, containing $z_{2i}(1)$, $z_{2i}(2)$ and $z_{2i}(3)$, is associated with linear quadratic gaussian (LQG) aspects or H_2 performance.

η_{1i}, η_{2i} and η_{3i} are constant weights that must be chosen by the designer to get the desired performance and consider practical constraints on the control action. Experience suggests that one can fix the weights η_{1i}, η_{2i} and η_{3i} to unity and use the method with the regional pole placement technique for performance tuning [6]. We can redraw Fig. 2 as shown in Fig. 3, where $G_i(s)$ and $K_i(s)$ correspond to the nominal dynamical model of the given control area and controller, respectively. Also, y_i is the measured output (performed by area control error ACE), u_i is the control input and w_i includes the perturbed and disturbance signals in the control area.

According to Fig. 3, the LFC as a multi-objective control problem can be expressed by the following optimization problem: design a controller that minimizes the 2 norm of the fictitious output signal z_{2i} under the constraints that the ∞ norm of the transfer function from w_{1i} to $z_{\infty i}$ is less than one. On the other hand, the LFC design is reduced to finding an internally stabilizing controller K_i that minimizes $\|T_{z_{2i}w_i}\|_2$ while maintaining $\|T_{z_{\infty i}w_{1i}}\|_\infty < 1$. This problem can be solved by convex optimization using linear matrix inequalities.

Considering Fig. 1 and the proposed control framework (Fig. 3), the state space model for control area i, $G_i(s)$, can be obtained as

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_{1i} w_i + B_{2i} u_i \\ z_{\infty i} &= C_{\infty i} x_i + D_{\infty 1i} w_i + D_{\infty 2i} u_i \\ z_{2i} &= C_{2i} x_i + D_{21i} w_i + D_{22i} u_i \\ y_i &= C_{yi} x_i + D_{y1i} w_i \end{aligned} \tag{11}$$

where

$$x_i^{\mathrm{T}} = \begin{bmatrix} \Delta f_i & \Delta P_{\text{tie}-i} & \int \text{ACE}_i & x_{ti} & x_{gi} \end{bmatrix} \tag{12}$$

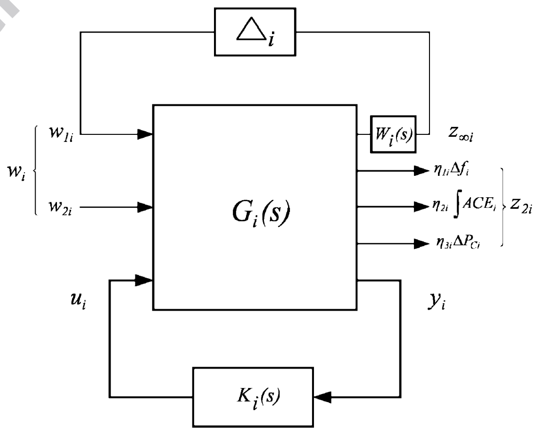

Fig. 3. Mixed H_2/H_∞ based control framework.

$$x_{ti} = [\,\Delta P_{t1i} \quad \Delta P_{t2i} \quad \ldots \quad \Delta P_{tni}\,] \tag{13}$$

$$x_{gi} = [\,\Delta P_{g1i} \quad \Delta P_{g2i} \quad \ldots \quad \Delta P_{gni}\,] \tag{14}$$

$$w_i^{\mathrm{T}} = [\,w_{1i} \quad w_{2i}\,], \quad w_{2i}^{\mathrm{T}} = [\,v_{1i} \quad v_{2i} \quad v_{3i} \quad v_{4i}\,], \quad v_{4i}^{\mathrm{T}} = [\,v_{4i-1} \quad v_{4i-2} \quad \ldots \quad v_{4i-n}\,] \tag{15}$$

$$y_i = \mathrm{ACE}_i \tag{16}$$

$$u_i = \Delta P_{Ci}, \quad z_{2i}^{\mathrm{T}} = \lfloor\,\eta_{1i}\Delta f_i \quad \eta_{2i}\int \mathrm{ACE}_i \quad \eta_{3i}\Delta P_{Ci}\,\rfloor \tag{17}$$

and

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} & A_{i13} \\ A_{i21} & A_{i22} & A_{i23} \\ A_{i31} & A_{i32} & A_{i33} \end{bmatrix}, \quad B_{1i} = \begin{bmatrix} B_{1i11} & B_{1i12} \\ B_{1i21} & B_{1i22} \\ B_{1i31} & B_{1i32} \end{bmatrix}, \quad B_{2i} = \begin{bmatrix} B_{2i1} \\ B_{2i2} \\ B_{2i3} \end{bmatrix}$$

