Bevrani, Hassan and Hiyama, Takashi (2006) On robust control of fixed pattern power rectifiers. In *Proceedings The Nordic Workshop on Power and Industrial Electronics (NORPIE)* CD-ROM, Lund, Sweden.

On Robust Control of Fixed Pattern Power Rectifiers

H. Bevrani, *Member, IEEE*, and T. Hiyama, *Senior Member, IEEE*

***Abstract*—This paper addresses a new systematic method to design a robust proportional-integral (PI) controller for a three-phase sinusoidal active power rectifier based on H∞ control technique. The control design problem is reduced to solve an H∞ based static output feedback control problem. To determine the optimal gains, an iterative linear matrix inequalities algorithm is used. A classical power active rectifier example is given to demonstrate the efficiency of developed approach. The proposed robust technique is shown to maintain the robust performance and minimize the effects of disturbances, properly.**

***Index Terms*—AC-DC power converters, PI, H∞ control, static output feedback, LMI.**

I. Introduction

BECAUSE of some important advantages, for example near sinusoidal ac input currents, near unity power factor and regenerative capabilities, the active front-end rectifiers are receiving more attention in comparison of diode front-end rectifiers during the last years. Over the years, many different control methods for the active rectifiers have been presented. Many of these methods show complex control stricture and requirements. Recently some useful and simple methodologies have been proposed to control synthesis/analysis of power rectifiers [1-5].

However, the most of given tuning procedures for the controller parameters are mainly based on the classical methods following some modifications. It is clear that meeting all design specifications and control objectives by a simple PI controller which is tuned based on classical and experiences/trial-error methods is difficult. Using the analytical model, it can be easily shown that the classical PI controllers yield poor dynamic results, especially at low load conditions [1, 6].

In this paper, a simple proportional-integral (PI) controller is designed. To calculate the PI parameters optimally, first the PI control problem for the assumed power rectifier is formulated as an H∞ static output feedback (H∞-SOF) control problem and then the control parameters are easily carried out using a developed iterative linear matrix inequalities (ILMI) algorithm.

This work was supported in part by Japan Society for Promotion of Science (JSPS) under grant P04346.

H. Bevrani is with the Department of Electrical and Computer Eng., Kurdistan University, Iran (corresponding author's e-mail: bevrani@st.eecs.kumamoto-u.ac.jp).

T. Hiyama is with the Department of Electrical and Computer Eng., Kumamoto University, Japan.

II. Control Background

This section gives a brief overview for the H∞ based static output feedback control design. Consider a linear time invariant system $G(s)$ with the following state-space realization.

$$G(s):\ \begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x \end{aligned} \qquad (1)$$

where x is the state variable vector, w is the disturbance and area interface vector, z is the controlled output vector and y is the measured output vector.

The H∞-SOF control problem for the linear time invariant system $G(s)$ with the state-space realization of (1) is to find a gain matrix K ($u = Ky$), such that the resulted closed-loop system is internally stable, and the H∞ norm from w to z (Fig. 1) is smaller than γ, a specified positive number, i.e.

$$\|T_{zw}(s)\|_\infty < \gamma \qquad (2)$$

It is notable that the H∞-SOF control problem can be transferred to a generalized SOF stabilization problem which is expressed via the following theorem [7].

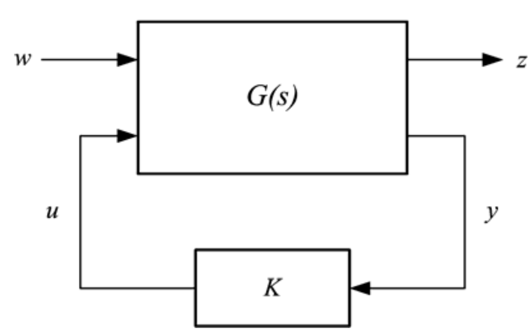

Fig. 1. Closed-loop system via H∞-SOF control

Theorem: The system (*A, B, C*) is stabilizable via SOF if and only if there exist $P>0$, $X>0$ and K satisfying the following quadratic matrix inequality

$$\begin{bmatrix} A^T X + XA - PBB^T X - XBB^T P + PBB^T P & (B^T X + KC)^T \\ B^T X + KC & -I \end{bmatrix} < 0 \qquad (3)$$

Since a solution for the consequent non convex optimization problem (3) can not be directly achieved by using general LMI technique [8], a variety of methods were proposed by many researchers with many analytical and numerical methods to approach a local/global solution. In this paper, to solve the resulted SOF problem, an iterative LMI is used based on the existence necessary and sufficient condition for SOF stabilization, via the H∞ control technique.

