Paper

An Effective Trade-off between Stability and Voltage Regulation

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This paper addresses a new robust control methodology to enhance the power system stability and voltage regulation as an integrated design approach. The automatic voltage regulation (AVR) and power system stabilizer (PSS) design problems are reduced to solve a single H∞ based static output feedback control problem. To determine the optimal gains, an iterative linear matrix inequalities (LMI) algorithm is used. A four-machine infinite-bus system example is given to demonstrate the efficiency of developed approach. The proposed robust technique is shown to maintain the robust performance and minimize the effects of disturbances, properly.

Keywords: H∞ control, static output feedback, LMI, voltage regulation, power system stabilizer, robust performance

1. Introduction

Power systems continuously experience changes in operating conditions due to variations in generation/load and a wide range of disturbances [1]. Power system stability and voltage regulation have been considered as an important problem for secure system operation over the years. Currently, because of expanding physical setups, functionality and complexity of power systems, the mentioned problem becomes a more significant than the past. That is why in recent years a great deal of attention has been paid to application of advanced control techniques in power system as one of the more promising application areas.

Conventionally, the automatic voltage regulation and power system stabilizer (AVR-PSS) design is considered as a sequential design including two separate stages. Firstly, the AVR is designed to meet the specified voltage regulation performance and then the PSS is designed to satisfy the stability and required damping performance. It is well known that the stability and voltage regulation are ascribed to different model descriptions, and it has been long recognized that AVR and PSS have inherent conflicting objectives[2]. That is why the successful achievement of both goals using nonintegrated design approach turns out to be very difficult. Therefore, it is reasonable to realize a compromise between the desired stability and regulation performances by a unique controller.

In the last two decades, some studies have considered an integrated design approach to AVR and PSS design using domain partitioning[3], robust pole-replacement[4] and adaptive control[5]. Recently, several control methods have been made to coordinate the various requirements for stabilization and voltage regulation within the one controller[6~11]. A desensitized controller based on Linear Quadratic Gaussian (LQG) optimal technique is used in Ref. (6). An approach used in Ref. (2, 7) involves use of Internal Model Control (IMC) method to make a trade-off between voltage regulation and power system stabilization. Although all above approaches have used linear control techniques, because of complexity of control structure, numerous unknown design parameters and neglecting real constraints, they are not well suited to meet the design objectives for a multi-machine power system. Some proposed scenarios apply a switching strategy of two different kinds of controller to cover the different behavior of system operation during transient period and post-transient period[8~10]. The performance of these schemes essentially depends upon the selection of switching time. Moreover, using different control surfaces through a highly nonlinear structure increases the complexity of designed controllers. As a preliminary step of this work, the authors have addressed the problem of a robust control methodology to enhance the stability and voltage regulation of a single-machine infinite bus in the presence of conventional PSS and AVR equipments[11].

In this paper, the stabilization and voltage regulation considering the practical constraints for feasibility are formulated via an H∞ static output feedback (H∞-SOF) control problem which it can be easily solved using an iterative linear matrix inequalities (LMI) algorithm. The resulting controller is not only robust but it also allows direct and effective trade-off between voltage regulation and damping performance. The proposed controller uses the measurable signals and has merely proportional gains; so gives considerable promise for implementation, especially in a multi-machine system. In fact the proposed control strategy attempts to make a bridge between the simplicity of control structure and robustness of stability and performance to satisfy the simultaneous AVR and PSS tasks. In order to show the effectiveness of this methodology, it is applied to a four-machine infinite-bus system. The obtained results are compared with a full-order dynamic H∞ output feedback control design.

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2. A Background on H∞-SOF Control Design

This section gives a brief overview for the H∞ based static output feedback (H∞-SOF) control design. Consider a linear time invariant system $G(s)$ with the following state-space realization.

$$G_i(s):\ \begin{aligned} \dot{x}_i &= A_i x_i + B_{1i} w_i + B_{2i} u_i \\ z_i &= C_{1i} x_i + D_{12i} u_i \\ y_i &= C_{2i} x_i \end{aligned} \qquad \text{…………....} \quad (1)$$

where x_i is the state variable vector, w_i is the disturbance and area interface vector, z_i is the controlled output vector and y_i is the measured output vector.