$$A_{i11} = \begin{bmatrix} -D_i/2\pi M_i & -1/2\pi M_i & 0 \\ \sum_{\substack{j=1 \\ j\neq i}}^{N} T_{ij} & 0 & 0 \\ B_i & 1 & 0 \end{bmatrix}, \quad A_{i12} = \begin{bmatrix} 1/2\pi M_i & \ldots & 1/2\pi M_i \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{bmatrix}_{3\times n}$$

$$A_{i22} = -A_{i23} = \mathrm{diag}[\,-1/T_{t1i} \quad -1/T_{t2i} \quad \ldots \quad -1/T_{tni}\,],$$
$$A_{i33} = \mathrm{diag}[\,-1/T_{g1i} \quad -1/T_{g2i} \quad \ldots \quad -1/T_{gni}\,]$$

$$A_{i31} = \begin{bmatrix} -1/(T_{g1i}R_{1i}) & 0 & 0 \\ \vdots & \vdots & \vdots \\ -1/(T_{gni}R_{ni}) & 0 & 0 \end{bmatrix}, \quad A_{i13} = A_{i21}^{\mathrm{T}} = 0_{3\times n}, A_{i32} = 0_{n\times n}$$

$$B_{1i12} = \begin{bmatrix} -1/2\pi M & 0 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 0 & \cdots & 0 \end{bmatrix}_{3\times(3+n)}$$

$$B_{1i22} = 0_{n\times(3+n)}, \quad B_{1i32} = [\,0_{n\times 3} \quad b\,], \quad b = \mathrm{diag}[\,1/T_{g1i} \quad 1/T_{g2i} \quad \cdots \quad 1/T_{gni}\,]$$

$$B_{2i1} = 0_{3\times 1}, \quad B_{2i2} = 0_{n\times 1}, \quad B_{2i3}^{\mathrm{T}} = [\,\alpha_{1i}/T_{g1i} \quad \alpha_{2i}/T_{g2i} \quad \cdots \quad \alpha_{ni}/T_{gni}\,]$$

$$B_{1i11} = 0_{3\times 1}, \quad B_{1i21} = 0_{n\times 1}, \quad B_{1i31}^{\mathrm{T}} = [\,\alpha_{1i}/T_{g1i} \quad \alpha_{2i}/T_{g2i} \quad \cdots \quad \alpha_{ni}/T_{gni}\,]$$

$$C_{\infty i} = 0_{1\times(2n+3)}, \quad D_{\infty 1i} = [\,-1 \quad 0_{1\times(3+n)}\,], \quad D_{\infty 2i} = 1$$

$$C_{2i} = [\,c_{2i1} \quad c_{2i2}\,], \quad c_{2i1} = \begin{bmatrix} \eta_{1i} & 0 & 0 \\ 0 & 0 & \eta_{2i} \\ 0 & 0 & 0 \end{bmatrix}, \quad c_{2i2} = 0_{3\times 2n}$$

$$D_{21i} = 0_{3\times(4+n)}, D_{22i} = \begin{bmatrix} 0 \\ 0 \\ \eta_{3i} \end{bmatrix}$$

The proposed control framework covers all mentioned LFC objectives. The H_2 performance is used to minimize the effects of disturbances on area frequency and area control error by introducing fictitious controlled outputs $z_{2i}(1)$ and $z_{2i}(2)$. As a result, the tie line power flow, which can be described as a linear combination of frequency deviation and ACE signals,

$$\Delta P_{\text{tie}-i,\text{error}} = \text{ACE}_i - B_i \Delta f_i \tag{18}$$

is controlled. Furthermore, the fictitious output $\eta_{3i}\Delta P_{Ci}$ sets a limit on the allowed control signal to penalize fast changes and large overshoots in the governor load set point with regard to corresponding practical constraints on power generation by the generator units. Also, in LFC design, it is important to maintain the frequency regulation and desired performance in the face of uncertainties affecting the control area [7]. The H_∞ performance is used to meet robustness against specified uncertainties and to reduce their impacts on closed loop system performance. Therefore, it is expected that the proposed strategy satisfy the main objectives of the LFC system under load disturbance and model uncertainties.

