An ILMI Based Solution for Robust Tuning of PI and PID Controllers

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Abstract: *This paper addresses a new method to bridge the gap between the power of optimal multiobjective control and PI/PID industrial controls. First the PI/PID control problem is reduced to a static output feedback control synthesis through the mixed H_2/H_∞ control technique, and then control parameters are easily carried out using an iterative linear matrix inequalities (ILMI) algorithm. Numerical examples on load-frequency control (LFC) design in a multi-area power system are given to illustrate the proposed methodology. The results are compared with genetic algorithm (GA) based multiobjective control and LMI based full order mixed H_2/H_∞ control designs.*

Keywords: Mixed H_2/H_∞ control, PI, PID, static output feedback control, LFC, LMI, robust performance, Time delay.

1 Introduction

The proportional-integral (PI) and proportional-integral-derivative (PID) controllers, because of their functional simplicity, are widely used in industrial applications. However, their parameters are often tuned using experiences or trial and error methods. On the other hand, the most of real-world control problems refer to multi-objective control designs that several objectives such as stability, disturbance attenuation and reference tracking with considering practical constraints must be followed by controller, simultaneously.

It is clear that meeting all design objectives by a simple PI/PID controller which is tuned based on experiences/trial-error methods is difficult. Over the years, many different parameter tuning methods have been presented for PI and PID controllers. A survey up to 2002 is given in Refs. [1] and [2]. Many of these methods are modifications of the frequency response method introduced by Ziegler and Nichols. Some efforts have also been made to find analytical approaches to tune the parameters. Several tuning methodology based on robust and optimal control techniques are introduced to design of PI/PID controllers [3-6]. In the most of proposed approaches, a single norm based performance criteria has been used to evaluate the robustness of resulted control systems.

It is well known that each robust method is mainly useful to capture a set of special specifications. For instance, the H_2 tracking design is more adapted to deal with transient performance by minimizing the linear quadratic cost of tracking error and control input, but H_∞ approach (and μ as a generalized H_∞ approach) is more useful to hold closed-loop stability in presence of control constraints and uncertainties.

Mixed H_2/H_∞ provides a powerful control design to meet different specified control objectives. However, it is usually complicated and not easily implemented for the real industrial applications. Recently, some efforts are reported to make a connection between the theoretical mixed H_2/H_∞ optimal control and classical PID control [7-9]. Ref. [7] has used a combination of different optimization criteria through a multiobjective technique to tune the PI parameters. A genetic algorithm (GA) approach to mixed H_2/H_∞ optimal PID control is given in [8]. A PID controller incorporating an adaptive control scheme for the mixed H_2/H_∞ tracking performance is developed for constrained non-holonomic mechanical systems in [9].

In this paper, the interesting combination of different objectives including H_2 and H_∞ tracking performances for a PI/PID controller has been addressed by a systematical, simple and fast algorithm. The multiobjective PI/PID control problem is formulated as a mixed H_2/H_∞ static output feedback (SOF) control problem to obtain a desired PI/PID controller. An iterative linear matrix inequalities (ILMI) algorithm is developed to tune the PI/PID control parameters to achieve mixed H_2/H_∞ optimal performance.

The proposed strategy is used to design of PI-based load-frequency control (LFC) system for a multi-area power system as a numerical example. The designed robust controllers (PI) are compared with the mixed H_2/H_∞ dynamic output feedback controllers (using general LMI technique [10]) and a GA based approach PI controllers [8]. The preliminary step of this work is given in [11].

2 Backgrounds

2.1 Transformation from PI/PID to SOF Control

In this section, the PI/PID control problem is transferred to a static output feedback (SOF) control problem. The main merit of this transformation is in possibility of using the well-known SOF control techniques to calculate the fixed gains, and once the SOF gain vector is obtained, the PI/PID gains are ready in hand and no additional computation is needed.