The H∞-SOF control problem for the linear time invariant system $G_i(s)$ with the state-space realization of (1) is to find a gain matrix K_i ($u_i = K_i y_i$), such that the resulted closed-loop system is internally stable, and the H∞ norm from w_i to z_i (Fig. 1) is smaller than γ, a specified positive number, i.e.

$$\left\| T_{z_i w_i}(s) \right\|_\infty < \gamma \qquad \text{………..........................} \ (2)$$

It is notable that the H∞-SOF control problem can be transferred to a generalized SOF stabilization problem which is expressed via the following theorem [12].

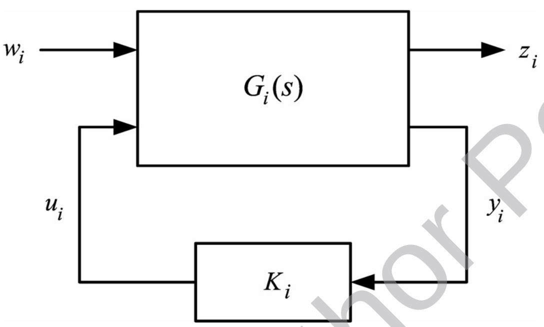

Fig. 1. Closed-loop system via H∞-SOF control

Theorem. The system (A, B, C) is stabilizable via SOF if and only if there exist $P>0$, $X>0$ and K_i satisfying the following quadratic matrix inequality

$$\begin{bmatrix} A^T X + XA - PBB^T X - XBB^T P + PBB^T P & (B^T X + K_i C)^T \\ B^T X + K_i C & -I \end{bmatrix} < 0$$

$$\text{………..........................} \ (3)$$

Since a solution for the consequent non convex optimization problem (3) can not be directly achieved by using general LMI technique [13], a variety of methods were proposed by many researchers with many analytical and numerical methods to approach a local/global solution. In this paper, to solve the resulted SOF problem, an iterative LMI is used based on the existence necessary and sufficient condition for SOF stabilization, via the H∞ control technique.

3. Proposed Control Strategy

3.1 Modeling In order to design a robust power system controller, it is first necessary to consider an appropriate linear mathematical description of multi-machine power system with two axis generator models. In the view point of generator unit "i", the state space representation model for such a system has the form

$$\begin{aligned} \dot{x}_{1i} &= x_{2i} \\ \dot{x}_{2i} &= -(D_i/M_i) x_{2i} - (1/M_i)\Delta P_{ei}(x) \\ \dot{x}_{3i} &= -(1/T'_{d0i}) x_{3i} - (\Delta x_{di}(x)/T'_{d0i})\Delta I_{di}(x) + u_i \\ \dot{x}_{4i} &= -(1/T'_{q0i}) x_{4i} - (\Delta x_{qi}(x)/T'_{q0i})\Delta I_{qi}(x) \end{aligned} \quad \cdots \quad (4)$$

where the states

$$x_i^T = \begin{bmatrix} x_{1i} & x_{2i} & x_{3i} & x_{4i} \end{bmatrix} = \begin{bmatrix} \delta_i & \omega_i & E'_{qi} & E'_{di} \end{bmatrix} \text{........} \ (5)$$

are defined as deviation form the equilibrium values

$$x_{ei}^T = \begin{bmatrix} \delta_{1i}^e & \omega_{2i}^e & E'^e_{qi} & E'^e_{di} \end{bmatrix}$$

and, here

$$\Delta x_{di} = x_{di} - x'_{di}\,,\ \Delta x_{qi} = x_{qi} - x'_{di} \qquad \text{............} \ (6)$$

$$\Delta P_{ei}(x) = (E'_{di} I_{di} + E'_{qi} I_{qi}) - (E'^e_{di} I^e_{di} + E'^e_{qi} I^e_{qi}) \ \text{......} \ (7)$$

$$\begin{aligned} I_{di} = &\sum_k [G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}]E'_{dk} \\ &+ \sum_k [G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}]E'_{qk} \end{aligned} \quad \text{............} \quad (8)$$

$$\begin{aligned} I_{qi} = &\sum_k [B_{ik}\cos\delta_{ik} - G_{ik}\sin\delta_{ik}]E'_{dk} \\ &+ \sum_k [G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}]E'_{qk} \end{aligned} \quad \text{............} \quad (9)$$