4. PI-based multi-objective LFC design

4.1. Transformation from PI to SOF control design

According to Fig. 1, in each control area, the ACE provides the input signal of the controller, which is the PI type practically used by the LFC system. Therefore, we have

$$u_i = \Delta Pci = k_{Pi}\text{ACE}_i + k_{Ii}\int \text{ACE}_i \tag{19}$$

By augmenting the system described (11) to include the ACE signal and its integral as a measured output vector, the PI control problem becomes one of finding a static output feedback (SOF) that satisfies prescribed performance requirements. Using this strategy, the PI based LFC design can be reduced to a H_2/H_∞ SOF control problem as shown in Fig. 4. In order to change Eq. (19) to a simple SOF control as

$$u_i = K_i y_i \tag{20}$$

we can rewrite Eq. (19) as follows [8]:

$$u_i = [\,k_{Pi} \quad k_{Ii}\,]\begin{bmatrix} \text{ACE}_i \\ \int \text{ACE}_i \end{bmatrix} \tag{21}$$

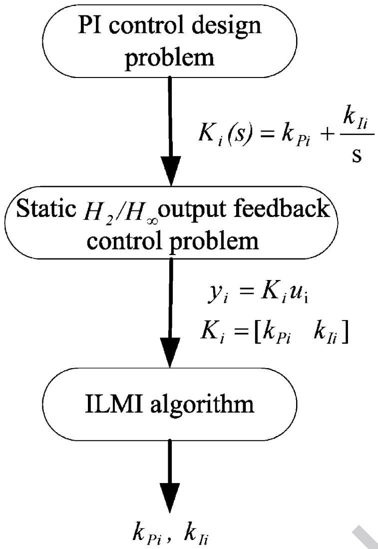

Fig. 4. Problem formulation.

Therefore, y_i in Eq. (20) can be augmented to the following form:

$$y_i^{\mathrm{T}} = \begin{bmatrix} \mathrm{ACE}_i & \int \mathrm{ACE}_i \end{bmatrix} \tag{22}$$

and for the corresponded coefficients in Eq. (11), we can write

$$C_{yi} = [\, c_{yi} \quad 0_{2\times 2n} \,], \quad c_{yi} = \begin{bmatrix} \beta_i & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D_{y1i} = 0_{2\times(4+n)}$$

4.2. H_2/H_∞ SOF design: Developed ILMI algorithm

Consider the linear time invariant system $G_i(s)$ with the state space realization of Eq. (11). A mixed H_2/H_∞ SOF control design can be expressed in the following optimization problem:

4.2.1. Optimization problem

Determine an admissible SOF law K_i belonging to a family of internally stabilizing SOF gains K_{sof},

$$u_i = K_i y_i, K_i \in K_{\mathrm{sof}} \tag{23}$$

such that

$$\inf_{K_i \in K_{\mathrm{sof}}} \|T_{z_{2i}} w_{2i}\|_2 \quad \text{subject to } \|T_{z_{\infty i}} w_{1i}\|_\infty < 1 \tag{24}$$

This problem defines a robust performance synthesis problem where the H_2 norm is chosen as the performance measure. Recently, several methods have been proposed to obtain the suboptimal solution for the H_2, H_∞ and H_2/H_∞ SOF control problems [9,10].

On substitution of Eq. (20) into Eq. (11), it is easy to find that for each control area the state space realization of the closed loop system will be given as

$$\begin{aligned} \dot{x}_i &= A_{ic}x_i + B_{1ic}w_i \\ z_{\infty i} &= C_{\infty ic}x_i \\ z_{2i} &= C_{2ic}x_i \\ y_i &= C_{yic}x_i \end{aligned} \tag{25}$$

and we can write [12],

$$\|T_{z_{2i}w_{2i}}\|_2^2 = \mathrm{trace}(C_{2ic}L_C C_{2ic}^{\mathrm{T}}) \tag{26}$$

where L_C denotes the controllability Gramian of the pair (A_{ic}, B_{1ic}). Lemma 1 (see Appendix A) gives a solution for the H_2 suboptimal SOF based on the given idea in Lemma 2. Following, a new ILMI algorithm is introduced to get a suboptimal solution for the above optimization problem. Specifically, the proposed algorithm formulates the H_2/H_∞ SOF control as a general SOF stabilization problem (see Lemma 2 in Appendix A) to get a family of H_2 stabilizing controllers K_{sof}. Then, the designed controller $K_i \in K_{\mathrm{sof}}$ will be chosen such that

$$|\gamma_2^* - \gamma_2| < \varepsilon, \quad \gamma_\infty = \|T_{z_{\infty i}w_{1i}}\|_\infty < 1 \tag{27}$$

where ε is a small real positive number, γ_2^* is the resulting H_2 performance by K_i subject to the given constraint in Eq. (27) and γ_2 is the resulting H_2 optimal performance index from the applied standard H_2/H_∞ dynamic output feedback control to the control area i as shown in Fig. 3.

Using Lemma 1, a family of H_2 stabilizing SOF gains K_{sof} can be obtained, but we are looking for the solution of such controller within this family that satisfies the given constraint in Eq. (27). The developed algorithm, which is mainly based on Lemmas 1 and 2, gives an iterative LMI suboptimal solution to obtain a H_2/H_∞ SOF controller for a given power system control area:

Step 1. Compute the state space model Eq. (11) for the given control area.