In a given PI/PID-based control system *i*, the measured output signal (y_{oi}) performs the input signal for the controller and we can write (for PID type)

$$u_i = k_{Pi} y_{oi} + k_{Ii} \int y_{oi} + k_{Di} \frac{dy_{oi}}{dt} \qquad (1)$$

Where k_{Pi} , k_{Ii} and k_{Di} are constant real numbers. Therefore, by augmenting the system description to include the y_{oi} , its integral and derivative as a new measured output vector (y_i), the PI/PID control problem becomes one of finding a static output feedback that satisfied prescribed performance requirements. In order to change Equation (1) to a simple SOF control as

$$u_i = K_i y_i \qquad (2)$$

We can rewrite Equation (1) as follows

$$u_i = [k_{Pi} \quad k_{Ii} \quad k_{Di}] \begin{bmatrix} y_{oi} & \int y_{oi} & \frac{dy_{oi}}{dt} \end{bmatrix}^T \qquad (3)$$

Therefore, y_i in Equation (2) can be augmented to following form.

$$y_i = \begin{bmatrix} y_{oi} & \int y_{oi} & \frac{dy_{oi}}{dt} \end{bmatrix}^T \qquad (4)$$

2.2 H_2/H_∞ SOF Design

A general control scheme using mixed H_2/H_∞ control technique is shown in Fig. 1. $G_i(s)$ is a linear time invariant system with the given state-space realization in Equation (5). where x_i is the state variable vector, w_i is disturbance and other external input vector, y_i is the augmented measured output vector and K_i is the controller. The output channel z_{2i} is associated with the LQG aspects (H_2 performance) while the output channel $z_{\infty i}$ is associated with the H_∞ performance.

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_{1i} w_i + B_{2i} u_i \\ z_{\infty i} &= C_{\infty i} x_i + D_{\infty 1i} w_i + D_{\infty 2i} u_i \\ z_{2i} &= C_{2i} x_i + D_{21i} w_i + D_{22i} u_i \\ y_i &= C_{yi} x_i + D_{y1i} w_i \end{aligned} \qquad (5)$$

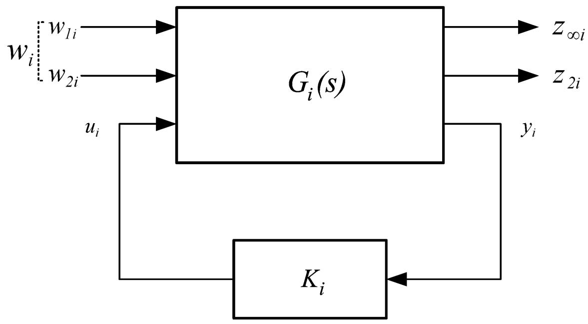

Figure 1: Closed-loop system via mixed H_2 / H_∞ control

Let $T_{z\infty i\, w1i}$ and $T_{z2i\, w2i}$ as the transfer functions from $w_i = [w_{1i} \quad w_{2i}]^T$ to $z_{\infty i}$ and z_{2i} respectively, and consider the following state-space realization for closed-loop system.

$$\begin{aligned} \dot{x}_i &= A_{ic} x_i + B_{1ic} w_i \\ z_{\infty i} &= C_{\infty ic} x_i + D_{\infty ic} w_i \\ z_{2i} &= C_{2ic} x_i + D_{2ic} w_i \\ y_i &= C_{yic} x_i + D_{yic} w_i \end{aligned} \qquad (6)$$

A mixed H_2/H_∞ SOF control design can be expressed as following optimization problem:

Optimization problem: Determine an admissible SOF law K_i , belong to a family of internally stabilizing SOF gains K_{sof} ,

$$u_i = K_i y_i \ , \ K_i \in K_{sof} \qquad (7)$$

such that

$$\inf_{K_i \in K_{sof}} \left\| T_{z2i\, w2i} \right\|_2 \ \text{subject to} \ \left\| T_{z\infty i\, w1i} \right\|_\infty < 1 \qquad (8)$$

The following lemma gives the necessary and sufficient condition for the existence of the H_2 based SOF controller to meet the following performance criteria.

$$\left\| T_{z_{2i}\, w_{2i}} \right\|_2 < \gamma_2 \qquad (9)$$

where, γ_2 is the H_2 optimal performance index.

