Multiobjective PI/PID Control Design Using an Iterative Linear Matrix Inequalities Algorithm

Hassan Bevrani and Takashi Hiyama

Abstract: Many real world control systems usually track several control objectives, simultaneously. At the moment, it is desirable to meet all specified goals using the controllers with simple structures like as proportional-integral (PI) and proportional-integral-derivative (PID) which are very useful in industry applications. Since in practice, these controllers are commonly tuned based on classical or trial-and-error approaches, they are incapable of obtaining good dynamical performance to capture all design objectives and specifications. This paper addresses a new method to bridge the gap between the power of optimal multiobjective control and PI/PID industrial controls. First the PI/PID control problem is reduced to a static output feedback control synthesis through the mixed H_2/H_∞ control technique, and then the control parameters are easily carried out using an iterative linear matrix inequalities (ILMI) algorithm. Numerical examples on load-frequency control (LFC) and power system stabilizer (PSS) designs are given to illustrate the proposed methodology. The results are compared with genetic algorithm (GA) based multiobjective control and LMI based full order mixed H_2/H_∞ control designs.

Keywords: LFC, LMI, Mixed H_2/H_∞ control, PI, PID, robust performance, static output feedback control, time delay.

1. INTRODUCTION

The proportional-integral (PI) and proportional-integral-derivative (PID) controllers, because of their functional simplicity, are widely used in industrial applications. However, their parameters are often tuned using experiences or trial and error methods. On the other hand, the most of real-world control problems refer to multi-objective control designs that several objectives such as stability, disturbance attenuation and reference tracking with considering of practical constraints must be followed by controller, simultaneously.

It is clear that meeting all design objectives by a simple PI/PID controller which is tuned based on experiences/trial-error methods is difficult. Over the years, many different parameter tuning methods have been presented for PI and PID controllers. A survey up to 2002 is given in Ref. [1,2]. Most of these methods present modifications of the frequency response method introduced by Ziegler and Nichols [3]. Some efforts have also been made to find analytical approaches to tune the parameters [4-6]. Several tuning methodology based on robust and optimal control techniques are introduced to design of PI/PID controllers [7-11]. In the most of proposed approaches, a single norm based performance criteria has been used to evaluate the robustness of resulted control systems.

It is well known that each robust method is mainly useful to capture a set of special specifications. For instance, the H_2 tracking design is more adapted to deal with transient performance by minimizing the linear quadratic cost of tracking error and control input, but H_∞ approach is more useful to stabilize the dynamical systems in the presence of control constraints and uncertainties. While the H_∞ norm is natural for norm-bounded perturbations, in many applications the natural norm for the input-output performance is the H_2 norm.

Mixed H_2/H_∞ provides a powerful control design to meet different specified control objectives. However, it is usually complicated and not easily implemented

Manuscript received July 27, 2005; revised August 23, 2006; accepted January 30, 2007. Recommended by Editorial Board member Jietae Lee under the direction of Editor Tae-Woong Yoon. This work is supported in part by Japan Society for the Promotion of Science (JSPS) under grant P04346 and the Research Office at the University of Kurdistan. Special thanks go to Prof. Y. Mitani from Kyushu Inst. of Technology and Prof. K. Tsuji from Osaka University for their valuable discussion and suggestions.

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for the real industrial applications. Recently, some efforts are reported to make a connection between the theoretical mixed H_2/H_∞ optimal control and classical PID control [12-14]. Ref. [12] has used a combination of different optimization criteria through a multi-objective technique to tune the PI parameters. A genetic algorithm (GA) approach to mixed H_2/H_∞ optimal PID control is given in [13]. A PID controller incorporating an adaptive control scheme for the mixed H_2/H_∞ tracking performance is developed for constrained non-holonomic mechanical systems in [14].

In this paper, an interesting combination of different objectives including H_2 and H_∞ tracking performances for a PI/PID controller is addressed by a systemic, simple and fast algorithm. The multi-objective PI/PID control problem is formulated as a mixed H_2/H_∞ static output feedback (SOF) control problem to obtain a desired PI/PID controller. An iterative linear matrix inequalities (ILMI) algorithm is developed to tune the PI/PID control parameters to achieve mixed H_2/H_∞ optimal performance.

The proposed strategy is used to design of PI-based load-frequency control (LFC) system and PID-based power system stabilizer (PSS) as numerical examples. The preliminary step of this work is given in [15].

2. BACKGROUNDS

2.1. Transformation from PI/PID to SOF control

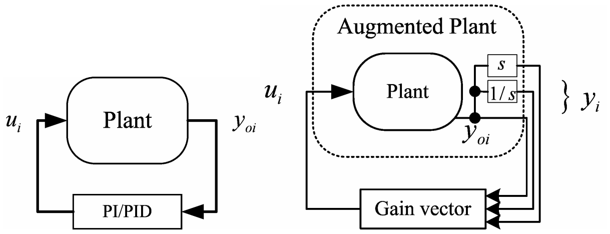
In this section, the PI/PID control problem is transferred to a static output feedback (SOF) control problem. The main merit of this transformation is in possibility of using the well-known SOF control techniques to calculate the fixed gains, and once the SOF gain vector is obtained, the PI/PID gains are ready in hand and no additional computation is needed.

In a given PI/PID-based control system i (Fig. 1(a)), the measured output signal (y_{oi}) performs the input signal for the controller and we can write (for PID type)

$$u_i = k_{Pi}y_{oi} + k_{Ii}\int y_{oi} + k_{Di}\frac{dy_{oi}}{dt}, \tag{1}$$

where k_{Pi}, k_{Ii} and k_{Di} are constant real numbers. Therefore, by augmenting the system description to include the y_{oi}, its integral and derivative as a new measured output vector (y_i), the PI/PID control problem becomes one of finding a static output feedback that satisfied prescribed performance requirements. In order to change (1) to a simple SOF control

$$u_i = K_i y_i, \tag{2}$$



(a) PI/PID control. (b) SOF control.