III. Proposed Control Strategy

A. Modeling and Problem Formulation

In order to design a robust power system controller, it is first necessary to consider an appropriate linear mathematical description of power rectifier. Consider the overall structure of a controlled three-phase sinusoidal active power rectifier as shown in Fig. 2 [1]. The output voltage v_{dc} is controlled by setting the phase angle θ to an appropriate value. The PI controller controls θ to reduce the output voltage error ($v_{dc} - v_{ref}$). The switches are governed by a fixed pulse width modulation (PWM) pattern and the PLL is used to detect the angle (ωt) of line voltage $v_{m,i}$.

The PI control problem can be transferred to a static output feedback (SOF) control problem by augmenting the measured output signal to include output voltage v_{dc} and its integral.

$$u(t) = Ky(t) \qquad (4)$$

$$u(t) = k_P v_{dc} + k_I \int v_{dc} = [k_P \quad k_I][v_{dc} \quad \int v_{dc}]^T \qquad (5)$$

k_P and k_I are constant real numbers (PI parameters). The main merit of this transformation is in possibility of using the well-known SOF control techniques to calculate the fixed gains, and once the SOF gain vector is obtained, the PI gains are ready in hand and no additional computation is needed.

Now, It is easy to calculate the linear time invariant dynamical model for the active rectifier with the state-space realization in form of (1). The mentioned variables and vectors for the problem at hand can be considered as follows:

$$x^T = [v_{dc} \quad i_d \quad i_q \quad \int v_{dc}] \qquad (4)$$

$$z^T = [\mu_1 \int v_{dc} \quad \mu_2 \theta] \qquad (5)$$

$$y^T = [v_{dc} \quad \int v_{dc}], \; u = \theta \qquad (6)$$

$$w^T = [i_o \quad v_m] \qquad (7)$$

Here, i_d and i_q are the transformed currents in a two-phase synchronous reference frame (*d*, *q*). The i_o is a fictitious current source for modeling of load variation. The H∞ based SOF control problem is to find a static output feedback law $u = Ky$, as shown in Fig. 1.

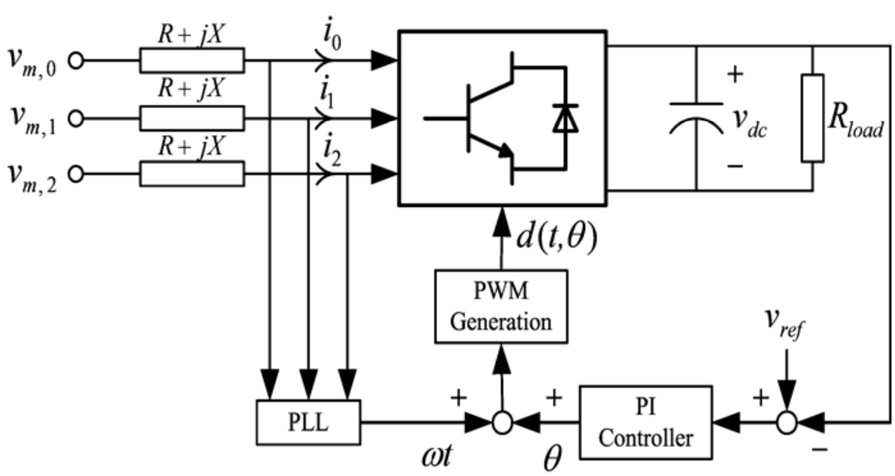

Fig. 2. The overall structure of a controlled active rectifier.

A. Control Strategy

As has described in the previous sections and shown in Fig. 3, to calculate the PI parameters optimally, first the PI control problem for the assumed power rectifier is formulated as an H∞-SOF control problem and then the control parameters are easily carried out using a developed ILMI algorithm.

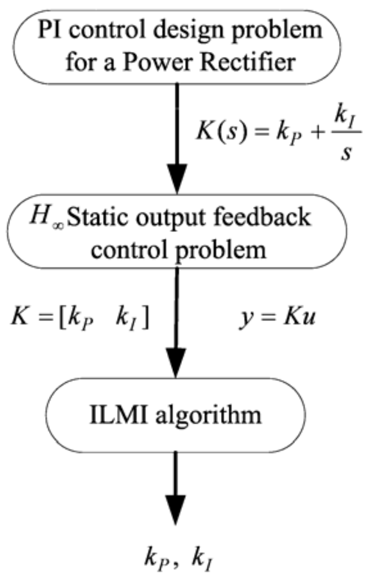

Fig. 3. Control strategy.