Step 2. Compute the optimal guaranteed H_2 performance index γ_2 using function *hinfmix* in the MATLAB based LMI control toolbox [2] to design a standard H_2/H_∞ output dynamic controller as descripted in Section 3, for the performed system in step 1. Set $i = 1$, $\Delta\gamma_2 = \Delta\gamma_0$ and let $\gamma_{2i} = \gamma_0 > \gamma_2$. $\Delta\gamma_0$ and γ_0 are positive real numbers.

Step 3. Select $Q = Q_0 > 0$, and solve for X from the following algebraic Riccati equation:

$$A_iX + XA_i^{\mathrm{T}} - XC_{yi}^{\mathrm{T}}C_{yi}X + Q = 0, \quad X > 0 \tag{28}$$

Set $P_1 = X$.

Step 4. Solve the following optimization problem for X_i, K_i and a_i:

Minimize a_i subject to the below LMI constraints:

$$\begin{bmatrix} A_iX_i + X_iA_i^{\mathrm{T}} + B_{1i}B_{1i}^{\mathrm{T}} - P_iC_{yi}^{\mathrm{T}}C_{yi}X_i - X_iC_{yi}^{\mathrm{T}}C_{yi}P_i + P_iC_{yi}^{\mathrm{T}}C_{yi}P_i - a_iX_i & B_{2i}K_i + X_iC_{yi}^{\mathrm{T}} \\ (B_{2i}K_i + X_iC_{yi}^{\mathrm{T}})^{\mathrm{T}} & -I \end{bmatrix} < 0 \tag{29}$$

$$\mathrm{trace}(C_{2ic}L_C C_{2ic}^{\mathrm{T}}) < \gamma_{2i} \tag{30}$$

$$X_i = X_i^{\mathrm{T}} > 0 \tag{31}$$

Denote a_i^* as the minimized value of a_i.

Step 5. If $a_i^* \leqslant 0$, go to step 9.

Step 6. For $i > 1$, if $a_{i-1}^* \leqslant 0$, $K_{i-1} \in K_{\mathrm{sof}}$ is an H_2 controller and go to step 10. Otherwise go to step 7.

Step 7. Solve the following optimization problem for X_i and K_i:

Minimize trace(X_i) subject to LMI constraints Eqs. (29)–(31) with $a_i = a_i^*$. Denote X_i^* as the X_i that minimized trace(X_i).

Step 8. Set $i = i + 1$ and $P_i = X_{i-1}^*$, then go to step 4.

Step 9. Set $\gamma_{2i} = \gamma_{2i} - \Delta\gamma_2$, $i = i + 1$. Then do steps 3–5.

Step 10. If

$$\gamma_{\infty,i-1} = \|W_i K_{i-1} G_i (I + K_{i-1} G_i)^{-1}\|_\infty \leqslant 1 \tag{32}$$

the K_{i-1} is an H_2/H_∞ SOF controller and $\gamma_2^* = \gamma_{2i} - \Delta\gamma_2$ indicates a lower H_2 bound such that the obtained controller satisfies Eq. (27). Otherwise, set $\gamma_{2i} = \gamma_{2i} + \Delta\gamma_2$, $i = i + 1$, then do steps 3–5.

In Section 5, two types of robust controllers are developed for a power system example including three control areas. The first one is an H_2/H_∞ dynamic controller based on the general robust LMI design and the second controller is based on the developed ILMI H_2/H_∞ SOF algorithm with the same assumed objectives and initializations to achieve the desired robust performance.

5. Application to a 3 control area power system

To illustrate the effectiveness of the proposed control strategy, a three control area power system, shown in Fig. 5, is considered as a test system. It is assumed that each control area includes two Gencos, which use the same ACE participation factor. The power system parameters are considered the same as Ref. [1].

5.1. Uncertainty and performance weights selection

In this example, with regards to uncertainties, it is assumed that the rotating mass and load pattern parameters have uncertain values in each control area. The variation range for the D_i and M_i parameters in each control area is assumed as ±20%. It is notable that we are not under obligation to consider the uncertainty in only a few parameters. Considering a more complete model by including additional uncertainties is possible and causes less conservative synthesis. However the complexity of computations and the order of the resulting controller will increase. Following, these uncertainties are modeled as an unstructured multiplicative uncertainty block that contains all the information available about the D_i and M_i variations.

