Robust Control Design and Implementation for a Quadratic Buck Converter

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***Abstract*-- A robust controller for a quadratic buck converter is designed using Kharitonov's theorem. Then, the controller is implemented and experimental results are given for a low power switching quadratic buck converter. The robust D-stability concept is used to achieve robust performance by clustering the roots of closed loop system characteristic polynomial equation in a specified angular region. The obtained results demonstrate a desirable reference voltage tracking, line disturbance rejection and show that the designed controller guarantees the robust stability and robust performance for a wide range of line voltage and load variation.**

***Index Terms*—Quadratic Buck Converter, Kharitonov's Theorem, Robust Control.**

I. Introduction

During the last few years, a great number of applications for quadratic buck converters (QBCs) have been reported [1-4]. In a QBC with a single switch, the dc conversion ratio has a quadratic dependence of the duty ratio. Its dynamical behavior is similar to two cascaded buck converters but using one active switching devise. The scheme of the QBC with a single switch is given in Fig. 1. In addition to the DC supply and load, the QBC circuit contains three parts: an active switch, two LC filters, and three passive switches.

It is generally known that switching QBCs are highly nonlinear systems which can be subjected to significant variation in the line voltage, load, uncertainty in the circuit parameters, and perturbations in switching times. To assure stable operation and acceptable performance despite the disturbances and inevitable uncertainty associated with such systems, highly accurate regulation schemes must be devised [5]. Indeed the routine application of most classical compensation techniques are severely limited when tight regulation measures have to be achieved [6]. These considerations in conjunction with increasing demand for cascading DC-DC switching regulators and QBCs has necessitated more systematic, precise and robust methodologies in control design for such systems.

In the present work a robust technique based on Kharitonov's theorem [7], is used to obtain an admissible proportional-integral (PI) controller which guarantee satisfactory operation of the system under realistic operating conditions. The D-stability concept is used to fine tuning of the designed PI loop parameters. The resulting controller is shown to minimize the effect of disturbances on the line-voltage/load, and achieve acceptable regulation.

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This paper is organized as follows: the dynamical modeling is briefly explained in Section 2, and controller design methodology is discussed in Section 3. Section 4 presents the experimental results, and finally the paper is concluded in Section 5.

II. Modeling

Since, for a QBC converter in the steady state the output DC voltage is a quadratic function of the switching duty ratio, we have

$$V_o = U^2 E \tag{1}$$

and, the nominal operating point of the converter can be derived as follows:

$$V_{c_1} = UE\ ,V_{c_2} = U^2 E\ , I_{l_1} = U^3 E, I_{l_2} = \frac{U^2 E}{R} \tag{2}$$

Considering the equivalent circuit model, which is shown in Fig. 2, and using the averaging techniques have been proposed in [2, 8], the linear state-space dynamic model for the QBC converter can be obtained. Then, using Laplace transform and after some algebraic manipulations, the corresponding transfer function from the output voltage to control signal (duty ratio) can be computed as follows.

$$G(s) = \frac{Vo}{\tilde{u}(s)} = \frac{Vo}{uC_2L_2}.\frac{s^2+d_1s+d_0}{s^4+a_3s^3+a_2s^2+a_1s+a_0} \tag{3}$$

where

$$a_3 = \frac{1}{RC_2}\ ;\ \ a_2 = \frac{u^2}{C_1L_1} + \frac{1}{L_2C_2} + \frac{1}{L_1C_1}$$

$$a_1 = \frac{u^2}{RC_1L_2C_2} + \frac{1}{RC_1L_1C_2};\ \ a_0 = \frac{1}{L_1C_1L_2C_2}$$

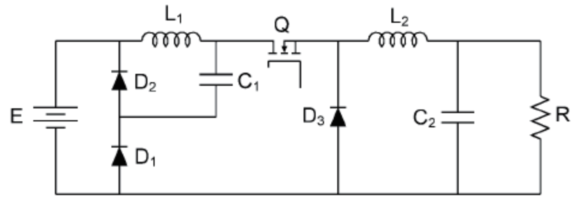


Fig. 1. QBC with a single switch.

$$d_1 = -\frac{u^2}{C_1 R}\ ;\ d_0 = \frac{2}{L_1 C_1} \tag{4}$$

Considering (3) and (4), it is seen that G(s) is a nonminimum phase transfer function, because there are two zeros in the right-half plane (RHP) as given in (5), for all possible parameter values [1].

$$S_{z1,2} = \sigma \pm j\omega \quad \sigma = \frac{U^2}{2C1R} \quad \omega = \sqrt{\frac{2}{L_1 C_1} - \sigma^2} \tag{5}$$

III. Control Design Methodology

A. Kharitonv's Theorem

We proceed to design a robust controller using the Kharitonov based synthesis approach. In this paper, our main focus is concentrated on robust performance e.g. minimize the effects of the line disturbances, desired reference tracking and holding stability in presence of load variation.