A detailed description of all symbols and quantities can be found in Ref. (14). Using the linearization technique and after some manipulation, the nonlinear state equations (5) can be expressed in the form of following linear state space model.

$$\dot{x}_i = A_i x_i + B_i u_i \qquad \text{…………....} \quad (10)$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & -\dfrac{D_i}{M_i} & a_{23} & a_{24} \\ a_{31} & 0 & a_{33} & -\dfrac{G_{ii}\Delta x_{di}}{T'_{d0i}} \\ a_{41} & 0 & \dfrac{G_{ii}\Delta x_{qi}}{T'_{q0i}} & a_{44} \end{bmatrix},\ B_i = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{T'_{d0i}} \\ 0 \end{bmatrix}$$

$$\text{………..........................} \ (11)$$

with elements that are given in Appendix.

3.2 Overall Control Framework The overall control structure is shown in Fig. 2, where the conventional power system stabilizer and automatic voltage regulator blocks are replaced by a single H∞-SOF controller including the following optimal gain vector.

$$K_i^T = \begin{bmatrix} k_{vi} & k_{pi} & k_{\omega i} \end{bmatrix} \qquad \text{……................} \quad (12)$$

The H∞-SOF controller uses the terminal voltage Δv_{ti}, electrical power Δp_{ei} and machine speed $\Delta\omega_i$ as input signals, which all of them are easily measurable in a real power system environment. Δv_{refi} and d_i show the reference voltage deviation and system disturbance input, respectively.

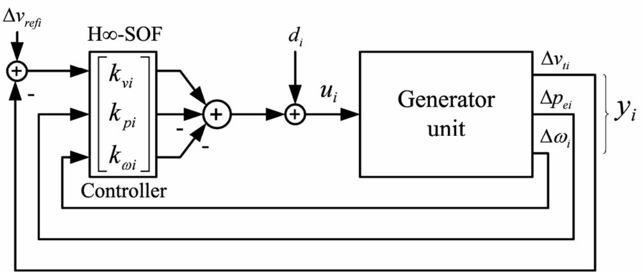

Fig. 2. Overall control structure

Using linearized model for a given power system unit "*i*" in the form of (1) and performing the standard H∞-SOF configuration (Fig. 1) with considering an appropriate controlled output signals results an effective control framework, which is shown in Fig. 3. This control structure adapts the H∞-SOF control technique with the described power system control targets and allows direct trade-off between voltage regulation and closed-loop stability by optimal tuning of a pure vector gain. Here, disturbance input vector w_i, controlled output vector z_i and measured output vector y_i are considered as follows:

$$w_i^T = \begin{bmatrix} \Delta v_{refi} & d_i \end{bmatrix} \qquad \text{……..................} \ (13)$$

$$z_i^T = \begin{bmatrix} \mu_{1i}\Delta v_{ti} & \mu_{2i}\Delta\delta_i & \mu_{3i}\Delta P_{ei} & \mu_{4i}u_i \end{bmatrix} \ \text{…….} \quad (14)$$

$$y_i^T = \begin{bmatrix} \Delta v_{ti} & \Delta p_{ei} & \Delta\omega_i \end{bmatrix} \qquad \text{……..................} \ (15)$$

Where $\mu_i = \begin{bmatrix} \mu_{1i} & \mu_{2i} & \mu_{3i} & \mu_{4i} \end{bmatrix}$ is a constant weight vector that must be chosen by designer to get the desired closed-loop performance. The selection of constant weights μ_{1i}, μ_{2i} and μ_{3i} is dependent on specified voltage regulation and damping performance goals. In fact an important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives. The selection of these weights entails a compromise among several performance requirements. Furthermore, μ_{4i} sets a limit on the allowed control signal to penalize fast changes, large overshoot with a reasonable control gain to meet the feasibility and the corresponded physical constraints.