The overall control framework to formulate the existing control problem via an H∞-SOF control design is shown in Fig. 4. The output channel z_∞ is associated with the H∞ performance while the y is the augmented measured output vector (performed by v_{dc} and its integral). μ_1 and μ_2 are constant weights that must be chosen by designer to get the desired closed-loop performance.

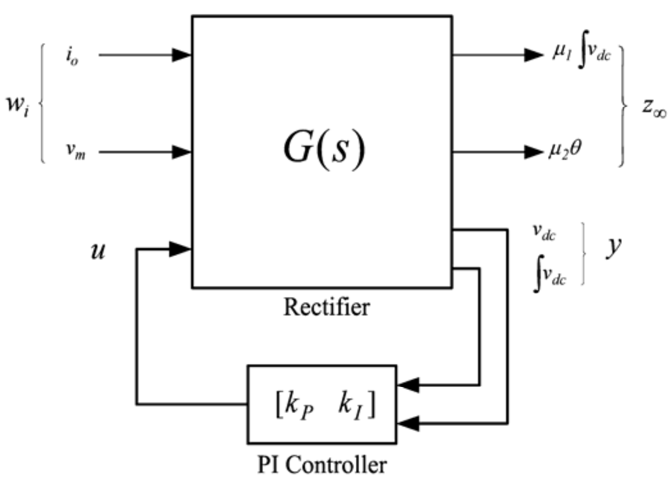

Fig. 4. H∞-SOF control framework.

Experience suggests that one can fix the weights μ_1 and μ_2 to unity and use the method with regional pole placement technique for performance tuning [9]. The first term of z_∞ output is used to minimize the effects of disturbances (due to line voltage and load variations) on output dc voltage by introducing an appropriate fictitious controlled output. Furthermore, fictitious output $\mu_2\theta$ sets a limit on the allowed control signal to penalize fast change and large overshoot in the control action signal [10, 11].

B. Iterative LMI Algorithm

In order to solve the H∞-SOF, an iterative LMI algorithm has been used. Similar to the given approach in [12], the key point is to formulate the H∞ problem via a generalized static output stabilization feedback such that all eigenvalues of (A-$B\,K\,C$) shift towards the left half plane through the reduction of a, a real number, to close to feasibility of (3). The described theorem in the previous section gives a family of internally stabilizing SOF gains is defined as K_{sof}. But the desirable solution K is an admissible SOF law

$$u = Ky \ , \ K \in K_{sof} \tag{8}$$

such that

$$\left\| T_{zw}(s) \right\|_\infty < \gamma^* , \ \left| \gamma - \gamma^* \right| < \varepsilon \tag{9}$$

where ε is a small positive number. Suboptimal performance index γ^* indicates a lower bound such that the closed-loop system is H∞ stabilizable. The optimal performance index (γ), can be obtained from the application of a full dynamic H∞ dynamic output feedback control method.

The proposed algorithm, which is described in Fig. 5, gives an iterative LMI suboptimal solution for above optimization problem. Here A_g , B_g and C_g are three generalized matrices of the following forms

$$A_g = \begin{bmatrix} A & B_1 & 0 \\ 0 & -\gamma I/2 & 0 \\ C_1 & 0 & -\gamma I/2 \end{bmatrix} \tag{10}$$

$$B_g = \begin{bmatrix} B_2 \\ 0 \\ D_{12} \end{bmatrix}, \ C_g = \begin{bmatrix} C_2 & 0 & 0 \end{bmatrix} \tag{11}$$