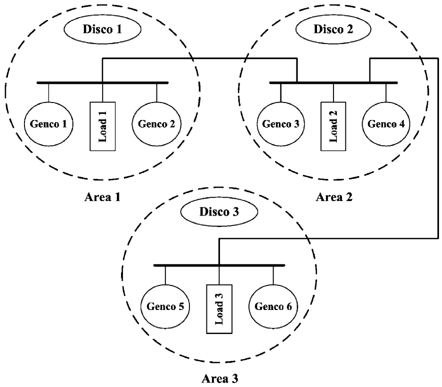

Fig. 5. Three control area power system.

As is mentioned in Section 3, we can consider the specified uncertainty in each area as a multiplicative uncertainty (W_i) associated with a nominal model. Let $\widehat{G}_i(s)$ denote the transfer function from the control input u_i to control output y_i at operating points other than the nominal point. Following a practice common in robust control, we will represent this transfer function as

$$| \Delta_i(s)W_i(s) |=| [\widehat{G}_i(s) - G_{0i}(s)]G_{0i}(s)^{-1} |; \quad G_{0i}(s) \neq 0 \tag{33}$$

where,

$$\|\Delta_i(s)\|_\infty = \sup_\omega | \Delta_i(s) | \leqslant 1 \tag{34}$$

$\Delta_i(s)$ shows the uncertainty block corresponding to the uncertain parameters and $G_{0i}(s)$ is the nominal transfer function model. Thus, $W_i(s)$ is such that its respective magnitude Bode plot covers the Bode plots of all possible plants. For example, using Eq. (33), some sample uncertainties corresponding to different values of D_i and M_i for area 1 can be obtained as shown in Fig. 6. Since the frequency responses of both sets of parametric uncertainties are close to each other, to keep

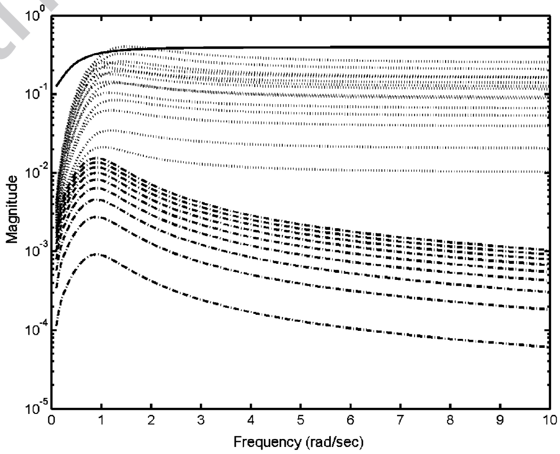

Fig. 6. Uncertainty plots due to parameters changes in area 1; D_i (dotted), M_i (dash-dotted) and W_1 (solid).

the complexity of the obtained controller low, we can model the uncertainties due to both sets of parameters variations by using a norm bonded multiplicative uncertainty to cover all possible plants as follows:

$$W_1(s) = \frac{0.3986s + 0.0786}{s + 0.6888} \tag{35}$$

Fig. 6 clearly shows that attempts to cover the uncertainties at all frequencies and finding a tighter fit (in low and high frequencies) using a higher order transfer function will result in a high order controller. The weight Eq. (35) used in our design provides a conservative design at low and high frequencies, but it gives a good trade off between robustness and controller complexity. Using the same method, the uncertainty weighting functions for areas 2 and 3 are computed as follows:

$$W_2(s) = \frac{0.3088s + 0.0487}{s + 0.6351}, \quad W_3(s) = \frac{0.3483s + 0.0751}{s + 0.7826} \tag{36}$$

The selection of performance constant weights η_{1i}, η_{2i} and η_{3i} is dependent on specified performance objectives and must be chosen by the designer. In fact, an important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives. The selection of these weights entails a trade off among several performance requirements. The coefficients η_{1i} and η_{2i} of controlled outputs set the performance goals, i.e., tracking the load variation and disturbance attenuation. η_{3i} sets a limit on the allowed control signal to penalize fast changes and large overshoots in the governor load set point signal. Here, a set of suitable values for constant weights is chosen as follows:

$$\eta_{1i} = 1.25, \quad \eta_{2i} = 0.001, \quad \eta_{3i} = 1.5 \tag{37}$$

5.2. Mixed H_2/H_∞ dynamic and SOF control design

For the sake of comparison, in addition to the proposed control strategy to synthesis the robust PI controller, a mixed H_2/H_∞ dynamic output feedback controller is designed for each area using the *hinfmix* function in the LMI control toolbox [2]. This function gives an optimal H_2/H_∞ controller through the mentioned optimization problem Eq. (24) and returns the controller $K(s)$ with optimal H_2 performance index γ_2. The resulting controllers are dynamic type and have the following state space form, whose orders are the same as the size of the generalized plant model (8th order in the present paper).