Since the vector z_i properly covers all significant controlled signals which must be minimized by an ideal AVR-PSS design, it is expected that the proposed robust controller to be able to satisfy the voltage regulation and stabilizing objectives, simultaneously.

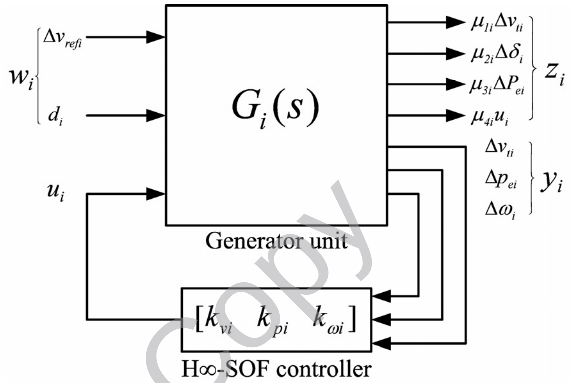

Fig. 3. The proposed H∞-SOF control framework

3.3 An Iterative LMI Algorithm In order to solve the H∞-SOF, an iterative LMI algorithm has been used. Similar to the given approach in Ref. (15, 16), the key point is to formulate the H∞ problem via a generalized static output stabilization feedback such that all eigenvalues of (A-$B\,K_i\,C$) shift towards the left half plane through the reduction of a, a real number, to close to feasibility of (3). The described theorem in the previous section gives a family of internally stabilizing SOF gains is defined as K_{sof}. But the desirable solution K_i is an admissible SOF law

$$u_i = K_i y_i\ ,\ K_i \in K_{sof} \qquad \text{............................} \ (16)$$

such that

$$\left\| T_{zi\,wi}(s) \right\|_\infty < \gamma^*,\ \left| \gamma - \gamma^* \right| < \varepsilon \qquad \text{.................} \quad (17)$$

where ε is a small positive number. Suboptimal performance index γ^* indicates a lower bound such that the closed-loop system is H∞ stabilizable. The optimal performance index (γ), can be obtained from the application of a full dynamic H∞ dynamic output feedback control method.

The proposed algorithm, which is described in Fig. 4, gives an iterative LMI suboptimal solution for above optimization problem. Here A_g, B_g and C_g are three generalized matrices of the following forms

$$A_g = \begin{bmatrix} A_i & B_{1i} & 0 \\ 0 & -\gamma I/2 & 0 \\ C_{1i} & 0 & -\gamma I/2 \end{bmatrix},\ B_g = \begin{bmatrix} B_{2i} \\ 0 \\ D_{12i} \end{bmatrix},\ C_g = \begin{bmatrix} C_{2i} & 0 & 0 \end{bmatrix}.$$