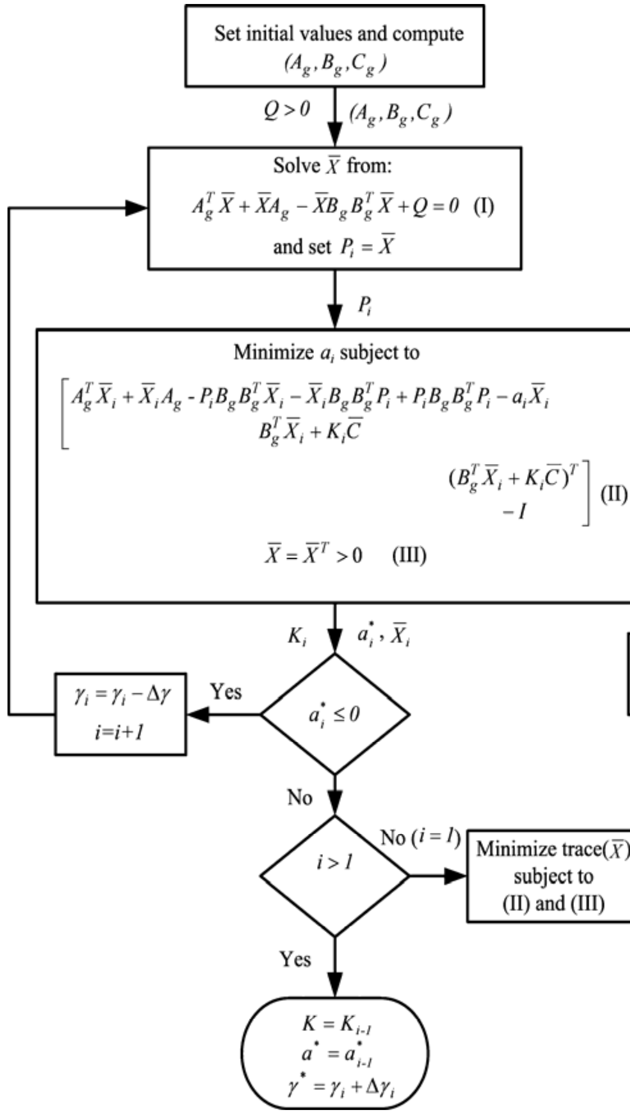

Fig.5. Iterative LMI algorithm.

I. CASE STUDY

In order to illustrate the effectiveness of the proposed control methodology, a 15 kW classical active rectifier is considered as a test system. The three-phase circuit diagram is shown in Fig. 6. The simulation data and circuit parameters are chosen same as given in [1].

For the given rectifier, in addition to the proposed control strategy to obtain the robust PI controller, a full order dynamic H∞ controller using LMI control toolbox in MATLAB is designed. Specifically, using *hinflmi* function in LMI toolbox of MATLAB software [13], a full order robust dynamic controller with the following structure is designed.

$$K(s): \begin{array}{l} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{array} \qquad (12)$$

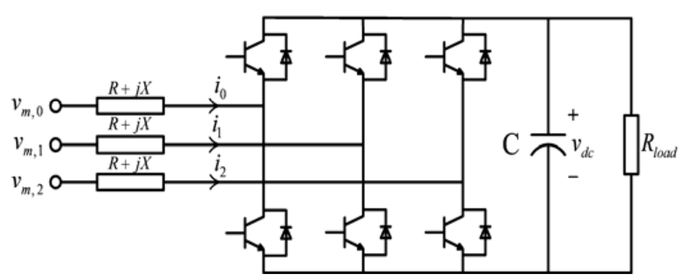

Fig. 6. Circuit diagram of active rectifier.

Then, applying the proposed H∞-SOF control methodology a robust PI controller is obtained for the problem at hand (13).

$$K = \begin{bmatrix} k_P & k_I \end{bmatrix} = \begin{bmatrix} 0.0472 & 4.6560 \end{bmatrix} \qquad (13)$$

The closed loop performance analysis shows that the resulted robust performance indices (γ and γ^*) of both synthesis methods are very close to each other. It indicates that although the proposed H∞-SOF approach gives a much simpler controller (pure gain) than the H∞ dynamic output feedback design, however it holds robust performance as well as dynamic H∞ controller.

II. Simulation Results

Nonlinear simulation is proposed using nonlinear model of given active rectifier. Figures 7 to 9 were obtained from simulation to demonstrate the effectiveness of the proposed design. The figures depict the waveform for the output voltage in the presence of changes in reference voltage, line voltage and rectifier load. In the all simulation, the step change is started at 0.15 sec.

Fig. 7 shows the output voltage for a serial step changes in the specified reference voltage. From this figure, it is evident that after few msec, the tracking error decays to zero; that is, the output voltage converges to the reference one for the H∞ based PI as well as full order dynamic output H∞ controller.

The problem of disturbance attenuation was taken into account in the control design. In particular, control laws have been synthesized to guarantee the disturbance attenuation. The output voltage time response to a large consequence step changes in line voltage V_m is shown in Fig. 8. Finally, Fig. 9 shows the effects of large step load variation on the dc output voltage.

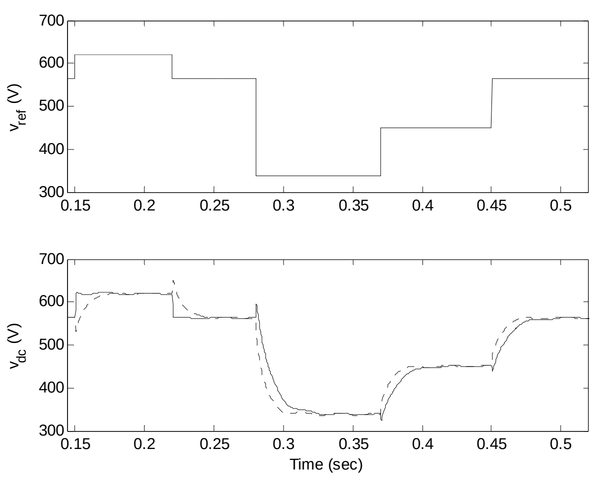

Fig. 7. Output voltage in the presence of changing in reference voltage.