Fig. 1. Transformation from PI/PID to SOF control system.

we can rewrite (1) as follows

$$u_i = [k_{Pi} \quad k_{Ii} \quad k_{Di}]\begin{bmatrix} y_{oi} & \int y_{oi} & \dfrac{dy_{oi}}{dt}\end{bmatrix}^T. \tag{3}$$

Therefore, y_i in (2) can be augmented to following form (Fig. 1(b)).

$$y_i = \begin{bmatrix} y_{oi} & \int y_{oi} & \dfrac{dy_{oi}}{dt}\end{bmatrix}^T. \tag{4}$$

2.2. H_2/H_∞ SOF Design

A general control scheme using mixed H_2/H_∞ control technique is shown in Fig. 2. $G_i(s)$ is a linear time invariant system with the given state-space realization in (5). where x_i is the state variable vector, w_i is disturbance and other external input vector, y_i is the augmented measured output vector and K_i is the controller. The output channel z_{2i} is associated with the LQG aspects (H_2 performance) while the output channel $z_{\infty i}$ is associated with the H_∞ performance.

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_{1i}w_i + B_{2i}u_i, \\ z_{\infty i} &= C_{\infty i}x_i + D_{\infty 1i}w_i + D_{\infty 2i}u_i, \\ z_{2i} &= C_{2i}x_i + D_{21i}w_i + D_{22i}u_i, \\ y_i &= C_{yi}x_i + D_{y1i}w_i. \end{aligned} \tag{5}$$

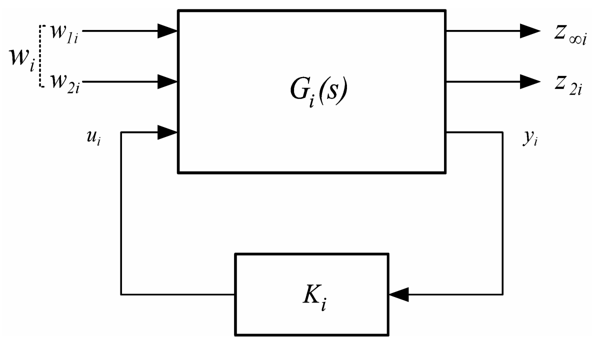


Fig. 2. Closed-loop system via mixed H_2/H_∞ control.

Let $T_{z_{\infty i} w_{1i}}$ and $T_{z_{2i} w_{2i}}$ as the transfer functions from $w_i = [w_{1i} \;\; w_{2i}]^T$ to $z_{\infty i}$ and z_{2i} respectively, and consider the following state-space realization for closed-loop system.

$$\begin{aligned} \dot{x}_i &= A_{ic} x_i + B_{1ic} w_i, \\ z_{\infty i} &= C_{\infty ic} x_i + D_{\infty_{ic}} w_i, \\ z_{2i} &= C_{2ic} x_i + D_{2ic} w_i, \\ y_i &= C_{y_{ic}} x_i + D_{y_{ic}} w_i. \end{aligned} \tag{6}$$

A mixed H_2/H_∞ SOF control design can be expressed as the following optimization problem.

Optimization problem: Determine an admissible SOF law K_i, belong to a family of internally stabilizing SOF gains K_{sof},

$$u_i = K_i y_i, \quad K_i \in K_{sof}, \tag{7}$$

such that

$$\inf_{K_i \in K_{sof}} \left\| T_{z_{2i} w_{2i}} \right\|_2 \text{subject to} \left\| T_{z_{\infty i} w_{1i}} \right\|_\infty < 1. \tag{8}$$

The following lemma (Lemma 1) gives the necessary and sufficient condition for the existence of the H_2 based SOF controller to meet the following performance criteria. Lemma 2 describes the generalized SOF stabilization for an assumed dynamic system.

$$\left\| T_{z_{2i} w_{2i}} \right\|_2 < \gamma_2, \tag{9}$$

where γ_2 is the H_2 optimal performance index.

Lemma 1 [16]**:** For fixed $(A_i, B_{1i}, B_{2i}, C_{yi}, K_i)$, there exist a positive definite matrix X which solves inequality

$$\begin{aligned} &(A_i + B_{2i} K_i C_{yi})X + X(A_i + B_{2i} K_i C_{yi})^T + B_{1i} B_{1i}^T < 0, \\ &X > L_C \end{aligned} \tag{10}$$

to satisfy (9), if and only if the following inequality has a positive definite matrix solution,

$$\begin{aligned} &A_i X + X A_i^T - X C_{yi}^T C_{yi} X \\ &+ (B_{2i} K_i + X C_{yi}^T)(B_{2i} K_i + X C_{yi}^T)^T + B_{1i} B_{1i}^T < 0, \end{aligned} \tag{11}$$

where L_C in (10) denotes the controllability Gramian of the pair (A_{ic}, B_{1ic}) and can be presented as follows [17].

$$\left\| T_{z_{2i} w_{2i}} \right\|_2^2 = trace(C_{2ic} L_C C_{2ic}^T). \tag{12}$$

It is notable that the Hurwitz property for $A_i + B_{2i} K_i C_{yi}$ is already implied by inequality (10). Thus if

$$trace(C_{2ic} X C_{2ic}^T) < \gamma_2^2, \tag{13}$$

the requirement (9) is satisfied.