$$\text{………….........................} \ (18)$$

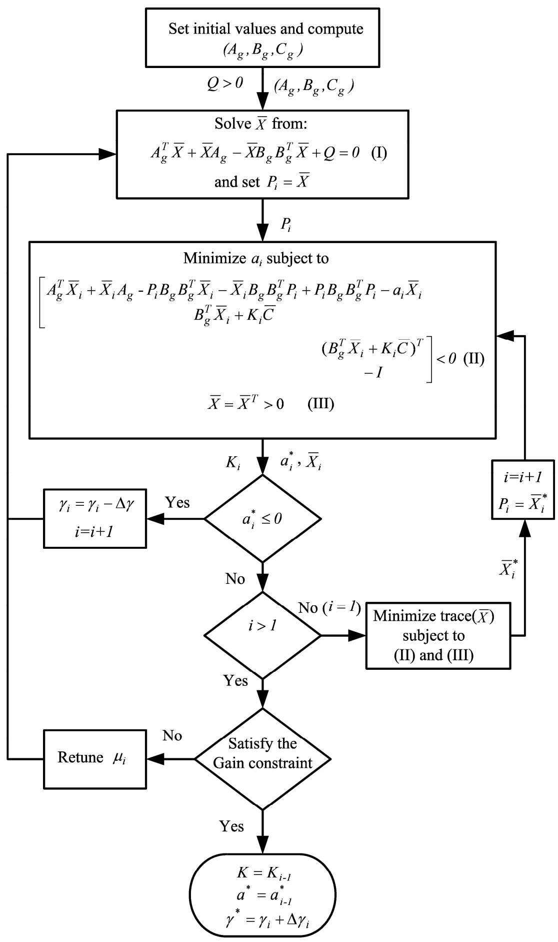

Fig. 4. Iterative LMI algorithm

4. Application to a 4-Machine Infinite Bus System

To illustrate the effectiveness of the proposed control strategy, a longitudinal four-machine infinite bus system, is considered as a test system[(17)(18)]. The study system is shown in Fig. 5. All units are thermal type, and units 2, 3 and 4 have a separately conventional excitation control system as shown in Fig. 6 and Fig. 7. The generators, lines and conventional excitation system parameters are given in Table 1, Table 2 and Table 3 (Appendix).

Unit 1 is selected to be equipped with robust control, and therefore our objective is to apply the control strategy developed in the previous section to controller design for unit 1. It is assumed this unit has an exciter part same as the shown "Exciter" block in Fig. 7.

First of all, using *hinflmi* function in LMI toolbox of MATLAB software[(19)], a full order robust dynamic controller with the following structure is designed.

$$K_1(s): \begin{aligned} \dot{x}_K &= A_K x_K + B_K y_1 \\ u_1 &= C_K x_K + D_K y_1 \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Then, applying the proposed H∞-SOF control methodology an optimal gain vector for the problem at hand is obtained as follows. The value of 10 is considered as upper limit for the gains of vector's arrays.

$$K_{1,SOF} = [9.9897 \quad 8.9987 \quad 1.5986] \ldots\ldots\ldots\ldots\ldots \quad (20)$$

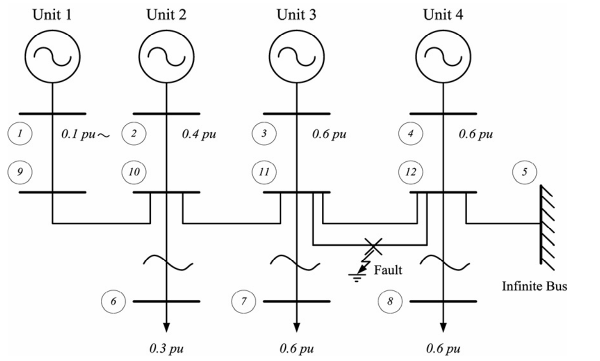

Fig. 5. Four-machine infinite-bus power system

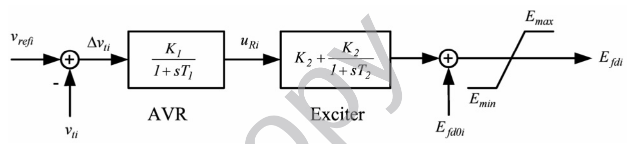

Fig. 6. Conventional excitation control system for units 2 and 3

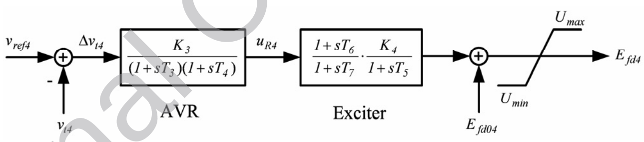

Fig. 7. Conventional excitation control system for unit 4

The closed loop performance analysis shows that the resulted robust performance indices (γ and γ^*) of both synthesis methods are very close to each other (Table 4). It indicates that although the proposed H∞-SOF approach gives a much simpler controller (pure gain) than the H∞ dynamic output feedback design, it holds robust performance as well as dynamic H∞ controller.