The results show the proposed control synthesis guarantees the robust performance for a wide range of operating conditions (due to variation in load, line and reference voltages) as well as full order dynamic H∞ control design.

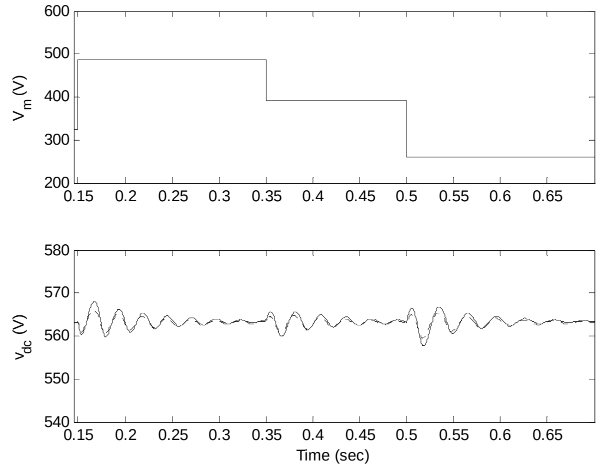

Fig. 8. Output voltage in the face of changing in line voltage.

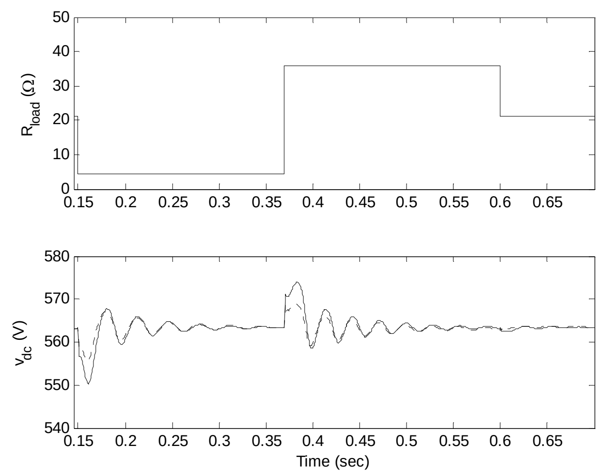

Fig. 9. Output voltage in the presence of load variation.

III. Conclusion

In this paper, we have addressed the problem of robust PI controller design for a three-phase, sinusoidal, active rectifier. The control problem is transferred to an H∞ static output feedback control problem and then the optimal gains are obtained using a developed iterative linear matrix inequalities algorithm. Design strategy includes enough flexibility to setting the desired level of stability and performance. The proposed method was applied to a 15 kW classical active rectifier. Nonlinear simulation results demonstrated the effectiveness of methodology. It was shown that the designed controller is capable to guarantee the robust stability and robust performance, i.e. precise reference tracking and disturbance attenuation under a wide range of reference/line voltage and load conditions as well as a high order H∞ controller.

ACKNOWLEDGMENT

H. Bevrani would like to thank Prof. Y. Mitani from Kyushu Institute of Technology, Prof. K. Tsuji and Prof. T. Ise from Osaka University, Dr. A. Salimi from Kurdistan University for their kind helps during the preliminary step of the present work.

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Hassan Bevrani received his B.S., M.S. (Hons.) and Ph.D. degrees in Electrical Engineering from Mashad University (Iran, 1991), K. N. Toosi University of Technology (Iran, 1997) and Osaka University (Japan, 2004), respectively. Currently, he is an assistant professor of Kurdistan University (Iran) and a postdoctoral research fellow in Kumamoto University (Japan). His special fields of interest include robust and intelligent control applications in Power electronic and Power system industry. He is a member of IEEE, the Institute of Electrical Engineers of Japan and IEE.

Takashi Hiyama received his B.E., M.S., and Ph.D. degrees all in Electrical Engineering from Kyoto University, in 1969, 1971, and 1980, respectively. Since 1989, he has been a Professor at the Department of the Electrical and Computer Engineering, Kumamoto University. His current interests include the application of intelligent systems to power system operation, management, and control. He is a senior member of IEEE, a member of the Institute of Electrical Engineers of Japan, Japan Solar Energy Society and SICE of Japan.