Lemma 2 (SOF stabilization) [18]**:** The system (A, B, C) is stabilizable via static output feedback if and only if there exist P>0, X>0 and K_i satisfying the following quadratic matrix inequality

$$\begin{bmatrix} A^T X + XA - PBB^T X - XBB^T P + PBB^T P & (B^T X + K_i C)^T \\ B^T X + K_i C & -I \end{bmatrix} < 0. \tag{14}$$

3. THE PROPOSED CONTROL STRATEGY

In the proposed control strategy, as summarized in Fig. 3, to design the PI/PID multiobjective control problem, the obtained SOF control problem is considered as a mixed H_2/H_∞ SOF control problem. Then, to solve the yielding nonconvex optimization problem, which can not be directly achieved by using general LMI techniques, an ILMI algorithm is developed.

3.1. Developed ILMI algorithm

The optimization problem given in (8) defines a robust performance synthesis problem where the H_2 norm is chosen as a performance measure. Recently, several LMI-based methods are proposed to obtain the suboptimal solution for the H_2, H_∞ and/or H_2/H_∞ SOF control problems [16,18-21]. Here, a new ILMI algorithm is introduced to get a desired solution for the above optimization problem. Specifically, the proposed algorithm formulates the H_2/H_∞ SOF control through a general SOF stabilization problem based on the given facts in Lemmas 1 and 2.

Using Lemma 1, it is directly difficult to achieve a

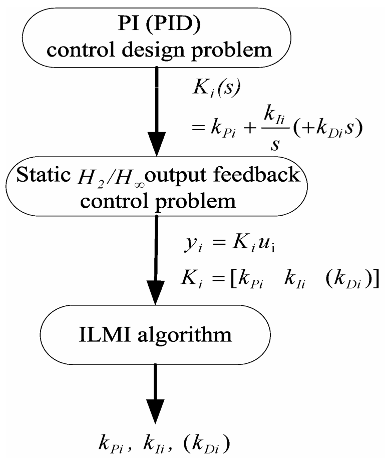


Fig. 3. Control strategy.

solution for (11) by the general LMI. Here, to get a simultaneous solution to satisfy (9) and H_∞ constraint, an iterative LMI algorithm is introduced. In summary, the proposed algorithm searches a desired mixed H_2/H_∞ SOF controller $K_i \in K_{sof}$ within a family of H_2 stabilizing controllers K_{sof}, such that

$$\left|\gamma_2^* - \gamma_2\right| < \varepsilon,\ \gamma_\infty = \left\|T_{z_{\infty i}\, w_{1i}}\right\|_\infty < 1, \quad (15)$$

where ε is a small real positive number, γ_2^* is the H_2 performance corresponded to H_2/H_∞ SOF controller K_i, and γ_2 is the reference optimal H_2 performance index provided by application of standard H_2/H_∞ dynamic output feedback control.

The key point is to formulate the H_2/H_∞ problem via the generalized static output stabilization feedback Lemma such that all eigenvalues of (A-BKC) shift towards the left half-plane through the reduction of a, a real number, to close to feasibility of (8). The proposed algorithm includes following Steps:

Step 1: Compute the state-space model (5) for the given control system.

Step 2: Compute the optimal guaranteed H_2 performance index γ_2 using function *hinfmix* in MATLAB based LMI control toolbox [22] to design standard H_2/H_∞ dynamic output controller for the performed system in Step 1.

Step 3: Set $j=1$, $\Delta\gamma_2 = \Delta\gamma_0$ and let $\gamma_{2j} = \gamma_0 > \gamma_2$. $\Delta\gamma_0$ and γ_0 are positive real numbers. Select $Q = Q_0 > 0$, and solve X from the following algebraic Riccati equation

$$A_i X + X A_i^T - X C_{yi}^T C_{yi} X + Q = 0, \quad X > 0. \quad (16)$$

Set $P_1 = X$.

Step 4: Solve the following optimization problem for X_j, K_j, and a_j: Minimize a_j subject to the bellow LMI constraints:

$$\begin{bmatrix} A_i X_j + X_j A_i^T + B_{1i} B_{1i}^T + \Sigma_j & B_{2i} K_j + X_j C_{yi}^T \\ (B_{2i} K_j + X_j C_{yi}^T)^T & -I \end{bmatrix} < 0, \quad (17)$$

$$trace(C_{2ic} X_j C_{2ic}^T) < \gamma_{2j}, \quad (18)$$

$$X_j = X_j^T > 0, \quad (19)$$

where

$$\Sigma_j = P_j C_{yi}^T C_{yi} P_j - P_j C_{yi}^T C_{yi} X_j - X_j C_{yi}^T C_{yi} P_j - a_j X_j.$$

Denote a_j^* as the minimized value of a_j.

Step 5: If $a_j^* \le 0$, go to Step 9.

Step 6: For $j > 1$ if $a_{j-1}^* \le 0$, $K_{j-1} \in K_{sof}$ and go to Step 10. Otherwise go to Step 7.

Step 7: Solve the following optimization problem for X_j and K_j: Minimize $trace(X_j)$ subject to LMI constraints (17-19) with $a_j = a_j^*$. Denote X_j^* as the X_j that minimized $trace(X_j)$.

Step 8: Set $j=j+1$ and $P_j = X_{j-1}^*$, then go to Step 4.

Step 9: Set $\gamma_{2j} = \gamma_{2j} - \Delta\gamma_2$, $j=j+1$. Then do Steps 3 to 5.