5. Simulation Results

In order to demonstrate the effectiveness of the proposed strategy, some simulations were carried out. The performance of the closed-loop system in comparison of a full-order dynamic H∞ output feedback controller is tested in the presence of voltage deviation, faults and system disturbance. During the simulation, the output setting of unit 1 is fixed to 0.6 pu.

Fig. 8 shows the electrical power, terminal voltage and machine speed of unit 1, and the electrical powers of other units, following a fault on the line between buses 11 and 12 at 2 *sec*. For the next test case, the performance of designed controllers was evaluated in the presence of a 0.05 pu step disturbance injected at the voltage reference input of unit 1 at 20 *sec*. Fig. 9 shows the closed-loop response of the power systems fitted with the dynamical H∞ controller and the proposed robust gain vector. Finally, Fig. 10 shows the system response in the face of a step disturbance (d_i) with one second duration in the closed-loop system at 20 *sec*.

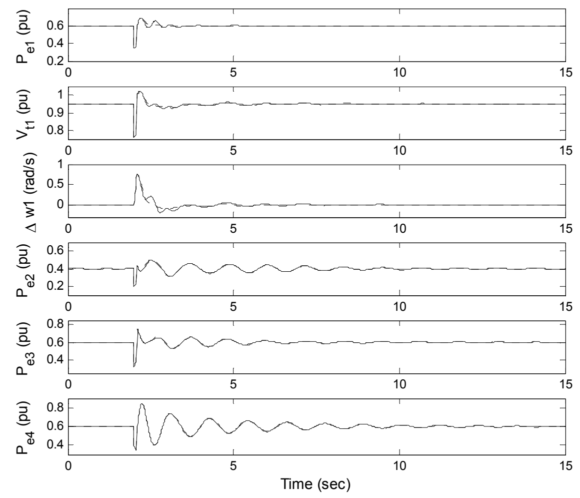

Fig. 8. System response for a fault between buses 11 and 12; Solid (H∞-SOF), dotted (H∞-Dynamic)

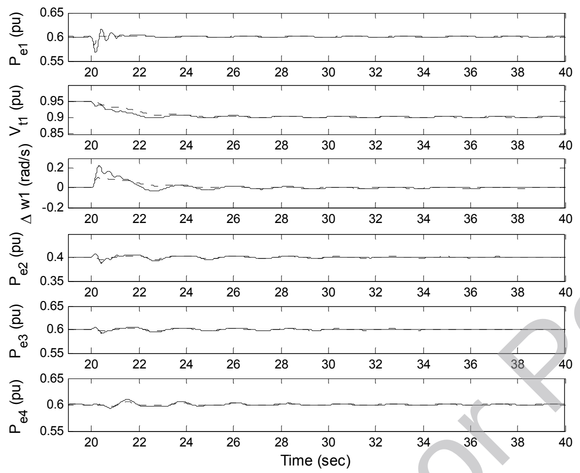

Fig. 9. System response for a 0.05 pu step change at the voltage reference input of unit 1; Solid (H∞-SOF), dotted (H∞-Dynamic)

Comparing the simulation results shows that the robust design achieves robustness against the voltage deviation, disturbance and line fault with a quite good performance as well as full dynamical H∞ controller. Furthermore, practically it is highly desirable, for reasons of simplicity, ease of maintainability and tune-ability. Table 4 shows a comparison between the proposed H∞-SOF and H∞-Dynamic approaches in view point of structure, robust performance indices and the critical power output from unit 1 for a three-phase to ground fault (between buses 11 and 12 in Fig. 5). To investigate the critical point, the real power output of unit 1 is increased from 0.1 pu (The setting of the real power output from the other units is fixed at the values shown in Fig. 5). The size of resulted stable region by both methods is approximately equal, and it is significantly enlarged in comparison of conventional PSS-AVR controller [17][18].