$$\begin{aligned} \dot{x}_{ki} &= A_{ki}x_{ki} + B_{ki}y_i \\ u_i &= C_{ki}x_{ki} + D_{ki}y_i \end{aligned} \tag{38}$$

At the next step, according to the synthesis methodology described in Section 4, a set of three decentralized robust PI controllers are designed. This control strategy is fully suitable for LFC applications that usually employ PI control, while most of the other robust and optimal control designs (such as the LMI approach) yield complex controllers whose size can be larger than real

Table 1
PI control parameters from ILMI design

Parameters	Area 1	Area 2	Area 3
k_{Pi}	−0.1250	−0.0015	−0.4278
k_{Ii}	−5.00E-04	−5.14E-04	−5.30E-04

Table 2
Robust performance indices

Performance index	Area 1	Area 2	Area 3
γ_{2i} (Dynamic)	2.1835	1.7319	2.1402
γ^*_{2i} (PI)	2.2900	1.8321	2.2370
$\gamma_{\infty i}$ (Dynamic)	0.4177	0.3339	0.3536
$\gamma^*_{\infty i}$ (PI)	0.3986	0.3088	0.3483

world LFC systems. Using the ILMI approach, the controllers are obtained following several iterations. The proposed control parameters for three control areas are shown in Table 1. The optimal performance indices for dynamic and PI controllers are listed in Table 2.

The resulting robust performance indices of both synthesis methods (γ_{2i} and γ^*_{2i}) are close to each other. This shows that although the proposed ILMI approach gives a set of much simpler controllers (PI) than the dynamic H_2/H_∞ design, however, they hold robust performance as well as the dynamic H_2/H_∞ controllers.

6. Simulation results

In order to demonstrate the effectiveness of the proposed strategy, some simulations were performed. In these simulations, the proposed PI controllers were applied to the three control area power system described in Fig. 5. The performance of the closed loop system using the designed PI controllers in comparison with full order H_2/H_∞ dynamic controllers is tested in the presence of load demands, disturbances and uncertainties.

6.1. Case 1

In this case, the closed loop performance is tested in the face of both step contracted load demand and uncertainties. It is assumed that a large load demand 100 MW (0.1 pu) is requested by each Disco, following a 20% decrease in uncertain parameters D_i and M_i. Furthermore, assume the Discos contract with the available Gencos according to the following GPM:

$$\mathrm{GPM}^{\mathrm{T}} = \begin{bmatrix} 0.25 & 0.5 & 0 & 0.25 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0.25 & 0 \\ 0 & 0 & 0.75 & 0 & 0 & 0.25 \end{bmatrix}$$

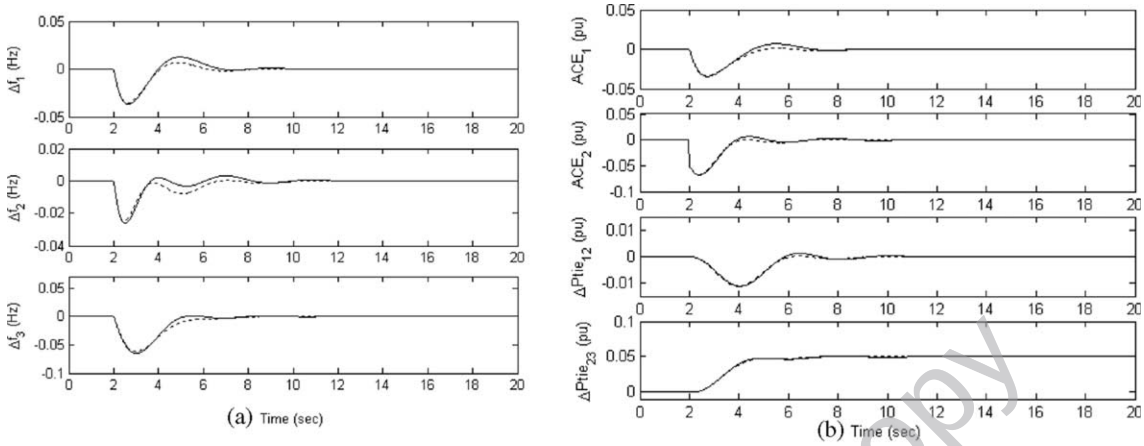

Fig. 7. (a) Frequency deviation; (b) area control error and tie line powers; solid (ILMI based PI), dotted (dynamic H_2/H_∞).