Lemma 1, [12]:

For fixed $(A_i, B_{1i}, B_{2i}, C_{yi}, K_i)$, there exist a positive definite matrix X which solves inequality

$$\begin{aligned} &(A_i + B_{2i} K_i C_{yi}) X + X (A_i + B_{2i} K_i C_{yi})^T + B_{1i} B_{1i}^T < 0 \\ &X > L_C \end{aligned} \qquad (10)$$

to satisfy (9), if and only if the following inequality has a positive definite matrix solution,

$$\begin{aligned} &A_i X + X A_i^T - X C_{yi}^T C_{yi} X \\ &+ (B_{2i} K_i + X C_{yi}^T)(B_{2i} K_i + X C_{yi}^T)^T + B_{1i} B_{1i}^T < 0 \end{aligned} \qquad (11)$$

where L_C in (10) denotes the controllability Gramian of the pair (A_{ic}, B_{1ic}) and can be presented as follows [13].

$$\left\| T_{z2i\, w2i} \right\|_2^2 = trace(C_{2ic} L_C C_{2ic}^T) \qquad (12)$$

Notice that the condition that $A_i + B_{2i} K_i C_{yi}$ is Hurwitz is implied by inequality (10). Thus if

$$trace(C_{2ic} X C_{2ic}{}^T) < \gamma_2{}^2 \qquad (13)$$

the requirement (9) is satisfied. The interested reader can find more detail in [12].

Lemma 2, [14] (SOF stabilization) :

The system (A, B, C) is stabilizable via static output feedback if and only if there exist P>0, X>0 and K_i satisfying the following quadratic matrix inequality

$$\begin{bmatrix} A^T X + XA - PBB^T X - XBB^T P + PBB^T P & (B^T X + K_i C)^T \\ B^T X + K_i C & -I \end{bmatrix} < 0 \qquad (14)$$

3 The Proposed Control Strategy

In the proposed control strategy, as shown in Fig. 2, to design the PI/PID multiobjective control problem, the obtained SOF control problem to be considered as a mixed H_2/H_∞ SOF control problem. Then to solve the yielding nonconvex optimization problem, which can not be directly achieved by using general linear matrix inequalities (LMI) techniques an ILMI algorithm is developed.

The optimization problem given in (8) defines a robust performance synthesis problem where the H_2 norm is chosen as a performance measure. Recently, several LMI-based methods are proposed to obtain the suboptimal solution for the H_2, H_∞ and/or H_2/H_∞ SOF control problems [12, 14]. Here, a new ILMI algorithm is introduced to get a desired solution for the above optimization problem. Specifically, the proposed algorithm formulates the H_2/H_∞ SOF control through a general SOF stabilization problem based on the given facts in lemmas 1 and 2.

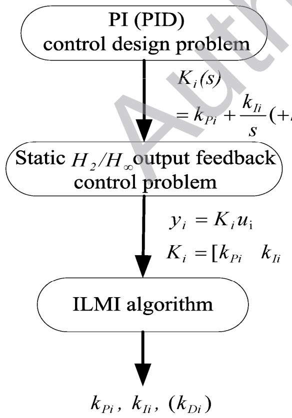

Figure 2: Control strategy

3.1 Developed ILMI Algorithm

Using lemma 1, it is difficult to achieve a solution for (11) by the general LMI, directly. Here, to get a simultaneous solution to meet (9) and H_∞ constraint, an iterative LMI algorithm is introduced. In the proposed strategy, based on the generalized static output stabilization feedback lemma (lemma 2), first the stability domain of (PI/PID parameters) space, which guarantees the stability of gain vector the closed-loop system, is specified.

In the second step, the subset of the stability domain in the PI/PID parameter space in step one is specified so that minimizes the H_2 tracking performance. Finally and in the third step, the design problem becomes, in the previous subset domain, what is the point with closest H_2 performance index to optimal one which meets the H_∞ constraint. In summary, the proposed algorithm searches a desired mixed H_2/H_∞ SOF controller $K_i \in K_{sof}$ within a family of H_2 stabilizing controllers K_{sof}, such that

$$\left|\gamma_2^* - \gamma_2\right| < \varepsilon\,, \quad \gamma_\infty = \left\|T_{z\infty i\, w1i}\right\|_\infty < 1 \qquad (15)$$

where ε is a small real positive number, γ_2^* is H_2 performance corresponded to H_2/H_∞ SOF controller K_i and γ_2 is the reference optimal H_2 performance index provided by application of standard H_2/H_∞ dynamic output feedback control.