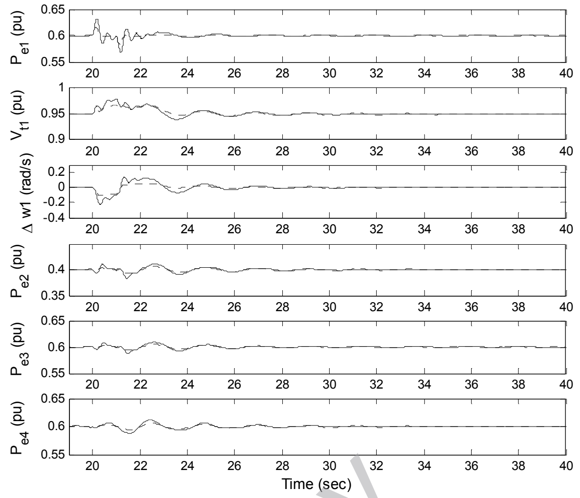

Fig. 10. System response for a step disturbance at 20 *sec*; Solid (H∞-SOF), dotted (H∞-Dynamic)

Table 4. Comparison of H∞-based proposed robust control designs

Control design	*Control structure*	*Robust Perf. index*	*Critical power output*
H∞-Dynamic	High order	γ *=455.1052*	*0.95* (*pu*)
H∞-SOF	Pure gain	γ^* *=456.3110*	*0.93* (*pu*)

6. Conclusion

In order to simultaneous enhancement of power system stability and voltage regulation, a new control strategy is developed using an H∞-SOF control technique via a developed iterative LMI algorithm. The proposed method was applied to a four-machine infinite bus power system, and the results are compared with a full-order dynamical H∞ control design. The performance of the resulting closed-loop system is shown to be satisfactory over a wide range of operating conditions. Making an effective and direct trade-off between voltage regulation and damping improvement, having a decentralized property and simplicity of structure are the main advantages of the developed methodology.

Acknowledgments

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Appendix

The elements of A_i matrix in (11):

$$a_{21} = -\frac{1}{M_i}\frac{\partial f_{1i}(x)}{\partial x_{1i}}\bigg|_{x_{ei}}$$

$$a_{23} = -\frac{\left[G_{ii}E'^{e}_{qi} - B_{ii}E'^{e}_{di} + I^{e}_{qi}\right]}{M_i} - \frac{1}{M_i}\frac{\partial f_{1i}(x)}{\partial x_{3i}}\bigg|_{x_{ei}}$$

$$a_{24} = -\frac{\left[G_{ii}E'^{e}_{di} + B_{ii}E'^{e}_{qi} + I^{e}_{di}\right]}{M_i} - \frac{1}{M_i}\frac{\partial f_{1i}(x)}{\partial x_{4i}}\bigg|_{x_{ei}}$$

$$a_{31} = -\frac{\Delta x_{di}}{T'_{d0i}}\frac{\partial f_{2i}(x)}{\partial x_{1i}}\bigg|_{x_{ei}}, \quad a_{33} = -\frac{1}{T'_{d0i}} + \frac{B_{ii}\Delta x_{di}}{T'_{d0i}}$$

$$a_{41} = -\frac{\Delta x_{qi}}{T'_{q0i}}\frac{\partial f_{3i}(x)}{\partial x_{1i}}\bigg|_{x_{ei}}, \quad a_{44} = -\frac{1}{T'_{q0i}} + \frac{B_{ii}\Delta x_{qi}}{T'_{q0i}}$$

where,

$$f_{1i}(x) = x_{4i}\Delta I_{di}(x) + x_{3i}\Delta I_{qi}(x) + \sum_{k\neq i}\Big\{[E'^{e}_{di}\eta_{ik}(\delta) + E'^{e}_{qi}\hat{\eta}_{ik}(\delta)]x_{4k}$$
$$+ [E'^{e}_{di}\nu_{ik}(\delta) + E'^{e}_{qi}\hat{\nu}_{ik}(\delta)]x_{3k} + [E'^{e}_{di}\upsilon_{ik}(\delta) + E'^{e}_{qi}\hat{\upsilon}_{ik}(\delta)]\sin\phi_{ik}\Big\}$$