Step 10: If $\gamma_{\infty,j-1} = \left\|T_{z_{\infty i}\, w_{1i}}\right\|_\infty \le 1$, $K_i = K_{j-1}$ is a

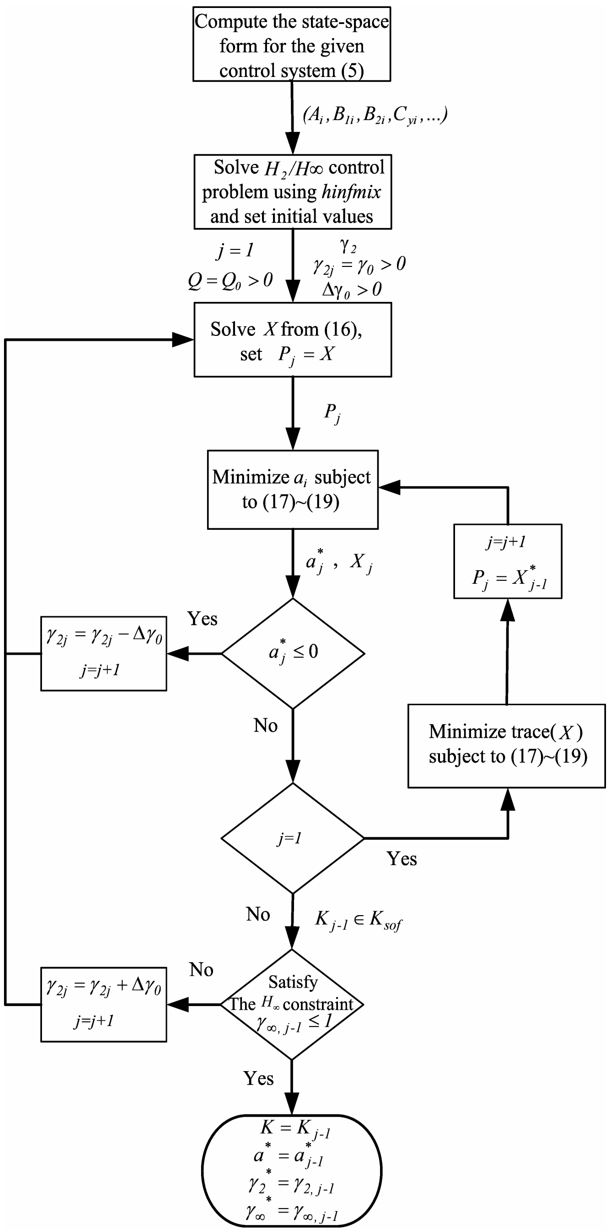


Fig. 4. Developed ILMI algorithm.

suboptimal H_2/H_∞ SOF controller and $\gamma_2^* = \gamma_{2j} - \Delta\gamma_2$ indicates a lower H_2 bound such that the obtained controller satisfies (15). Otherwise go to 7.

The proposed ILMI algorithm is summarized in Fig. 4. The proposed iterative LMI algorithm shows that if we simply perturb A_i to $A_i - (a/2)I$ for some $a > 0$, then we will find a solution (X >0, K) of the matrix inequality (14) for the performed generalized plant. That is, there exist a real number ($a > 0$) and a matrix $P > 0$ to satisfy inequality (17).

Consequently, the closed-loop system matrix $A_i - B_i K C_i$ has eigenvalues on the left-hand side of the line $\Re(s) = a$ in the complex s-plane. Based on the idea that all eigenvalues of $A_i - B_i K C_i$ are shifted progressively towards the left half plane through the reduction of a. The given generalized eigenvalue minimization in the developed iterative LMI algorithm guarantees this progressive reduction.

In summary, the proposed control algorithm first specifies the stability domain of (PI/PID parameters) space, which guarantees the stability of the closed-loop system, using the generalized static output stabilization feedback lemma (Lemma 2). In the second step, the subset of the stability domain in the PI/PID parameter space in step one is specified so that minimizes the H_2 tracking performance. Finally and in the third step, the design problem reduced to search within the previous subset domain and to find the point with closest H_2 performance index to the optimal one which meets the H_∞ constraint.

3.2. Applicable to time-delay systems

It is significant to note that because of using simple constant gains, pertaining to SOF synthesis for dynamical systems in the presence of strong constraints and tight objectives are few and restrictive [23]. Under such conditions, the addressed optimization problem may not approach to a strictly feasible solution. However, in the most cases, reaching to a near optimal solution is possible by effective and flexible search techniques such as descript algorithm in the previous section. In order to adopt the proposed control procedure to time-delayed systems, it is enough to consider the time-delays effects as model uncertainties.

A delay term can be expressed by the exponential function $e^{-s\tau}$ where τ gives the delay time. To use linear robust control techniques, an exponential delay term can be expressed in the form of low-order Pade approximation for the related Taylor series expansion.

The uncertainties due to time delays can be modeled as an unstructured multiplicative uncertainty block W_i that contains all possible variations in the assumed delays range. Let $\hat{G}_i(s)$ denotes the transfer function of time-delayed system from the control input u_i to control output y_i at operating points other than nominal point. Following a practice common in robust control, we can represent this transfer function as

$$\left|\Delta_i(s) W_i(s)\right| = \left|[\hat{G}_i(s) - G_{0i}(s)] G_{0i}(s)^{-1}\right|, \qquad (20)$$

where

$$\|\Delta_i(s)\|_\infty = sup_\omega \left|\Delta_i(s)\right| \le 1; \quad G_{0i}(s) \ne 0.$$

$\Delta_i(s)$ shows the uncertainty block corresponding to delayed terms and $G_{0i}(s)$ is the nominal transfer function model. Thus, $W_i(s)$ is such that its respective magnitude bode plot covers the bode plots of all possible open-loop structures (including time delays). Finally the developed ILMI algorithm can be run to obtain the robust PI/PID controllers as descript in above.

4. NUMERICAL EXAMPLES

4.1. Example 1: Load-frequency control(LFC) design

4.1.1 PI-based LFC

To illustrate the effectiveness of the proposed control strategy, the decentralized PI-based load-frequency control (LFC) design in a three control area power system, shown in Fig. 5, is considered as an example. Each control area can be approximate to a linear system described in Fig. 6 and (here) includes three generation companies (Gencos) with 9th order. The power system data and parameters are considered the same as in [24].