The key point is to formulate the H_2/H_∞ problem via the generalized static output stabilization feedback lemma such that all eigenvalues of (*A-BKC*) shift towards the left half-plane through the reduction of *a*, a real number, to close to feasibility of (8). The proposed algorithm is simply shown in Fig. 3. $\sum_i$ in (17) is:

$$\sum_i = P_i C_{yi}{}^T C_{yi} P_i - P_i C_{yi}{}^T C_{yi} X_i - X_i C_{yi}{}^T C_{yi} P_i - a_i X_i$$

3.2 Applicable to Time-delay Systems

It is significant to note that because of using simple constant gains, pertaining to SOF synthesis for dynamical systems in presence of strong constraints and tight objectives are few and restrictive [15]. Under such conditions, the addressed optimization problem may be not approach to a strictly feasible solution. However, in the most of cases, approach to a near optimal solution is possible by effective and flexible search techniques such as descript algorithm in the previous section. In order to adopt the proposed control procedure to time-delayed systems, it is enough to consider the time-delays effects as model uncertainties.

A delay term can be expressed by the exponential function $e^{-s\tau}$ where τ gives the delay time. To use linear robust control techniques, an exponential delay term can be expressed in the form of low-order Pade approximation for the related Taylor series expansion.

The uncertainties due to time delays can be modeled as an unstructured multiplicative uncertainty block W_i that contains all possible variations in the assumed delays range. Let $\hat{G}_i(s)$ denotes the transfer function of time-delayed system from the control input u_i to control output y_i at operating points other than nominal point. Following a practice common in robust control, we can represent this transfer function as

$$\left|\Delta_i(s) W_i(s)\right| = \left|[\hat{G}_i(s) - G_{0i}(s)] G_{0i}(s)^{-1}\right| \qquad (20)$$

where, $\|\Delta_i(s)\|_\infty = sup_\omega |\Delta_i(s)| \le 1;\ \ G_{0i}(s) \neq 0$

$\Delta_i(s)$ shows the uncertainty block corresponding to delayed terms and $G_{0i}(s)$ is the nominal transfer function model. Thus, $W_i(s)$ is such that its respective magnitude bode plot covers the bode plots of all possible open-loop structures (including time delays). Finally the developed ILMI algorithm can be run to obtain the robust PI/PID controllers as descript in above.