$$f_{2i}(x) = \sum_{k\neq i}\left[\eta_{ik}(\delta)x_{4k} + \nu_{ik}(\delta)x_{3k} + \upsilon_{ik}(\delta)\sin\phi_{ik}\right]$$

$$f_{3i}(x) = \sum_{k\neq i}\left[\hat{\eta}_{ik}(\delta)x_{4k} + \hat{\nu}_{ik}(\delta)x_{3k} + \hat{\upsilon}_{ik}(\delta)\sin\phi_{ik}\right]$$

$$\eta_{ik}(\delta) = G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}, \quad \hat{\eta}_{ik}(\delta) = B_{ik}\cos\delta_{ik} - G_{ik}\sin\delta_{ik}$$

$$\nu_{ik}(\delta) = G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}, \quad \hat{\nu}_{ik}(\delta) = B_{ik}\sin\delta_{ik} - G_{ik}\cos\delta_{ik}$$

$$\upsilon_{ik}(\delta) = 2g1_{ik}\sin\frac{\delta^{e}_{ik} + \delta_{ik}}{2} + 2g2_{ik}\cos\frac{\delta^{e}_{ik} + \delta_{ik}}{2}, \quad \phi_{ik} = 0.5(x_{1i} - x_{1k})$$

$$\hat{\upsilon}_{ik}(\delta) = 2g2_{ik}\sin\frac{\delta^{e}_{ik} + \delta_{ik}}{2} - 2g1_{ik}\cos\frac{\delta^{e}_{ik} + \delta_{ik}}{2}, \quad \delta_{ik} = \delta_i - \delta_k$$

$$g1_{ik} = G_{ik}E'^{e}_{dk} - B_{ik}E'^{e}_{qk}, \quad g2_{ik} = G_{ik}E'^{e}_{qk} + B_{ik}E'^{e}_{qk}$$

Table 1. Generator constants

Unit No.	M_i (*sec*)	D_i	x_{di} (*pu*)	x'_{di} (*pu*)	x_{qi} (*pu*)	x'_{qi} (*pu*)	T'_{d0i} (*sec*)	T'_{q0i} (*sec*)
1	8.05	0.002	1.860	0.440	1.350	1.340	0.733	0.0873
2	7.00	0.002	1.490	0.252	0.822	0.821	1.500	0.1270
3	6.00	0.002	1.485	0.509	1.420	1.410	1.550	0.2675
4	8.05	0.002	1.860	0.440	1.350	1.340	0.733	0.0873

Table 2. Line parameters

Line No.	*Bus-Bus*	R_{ij} (*pu*)	X_{ij} (*pu*)	S_{ij} (*pu*)
1	1-9	0.02700	0.1304	0.0000
2	2-10	0.07000	0.1701	0.0000
3	3-11	0.04400	0.1718	0.0000
4	4-12	0.02700	0.1288	0.0000
5	10-6	0.02700	0.2238	0.0000
6	11-7	0.04000	0.1718	0.0000
7	12-8	0.06130	0.2535	0.0000
8	9-10	0.01101	0.0829	0.0246
9	10-11	0.01101	0.0829	0.0246
10	11-12	0.01468	0.1105	0.0328
11	12-5	0.12480	0.9085	0.1640

Table 3. Excitation parameters for units 2, 3 and 4

K_1	K_2	K_3	K_4	$\lvert E_{max(min)}\rvert$	U_{max}	U_{min}
1.00	19.21	10.00	6.48	5.71	7.60	-5.20
T_1 (*sec*)	T_2 (*sec*)	T_3 (*sec*)	T_4 (*sec*)	T_5 (*sec*)	T_6 (*sec*)	T_7 (*s*)
0.010	1.560	0.013	0.013	0.200	3.000	10.000