According to (5), we can calculate the state-space model for each control area as follows:

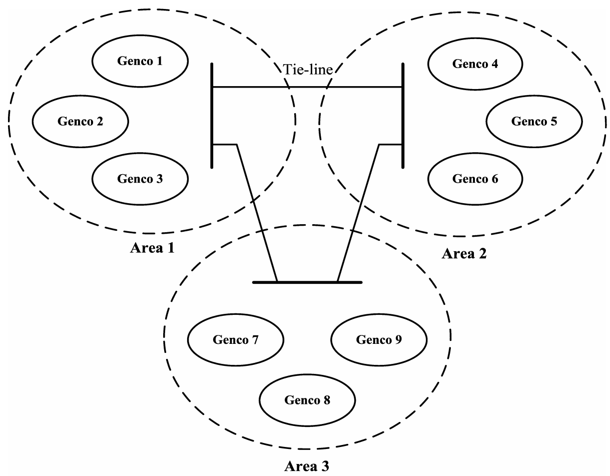


Fig. 5. Three control area power system.

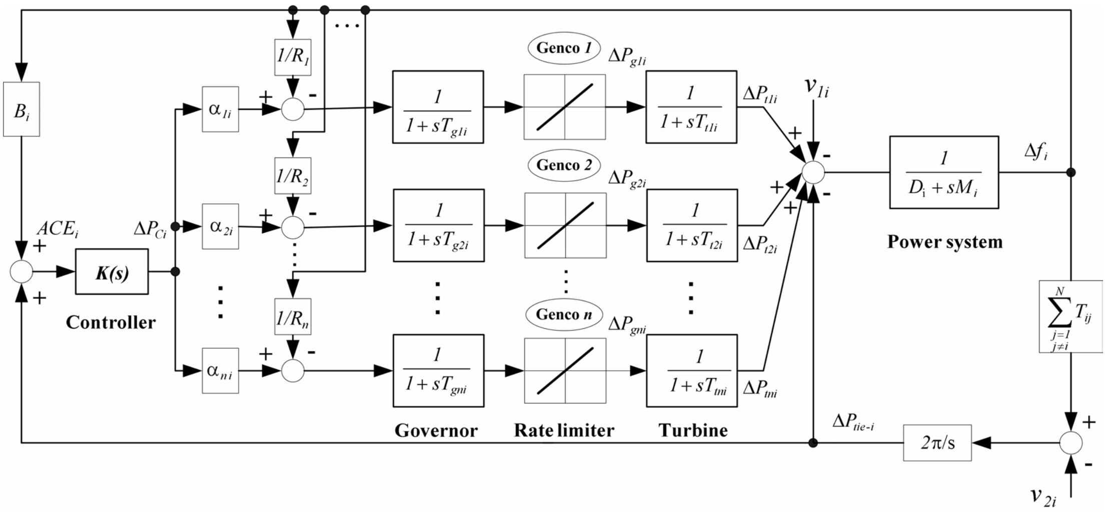


Fig. 6. A general control area (the explanation of labels and parameters is available in [24]).

$$\begin{aligned}
\dot{x}_i &= A_i x_i + B_{1i} w_i + B_{2i} u_i, \\
z_{\infty i} &= C_{\infty i} x_i + D_{\infty 1i} w_i + D_{\infty 2i} u_i, \\
z_{2i} &= C_{2i} x_i + D_{21i} w_i + D_{22i} u_i, \\
y_i &= C_{yi} x_i + D_{y1i} w_i, \quad i = 1, 2, 3.
\end{aligned} \tag{21}$$

A suitable control framework in order to LFC design for each control area via a mixed H_2/H_∞ control technique is shown in Fig. 7 [25], where Δf_i, ACE_i and ΔP_{ci} are frequency deviation, area control error (measured output) and governor load setpoint, respectively. $G_i(s)$ corresponds to the nominal augmented model of the given control area. y_i is the measured output (performed by ACE and its integral), u_i is the control input and w_i includes the perturbed and disturbance signals in the given control area.

The w_i can be obtained as follows:

$$w_i = [w_{1i} \quad w_{2i}], \; w_{1i} = [v_{1i} \quad v_{2i}]. \tag{22}$$

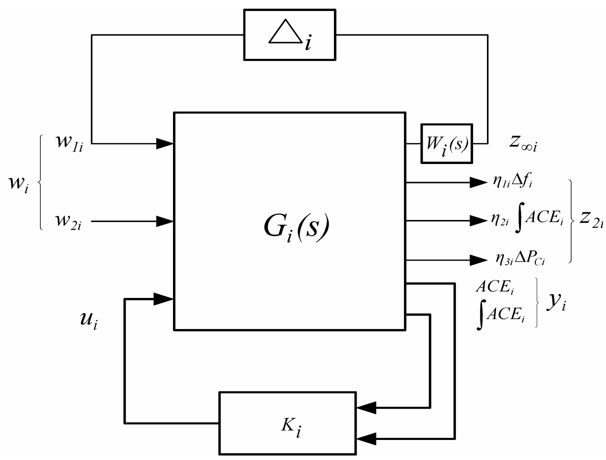


Fig. 7. H_2/H_∞ SOF-based LFC.

w_{2i} is the fictitious perturbed input signal associated with the uncertainty loop in Fig. 7. v_{1i} and v_{2i} demonstrate the area load disturbance and interconnection effects (area interface) respectively, and can be defined as follows.

$$v_{1i} = \Delta P_{di}, \quad v_{2i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} T_{ij} \Delta f_j. \tag{23}$$

Here, ΔP_{di} and T_{ij} denote local load disturbance and tie-line synchronizing coefficient for area i and j. The Δ_i block models the structured uncertainty set in the form of multiplicative type and W_i includes the associated weighting function. η_{1i}, η_{2i}, and η_{3i} are constant weights that must be chosen by designer to get the desired performance. The selection of these constants is dependent on specified voltage regulation and damping performance goals. In fact an important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives. Furthermore, η_{3i} sets a limit on the allowed control signal to penalize fast changes, large overshoot with a reasonable control gain to meet the feasibility and the corresponded physical constraints.