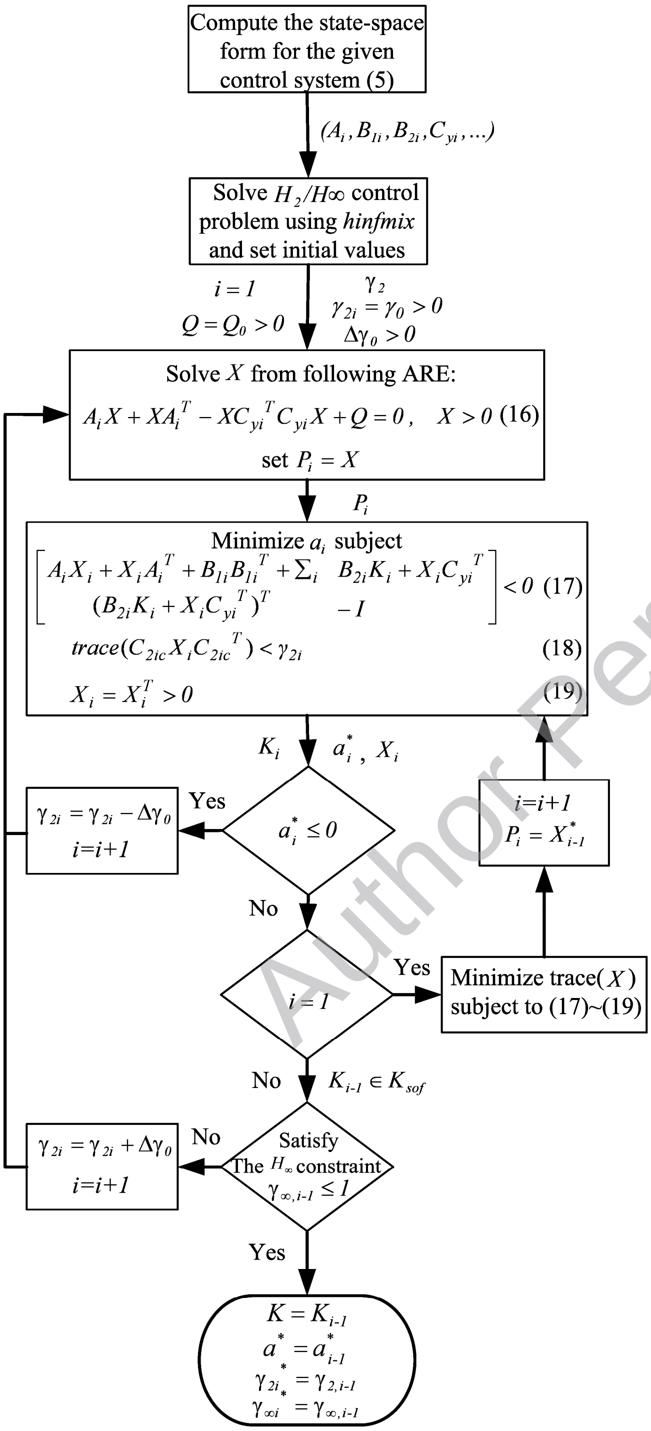

Figure 3: Developed ILMI algorithm

4 Numerical Examples

4.1 Example 1: Load-Frequency Control (LFC) Design

4.1.1 PI-based LFC

To illustrate the effectiveness of the proposed control strategy, the decentralized PI-based load-frequency control (LFC) design in a three control area power system, given in [16] and [17], is considered as an example. Each control area includes three generation companies (Gencos) with 9th order. The power system data and parameters are considered the same as in [16].

According to Equation (5), we can calculate the state-space model for each control area. A suitable control framework in order to LFC design for each control area via a mixed H_2/H_∞ control technique is shown in Fig. 4 [18], where Δf_i, ACE_i and ΔP_{ci} are frequency deviation, area control error (measured output) and governor load setpoint, respectively. $G_i(s)$ corresponds to the nominal augmented model of the given control area. y_i is the measured output (performed by ACE and its integral), u_i is the control input and w_i includes the perturbed and disturbance signals in the given control area. Δ_i block models the structured uncertainty set in the form of multiplicative type and W_i includes the associated weighting function. η_{1i}, η_{2i} and η_{3i} are constant weights that must be chosen by designer to get the desired performance. It is assumed that the parameters of inertia constant and damping coefficient in each area have uncertain values ($\pm 20\%$ of nominal values). For the example at hand, a set of suitable weighting functions is assumed as follows:

$$W_1(s) = \frac{0.3619s + 0.1613}{s + 1.6242},\ W_2(s) = \frac{0.2950s + 0.1073}{s + 1.6814}$$

$$W_3(s) = \frac{0.3497s + 0.3515}{s + 3.4815},\ \eta_{1i} = 0.12,\ \eta_{2i} = 0.35,\ \eta_{3i} = 0.42$$

The H_2 performance is used to minimize the effects of disturbances on area frequency, area control error and penalize fast changes and large overshoot in the governor load set-point. The H_∞ performance is used to meat the robustness against specified uncertainties and reduction of its impact on closed-loop system performance.

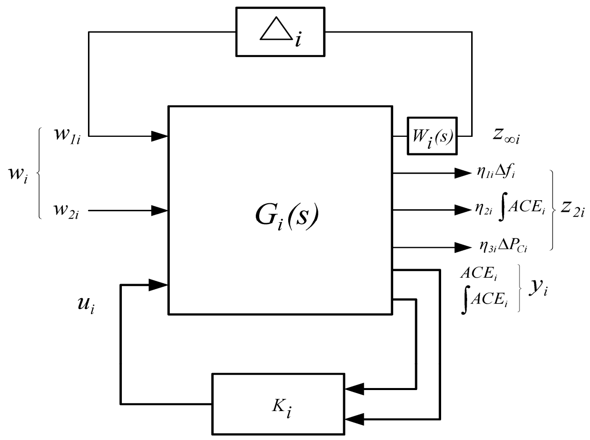

Figure 4: H_2/H_∞ SOF-based LFC

4.1.2 ILMI based PI Controllers

At the first step, a mixed H_2/H_∞ dynamic controller is designed for each control area, using *hinfmix* function in LMI control toolbox [10]. In this case, the resulted controller is dynamic type, whose order is the same as size of generalized plant model (10th order in the present paper). At the next step, according to synthesis methodology described in section 3, a set of three decentralized robust PI controllers are designed. Using developed ILMI algorithm, the controllers are obtained following several iterations. The proposed control parameters are shown in table 1. The guaranteed optimal H_2 and H_∞ indices for dynamic and PI controllers are listed in table 2.