All Gencos participate in the LFC task. Gencos 2 and 6 only participate for performing the LFC in their areas, while other Gencos track the load demand in their areas and/or others. Frequency deviation, area control errors (ACE1 and ACE2) and tie line power changes are shown in Fig. 7. Using the proposed method, the area control error and frequency deviation of all areas are quickly driven back to zero. The tie line power flows are properly convergent to the specified values Eq. (4). The generated powers are shown in Fig. 8. The actual generated powers of Gencos reach the desired values in the steady state (10) as given in Table 3.

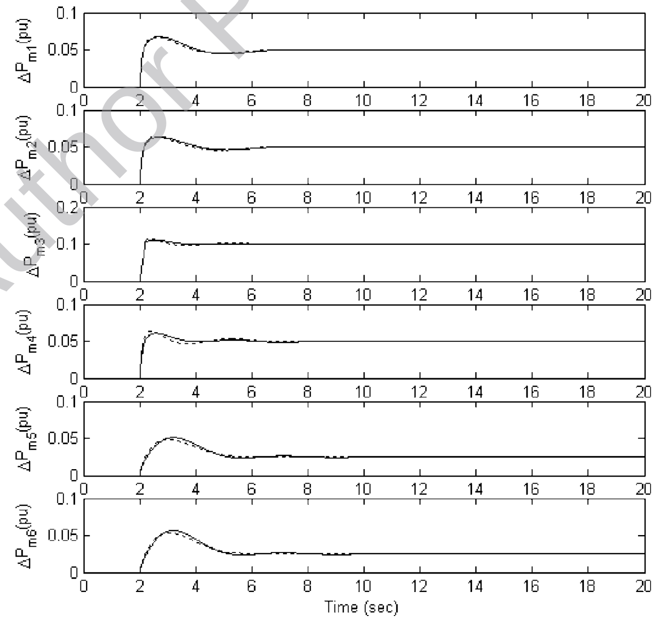

Fig. 8. Mechanical power changes; solid (ILMI based PI), dotted (dynamic H_2/H_∞).

Table 3
Generated power in response to case 1

Genco	1	2	3	4	5	6
ΔP_{mi} (*pu*)	0.05	0.05	0.1	0.05	0.025	0.025

6.2. Case 2

Consider case 1 again. Assume, in addition to the specified contracted load demands (0.1 pu) and 20% decrease in D_i and M_i, a bounded random step load change as a large uncontracted demand (shown in Fig. 9a) appears in each control area, where