It is assumed that the parameters of inertia constant and damping coefficient in each area have uncertain values ($\pm 20\%$ of nominal values). For the example at hand, a set of suitable weighting functions is assumed as follows:

$$W_1(s) = \frac{0.3619s + 0.1613}{s + 1.6242}, \; W_2(s) = \frac{0.2950s + 0.1073}{s + 1.6814},$$
$$W_3(s) = \frac{0.3497s + 0.3515}{s + 3.4815}, \; \eta_{1i}=0.12, \; \eta_{2i}=0.35, \; \eta_{3i}=0.42.$$

Using (20), some sample uncertainties due to delays variation, within the assumed delay range, can be obtained. we can model uncertainties from both delayed channels by using a norm bonded multiplicative uncertainty to cover all possible plants as given above.

The H_2 performance is used to minimize the effects of disturbances on area frequency, area control error and penalize fast changes and large overshoot in the governor load set-point. The H_∞ performance is used to meat the robustness against specified uncertainties and reduction of its impact on closed-loop system performance.

4.1.2 ILMI based PI controllers

According to the proposed algorithm described in section 3, first a mixed H_2/H_∞ dynamic controller is designed for each control area, using *hinfmix* function in LMI control toolbox [22]. In this case, the resulted controller is dynamic type, whose order is equal to the size of generalized plant model (10th order in the present example). In the next step, a set of three decentralized robust PI controllers are designed. Using developed ILMI algorithm, the controllers are obtained following several iterations. The proposed control parameters are shown in Table 1. The guaranteed optimal H_2 and H_∞ indices for dynamic and PI controllers are listed in Table 2.

The resulted optimal H_∞ indices (γ_∞ and γ_∞^*) and robust performance H_2 indices (γ_2 and γ_2^*) of both synthesis methods are very close to each other. It shows that although the proposed ILMI approach gives a set of much simpler controllers (PI) than the dynamic H_2/H_∞ design, however they holds robustness as well as dynamic H_2/H_∞ controllers.

4.1.3 GA based PI controllers

Since the PI/PID control problem is reduced to a minimization problem, and in the other hand, genetic algorithm (GA) is well-known as a powerful tool to solve such kind of optimization problems, in addition to proposed control strategy to synthesis the robust PI controllers, the GA has been used to evaluate our proposed control strategy to track the guaranteed optimal performance indices, using the given approach in [13]. GAs represent a heuristic search technique based on the evolutionary ideas of natural selection and genetics. GAs solve optimization problems by exploitation of a random search.

In this approach the GA is employed as an optimization engine to produce the PI controllers with performance indices near to optimal ones. The obtained control parameters and performance indices are shown in Table 3. The indices are comparable to the given results by the proposed ILMI algorithm.

In order to demonstrate the effectiveness of the proposed strategy, some simulations were carried out. Fig. 8 shows the closed-loop response (frequency deviation, area control error and control action signals) in the face of both step load disturbance (0.1 pu) and uncertainties (20% decrease in uncertain parameters). The simulation results demonstrate that the proposed ILMI-based PI controllers track the load fluctuations and meet robustness for a wide range of load disturbances as well as GA based PI and H_2/H_∞ dynamic controllers.

Table 1. PI control parameters from ILMI design.

Parameters	Area 1	Area 2	Area 3
k_{Pi}	-2.00E-04	-4.80E-03	-2.50E-03
k_{Ii}	-0.3908	-0.4406	-0.4207

Table 2. Guaranteed H_2 and H_∞ indices.

Indices	Area 1	Area 2	Area 3
γ_2 (Dynamic)	1.0700	1.0300	1.0310
γ_∞ (Dynamic)	0.3919	0.2950	0.3497
γ_2^* (PI)	1.0976	1.0345	1.0336
γ_∞^* (PI)	0.3920	0.2950	0.3498

Table 3. Control design using GA approach.

Areas	Area 1	Area 2	Area 3
k_{Pi}	-1.00E-04	-0.0235	-1.00E-04
k_{Ii}	-0.2309	-0.2541	-0.2544
γ_2^*	1.0371	0.9694	0.9807
γ_∞^*	0.3619	0.2950	0.3497

4.2. Example 2: Time-Delayed System

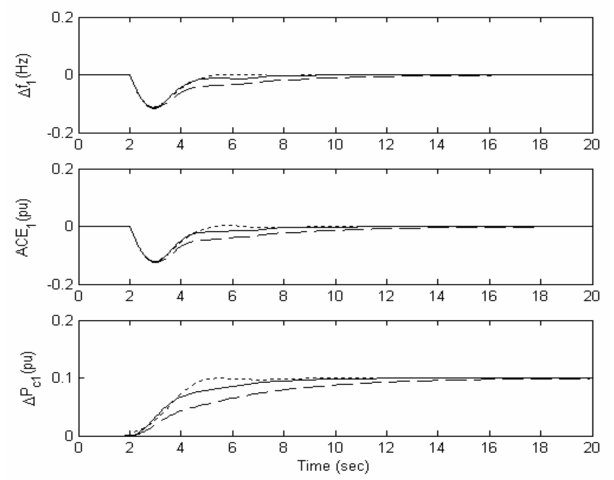
Consider the LFC system shown in Fig. 6 with delays in the communication channels ACE ($\tau_{di} \in [0\ \ 2.5]s$) from Gencos and tie-line to the control center and control effort ($\tau_{hi} \in [0\ \ 2.8]s$) from control center to Gencos.