Table 1: PI control parameters from ILMI design

Parameters	*Area 1*	*Area 2*	*Area 3*
k_{Pi}	-2.00E-04	-4.80E-03	-2.50E-03
k_{Ii}	-0.3908	-0.4406	-0.4207

Table 2: Guaranteed H_2 and H_∞ indices

Indices	*Area 1*	*Area 2*	*Area 3*
γ_{2i} (Dynamic)	1.0700	1.0300	1.0310
$\gamma_{\infty i}$ (Dynamic)	0.3919	0.2950	0.3497
γ^*_{2i} (PI)	1.0976	1.0345	1.0336
$\gamma^*_{\infty i}$ (PI)	0.3920	0.2950	0.3498

The resulted optimal H_2 and H_∞ indices ($\gamma_{\infty i}$ and $\gamma^*_{\infty i}$) and robust performance indices (γ_{2i} and γ^*_{2i}) of both synthesis methods are very close to each other. It shows that although the proposed ILMI approach gives a set of much simpler controllers (PI) than the dynamic H_2/H_∞ design, however they holds robustness as well as dynamic H_2/H_∞ controllers.

4.1.3 GA based PI Controllers

For the sake of comparison, in addition to proposed control strategy to synthesis the robust PI controllers, a mixed H_2/H_∞ -based PI controller is designed for each control area, using the given approach in [8]. GAs represent a heuristic search technique based on the evolutionary ideas of natural selection and genetics. GAs solve optimization problems by exploitation of a random search.

In this approach the genetic algorithm (GA) is employed as the optimization engine to produce the PI controllers with performance indices near to optimal ones. The obtained control parameters and performance indices are shown in table 3. The indices are comparable to the given results by the proposed ILMI algorithm.

In order to demonstrate the effectiveness of the proposed strategy, some simulations were carried out. Fig. 5 shows the closed-loop response (frequency deviation, area control error and control action signals) in face of both step load disturbance (0.1 pu) and uncertainties (*20*% decrease in uncertain parameters). The simulation results demonstrate that the proposed ILMI-based PI controllers track the load fluctuations and meet robustness for a wide range of load disturbances as well as H_2/H_∞ dynamic controllers. In addition the proposed controllers achieve better performance compared with the GA based controllers.