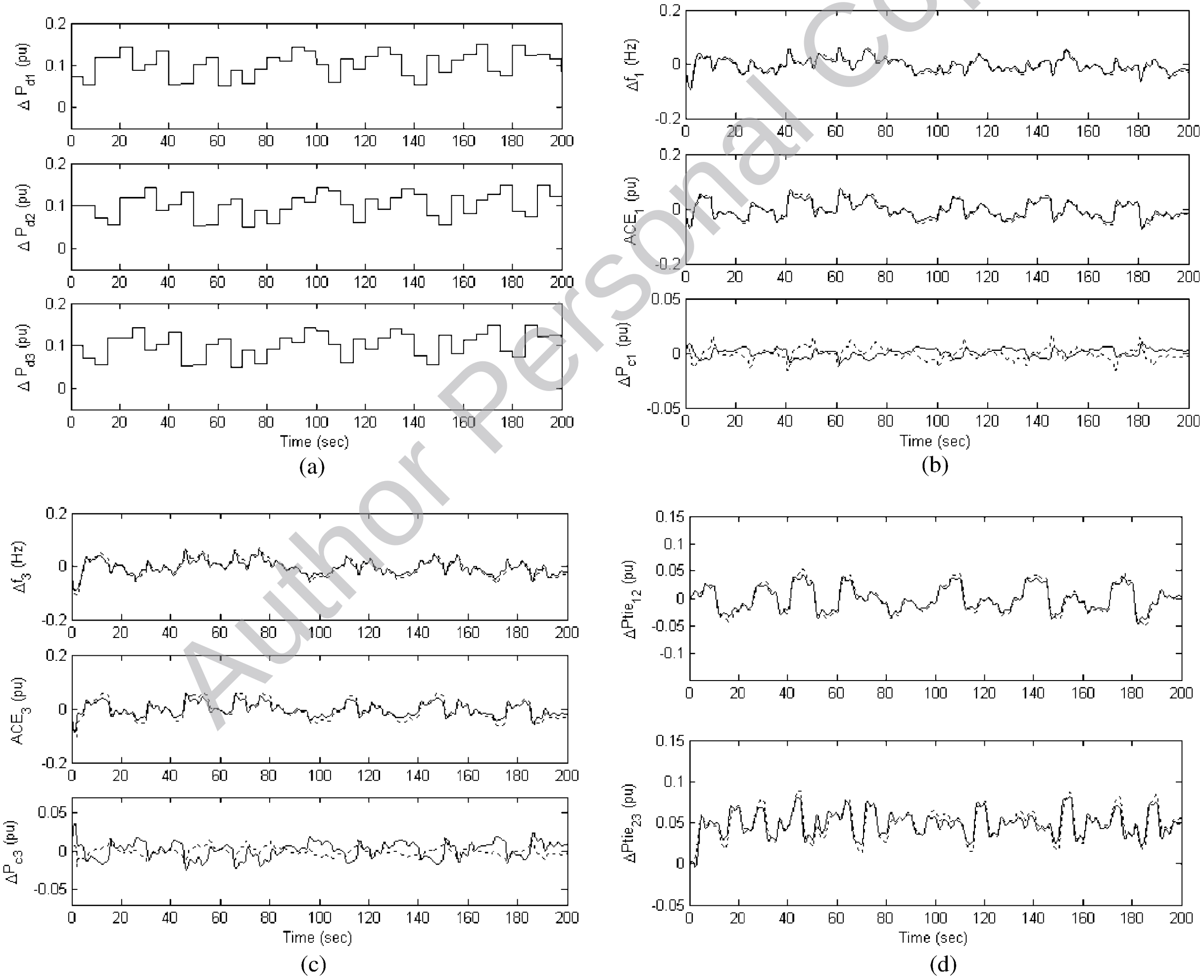

Fig. 9. Power system response for case 2. (a) Random load patterns, (b) area-1, (c) area-3, (d) tie line powers; solid (ILMI based PI), dotted (dynamic H_2/H_∞).

$$-50\ \text{MW}(-0.05\ \text{pu}) \leqslant \Delta P_{di} \leqslant +50\ \text{MW}(+0.05\ \text{pu})$$

The purpose of this scenario is to test the robustness of the proposed controllers against uncertainties and random large load disturbances. The closed loop response for areas 1 and 3 are shown in Fig. 9b and c. Fig. 9d shows the tie line power flows.

The simulation results demonstrate that the proposed ILMI based PI controllers track the load fluctuations and meet robustness for a wide range of load disturbances and possible bilateral contract scenarios as well as the H_2/H_∞ dynamic controllers.

7. Conclusion

Since, in real world restructured power systems, each control area is faced with various uncertainties and disturbances, the LFC problem in a multi-area power system is formulated as a decentralized multi-objective optimization control problem via a mixed H_2/H_∞ technique. An iterative LMI approach has been proposed for a bilateral based LFC scheme. The design strategy includes enough flexibility to set the desired level of performance and gives a set of simple PI controllers, which are commonly useful in real world power systems.

The proposed method was applied to a three control area power system and is tested under various operating conditions. The results are compared with the results of applied H_2/H_∞ dynamic output controllers. It was shown that the proposed simple ILMI based PI controllers are capable to guarantee the robust performance, such as precise reference, frequency tracking and disturbance attenuation, under a wide range of area load disturbances and specified uncertainties as well as the H_2/H_∞ dynamic controllers.

Appendix A

Lemma 1. [H_2 Suboptimal SOF, [10].] *For fixed* $(A_i, B_{1i}, B_{2i}, C_{yi}, K_i)$, *there exist a positive definite matrix X which solves inequality*

$$(A_i + B_{2i}K_iC_{yi})X + X(A_i + B_{2i}K_iC_{yi})^{\mathrm{T}} + B_{1i}B_{1i}^{\mathrm{T}} < 0, \quad X > L_C \tag{A.1}$$

to satisfy $\|T_{z_{2i}w_{2i}}\|_2 < \gamma$, *if and only if the following inequality has a positive definite matrix solution*,

$$A_iX + XA_i^{\mathrm{T}} + XC_{yi}^{\mathrm{T}}C_{yi}X + (B_{2i}K_i + XC_{yi}^{\mathrm{T}})(B_{2i}K_i + XC_{yi}^{\mathrm{T}})^{\mathrm{T}} + B_{1i}B_{1i}^{\mathrm{T}} < 0 \tag{A.2}$$

Lemma 2. [General stabilizing SOF, [11]]. *The system* (A, B, C) *that may also be identified by the following representation*:

$$\dot{x} = Ax + Bu \quad y = Cx \tag{A.3}$$

is stabilizable via static output feedback if and only if there exist $P > 0$, $X > 0$ *and* K_i *satisfying the following matrix inequality*

$$A^{\mathrm{T}}X + XA - PBB^{\mathrm{T}}X - XBB^{\mathrm{T}}P + PBB^{\mathrm{T}}P + (B^{\mathrm{T}}X + K_iC)^{\mathrm{T}}(B^{\mathrm{T}}X + K_iC) < 0 \tag{A.4}$$

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