Based on the given simple stability condition in [26], the open loop system with real matrices is stable if

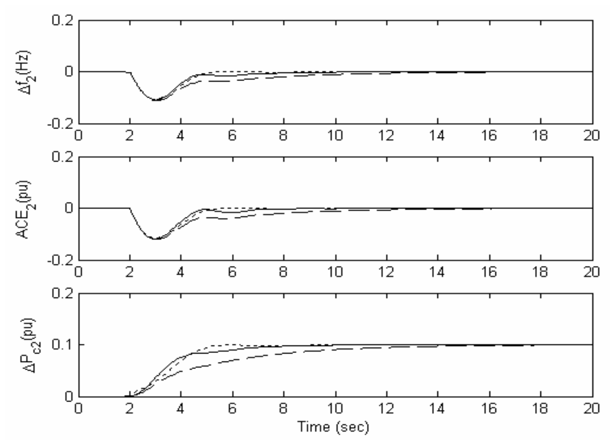
$$\mu(A_i) + \|A_{di}\| < 0, \tag{24}$$

where $\mu(A_i) = 0.5 \max_j \lambda_j (A_i^T + A_i)$.

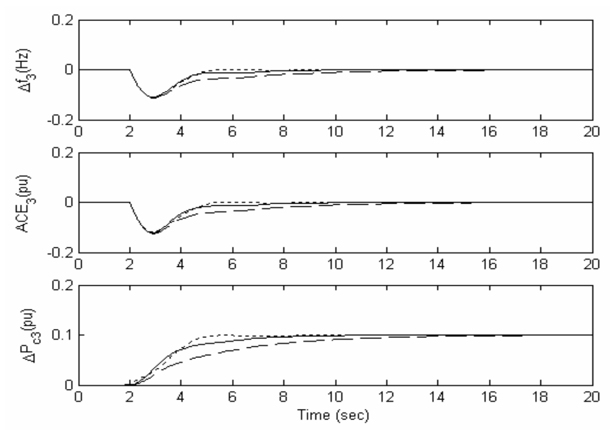
Here, λ_j denotes the jth eigenvalue of $(A_i^T + A_i)$. In light of above stability rule, we note that for the example at hand, the control areas are unstable for the assumed maximum delays:



(a)



(b)



(c)

Fig. 8. Closed-loop system response: (a) Area 1, (b) Area 2 and (c) Area 3; Solid (proposed ILMI), dotted (dynamic H_2/H_∞), dash line (GA).

$$\mu(A_1)+\|A_{d1}\|=10.4736>0,$$
$$\mu(A_2)+\|A_{d2}\|=12.2615>0,$$
$$\mu(A_3)+\|A_{d3}\|=10.2285>0.$$

Using (20), some sample uncertainties due to delay

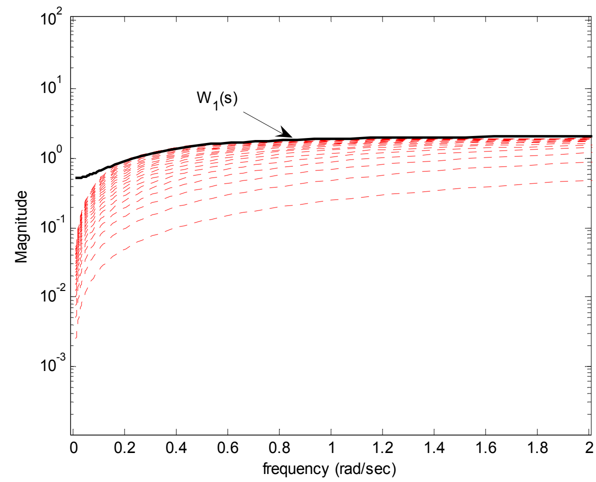


Fig. 9. Uncertainty plots (dotted) due to communication delays and the upper bound (solid) in area 1.

domain for area 1 are shown in Fig. 9. To keep the complexity of obtained controller low, we can model uncertainties from both channels delays by using a norm bonded multiplicative uncertainty to cover all possible plants as follows

$$W_1(s)=\frac{2.1339s+0.2557}{s+0.4962}.$$

Using the same method, the uncertainty weighting functions for areas 2 and 3 can be computed.

$$W_2(s)=\frac{2.0558s+0.2052}{s+0.3869},\ W_3(s)=\frac{2.0910s+0.2129}{s+0.5198}.$$

According to the synthesis methodology described in Section 3, a set of three decentralized robust PI controllers are designed as shown in Table 4.

Increasing the delays will degrade the LFC system performance seriously. In Fig. 10, the performance of the closed-loop system is compared with the delay-less control design (nominal design, Table 1) and full orders dynamic H_2/H_∞ controllers in the presence of load disturbances and communication delays.

Fig. 10 shows the frequency deviation for control areas in the face of following delays in the communication channels:

$$d_i=1.5s,\ h_i=2s;\quad i=1,2,3.$$

Following a 0.1 *pu* step load disturbance at *5s* in each control area. It shows that the nominal control design

Table 4. Control parameters for time delayed LFC system.