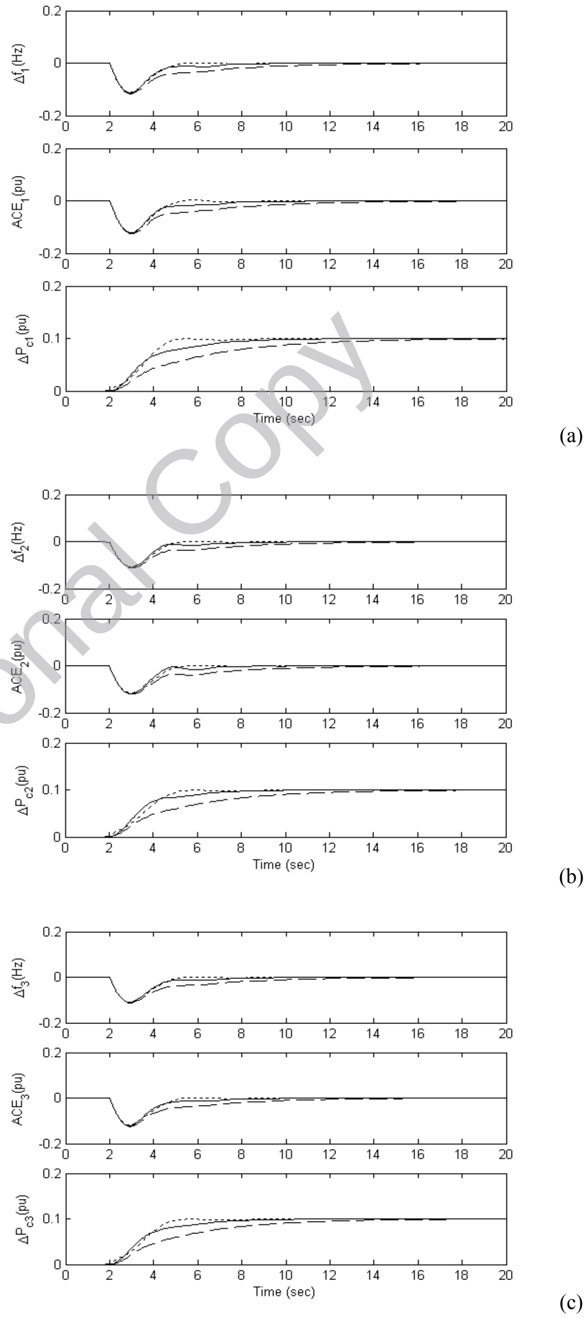

Figure 5: Closed-loop system response: a) Area 1, b) Area 2 and c) Area 3; Solid (proposed ILMI), dotted (dynamic H_2/H_∞), dash line (GA)

Table 3: Control design using GA approach

Areas	*Area 1*	*Area 2*	*Area 3*
k_{Pi}	-1.00E-04	-0.0235	-1.00E-04
k_{Ii}	-0.2309	-0.2541	-0.2544
γ^*_{2i}	1.0371	0.9694	0.9807
$\gamma^*_{\infty i}$	0.3619	0.2950	0.3497

4.2 Example 2: Time-Delayed System

Consider the LFC system of Example 1 with delays in the communication channels ACE ($\tau_{di} \in [0 \quad 2.5]\,s$) from Gencos and tie-line to the control center and control effort ($\tau_{hi} \in [0 \quad 2.8]\,s$) from control center to Gencos.

Based on the given simple stability condition in [19], the open loop system with real matrices is stable if

$$\Psi(A_i) = \mu(A_i) + \|A_{di}\| < 0 \qquad (22)$$

where $\mu(A_i) = \frac{1}{2}\max_j \lambda_j(A_i^T + A_i)$.

Here, λ_j denotes the jth eigenvalue of $(A_i^T + A_i)$. In light of above stability rule, we note that for the example at hand, the control areas are unstable for the assumed maximum delays:

$\Psi(A_1) = 10.4736 > 0$, $\Psi(A_2) = 12.2615 > 0$, $\Psi(A_3) = 10.2285 > 0$

Using Equation (20), some sample uncertainties due to delay domain for area 1 are shown in Fig. 6. To keep the complexity of obtained controller low, we can model uncertainties from both channels delays by using a norm bonded multiplicative uncertainty to cover all possible plants. Using the same method, the uncertainty weighting functions for areas 2 and 3 can be computed.

$$W_1(s) = \frac{2.1339s + 0.2557}{s + 0.4962},\quad W_2(s) = \frac{2.0558s + 0.2052}{s + 0.3869},\quad W_3(s) = \frac{2.0910s + 0.2129}{s + 0.5198}$$

According to synthesis methodology described in section 3, a set of three decentralized robust PI controllers are designed.

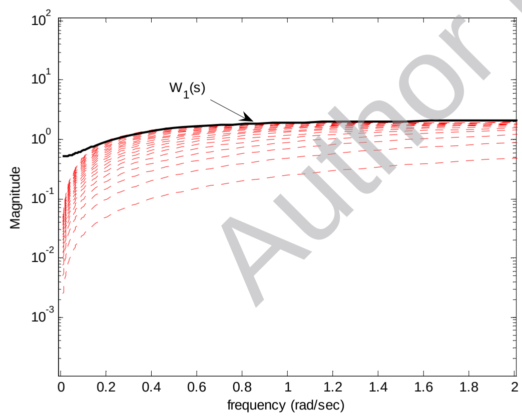

Figure 6: Uncertainty plots (dotted) due to communication delays and the upper bound (solid) in area 1

5 Conclusion

An H_2/H_∞ SOF-based ILMI algorithm is developed to design a simple PI/PID controller, which is useful in the real-world control systems. The proposed method was successfully applied to LFC synthesis in a three control area power system with and without communication time delays. The results are compared with the results of applied H_2/H_∞ dynamic controllers and a GA-based approach. It was shown that the desired performance can be achieved using the proposed control strategy.