Parameters	Area 1	Area 2	Area 3
k_{Pi}	-0.2728	-0.1475	-0.2142
k_{Ii}	-0.2296	-0.1773	-0.2397

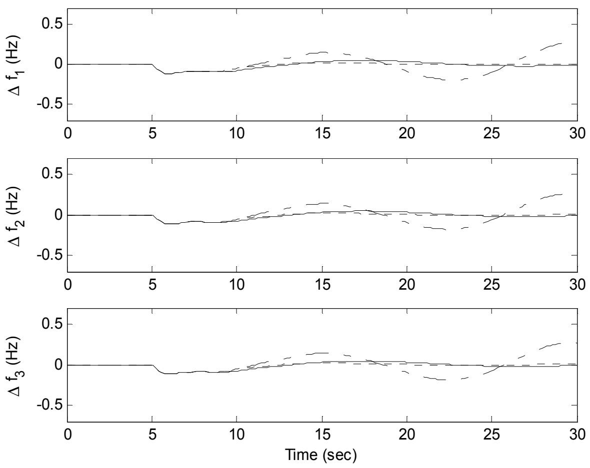


Fig. 10. A simulation test for Example 2; Solid (proposed design), dotted (GA), dash-dotted (nominal design).

is not capable to hold the stability of closed-loop system, while the proposed controllers track the load variation as well as full order dynamic robust controllers.

4.3. Example 3: PID based PSS design

This section addresses the application of the proposed control methodology to synthesis of a PID based robust power system stabilizers (PSS). For this purpose, a single-machine infinite-bus system is considered to illustrate the developed control approach.

A single line representation of the power system is shown in Fig. 11(a) and the block diagram of the closed-loop system is shown in Fig. 11(b). The electrical power signal is considered as input signal for PID based PSS. The power system parameters with extended explanation are given in [27].

With regard to uncertainty, it is assumed that the parameters of connected line to infinite bus (R_i and X_i) to be uncertain parameters. The state variables and the measured output signal are chosen as (25), where ω, δ, v_R, E_{fd}, and e_q' are machine speed, angle, AVR (automatic voltage regulator) voltage, field excitation voltage and the quadratic-axis transient voltage, respectively.

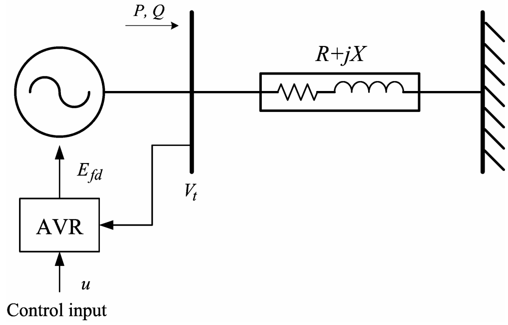
$$x^T = \left[\Delta\omega \quad \Delta\delta \quad \Delta E_{fd} \quad \Delta e_q' \quad \Delta v_R\right], \qquad y = \Delta P_e. \tag{25}$$

Using the proposed synthesis methodology described in Section 3, a robust PID controller is obtained with the following parameters.

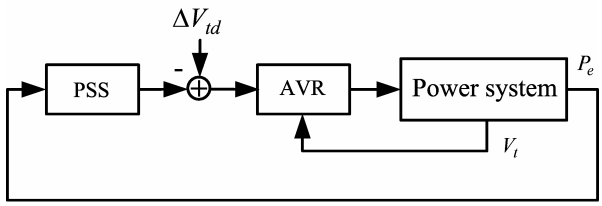
$$K_{PSS} = [k_P \quad k_I \quad k_D] = [0.4300 \quad 5.7888 \quad 0.9181]. \tag{26}$$

The performance of the closed-loop system in comparison of a well tuned conventional PSS [28] is tested in the presence of voltage disturbances, short circuit fault on transmission line and parameter variations. Here, because of space limitation, only the system response to the line voltage disturbance is given (Fig. 12).

As shown in Fig 12, the performance of two controllers were evaluated in the presence of a 0.1 pu step disturbance injected at the voltage reference input of the AVR at 1 second. The robust PSS is shown to



(a) Single line representation.



(b) Closed-loop block diagram.

Fig. 11. Single-machine infinite-bus power system.

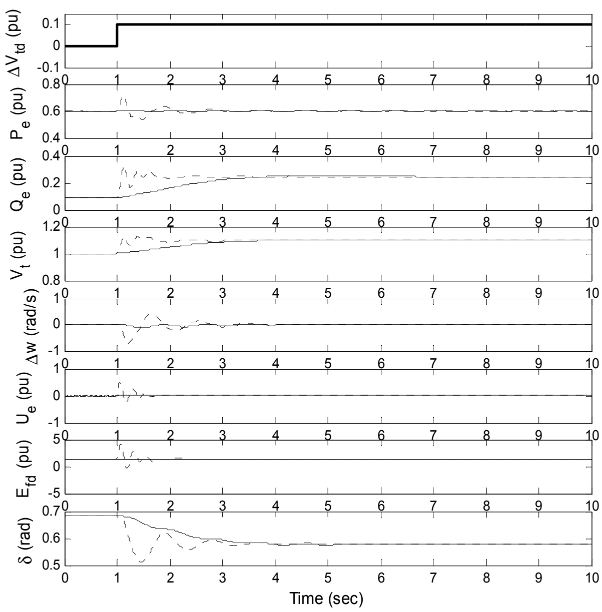


Fig. 12. System response to a step disturbance at the voltage reference input; Solid (Proposed PID based PSS), dotted (Well-tuned conventional PSS).

maintain the robust performance and minimize the effect of voltage disturbance properly.

5. CONCLUSION

An H_2/H_∞ SOF-based ILMI algorithm is developed to design a simple PI/PID controller, which is useful in the real-world control systems. The proposed method was successfully applied to LFC synthesis (in a three control area power system with and without communication time delays) and PSS design (for a single machine infinite bus system). The results are compared with the results of applied H_2/H_∞ dynamic controllers, GA-based approach and well tuned conventional designs. It was shown that the desired performance can be achieved using the proposed control strategy.

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