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References

[1] K. J. Astrom, T. Hagglund, C. C. Hang and W. K. Ho, “Automatic tuning and adaptation for PID controllers – a survey,” *Control Eng. Pract.*, No. 1, pp. 699-714, 1993.
[2] P. Cominos and N. Munro, “PID controllers: recent tuning methods and design to specification,” *IEE Proc. Control Theory Appl.*, Vol. 149, No. 1, pp. 46-53, 2002.
[3] B. Kristiansson and B. Lennartson, “Robust and optimal tuning of PI and PID controllers,” *IEE Proc. On Control Theory and Applications*, Vol. 149, No. 1, pp. 17-25, 2002.
[4] C. Lin, Q. G. Wang and T. H. Lee, “An improvement on multivariable PID controller design via iterative LMI approach,” *Automatica*, Vol. 40, pp. 519-525, 2004.
[5] F. Zheng, Q. G. Wang and T. H. Lee, “On the design of multivariable PID controllers via LMI approach,” *Automatica*, Vol. 38, pp. 517-526, 2002.
[6] M. T. Ho, “Synthesis of H∞ PID controllers: a parametric approach,” *Automatica*, Vol. 39, pp. 1069-1075, 2003.
[7] R. H. C. Takahashi, P. L. D. Peres and P. A. V. Ferreira, “Multiobjective H_2/H_∞ guaranteed cost PID design,” *IEEE Control Systems*, Vol. 17, No. 5, pp. 37-47, 1997.
[8] B. S. Chen, Y. M. Cheng and C. H. Lee, “A genetic approach to mixed H_2/H_∞ optimal PID control,” *IEEE Control Systems*, Vol. 15, No. 5, pp. 51-60, 1998.
[9] C. S. Tseng and B. S. Chen, “A mixed adaptive tracking control for constrained non-holonomic systems,” *Automatica*, Vol. 39, pp. 1011-1018, 2003.
[10] P. Gahinet, A. Nemirovski, A. J. Laub and M. Chilali. LMI Control Toolbox. The MathWorks, Inc., 1995.
[11] H. Bevrani and T. Hiyama, “PI/PID based multi-objective control design: an ILMI approach,” In Proc. Of IEEE Int. Conf. on Networking, Sensing and Control (CD ROM), USA, 2005.
[12] F. Zheng, Q. G. Wang and H. T. Lee, “On the design of multivariable PID controllers via LMI approach,” *Automatica,* Vol. 38, pp. 517-526, 2002.
[13] K. Zhou, J. C. Doyle and K. Glover. Robust and optimal control. Englewood Cliffs, NJ: Prentice-Hall, 1996.
[14] Y. Y. Cao, J. Lam, Y. X. Sun and W. J. Mao, “Static output feedback stabilization: an ILMI approach,” *Automatica*, Vol. 34, No. 12, pp. 1641-1645, 1998.
[15] M. S. Mahmoud. Robust control and filtering for time-delay systems. Marcel Dekker Inc., New York, 2000.
[16] D. Rerkpreedapong, A. Hasanovic and A. Feliachi, “Robust load frequency control using genetic algorithms and linear matrix inequalities,” *IEEE Trans. On Power Systems*, Vol. 18, No. 2, pp. 855-861, 2003.
[17] H. Bevrani, Y. Mitani and K. Tsuji, “Robust decentralized load-frequency control using an iterative linear matrix inequalities algorithm,” *IEE Proc. Gener. Transm. Distrib.*, vol. 150, no. 3, pp. 347-354, 2004.
[18] H. Bevrani, Y. Mitani and K. Tsuji, “Robust LFC in a deregulated environment: multi-objective control approach,” *IEEJ Trans on Power and energy*, Vol. 24, No. 12, 2004.
[19] T. Mori and H. Kokame, “Stability of $\dot{x}(t) = Ax(t) + Bx(t-\tau)$,” *IEEE Trans. On Automatic Control*, Vol. 34, pp. 460-462, 1989.