Based on Khartonov's theorem, every polynomial such $K(s)$,

$$K(s) = a_0 s + a_1 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \cdots \tag{6}$$

with real coefficients is Hurwitz if and only if the following four extreme polynomials are Hurwitz [7]:

$$\begin{aligned} K_1(s) &= a_0{}^+ + a_1{}^+ s + a_2{}^- s^2 + a_3{}^- s^3 + \cdots \\ K_2(s) &= a_0{}^- + a_1{}^- s + a_2{}^+ s^2 + a_3{}^+ s^3 + \cdots \\ K_3(s) &= a_0{}^- + a_1{}^+ s + a_2{}^+ s^2 + a_3{}^- s^3 + \cdots \\ K_4(s) &= a_0{}^+ + a_1{}^- s + a_2{}^- s^2 + a_3{}^+ s^3 + \cdots \end{aligned} \tag{7}$$

The "-" and "+" show the minimum and maximum bounds. The (3) can be rewritten as follows.

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{8}$$

where

$$a_3 = \frac{1}{RC_2},\quad a_2 = \frac{U^2}{C_1 L_2} + \frac{1}{L_2 C_2} + \frac{1}{L_1 C_1}$$

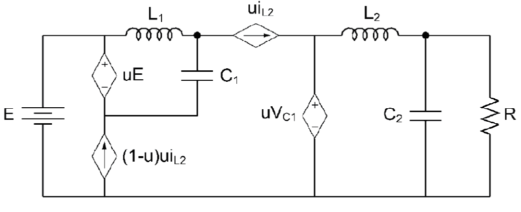


Fig. 2. Equivalent circuit for a QBC with a single switch.

$$b_2 = \frac{UE}{L_2 C_2},\quad a_1 = \frac{U^2}{C_1 L_2 C_2 R} + \frac{1}{L_1 C_1 C_2 R}$$

$$b_1 = -\frac{U^3 E}{C_1 L_2 C_2 R},\quad a_0 = \frac{1}{L_1 C_1 L_2 C_2}$$

$$b_0 = \frac{2UE}{L_1 C_1 L_2 C_2}$$

In the presence of a PI controller,

$$C(s) = -(K_p + \frac{K_i}{s}) \tag{9}$$

the order of closed-loop system is 5. Based on a lemma [7], to check the stability of a 5th order characteristic polynomial, it is just needed to test the Hurwitz criteria for the following three polynomials:

$$\begin{aligned} K_1(s) &= a_0{}^+ + a_1{}^+ s + a_2{}^- s^2 + a_3{}^- s^3 + a_4{}^+ s^4 + a_5{}^+ s^5 \\ K_3(s) &= a_0{}^- + a_1{}^+ s + a_2{}^+ s^2 + a_3{}^- s^3 + a_4{}^- s^4 + a_5{}^+ s^5 \\ K_4(s) &= a_0{}^+ + a_1{}^- s + a_2{}^- s^2 + a_3{}^+ s^3 + a_4{}^+ s^4 + a_5{}^- s^5 \end{aligned} \tag{10}$$

B. Case Study and Robust PI Design

A low power QBC (28 W) with the same parameter values that given in [1, 2, 10] is considered as a test system. Here, the nominal input voltage and output load are fixed at 15 V and 5Ω, respectively.

It is assumed that the coefficients of the open-loop system (8) being bounded as follows.

$$\begin{aligned} [b_0{}^-, b_0{}^+] &= [1.9080e+017,\ \ 2.3320e+017] \\ [b_1{}^-, b_1{}^+] &= [-2.7200e+012, -1.8208e+012] \\ [b_2{}^-, b_2{}^+] &= [680000000,\ \ 8.3111e+008] \\ [a_0{}^-, a_0{}^+] &= [1.1547e+016\ ,\ \ 1.1547e+016] \\ [a_1{}^-, a_1{}^+] &= [5.0380e+011,\ \ 6.1576e+011] \\ [a_2{}^-, a_2{}^+] &= [3.3169e+008,\ \ 3.3169e+008] \\ [a_3{}^-, a_3{}^+] &= [2.0202e+003,\ \ 2.4691e+003] \end{aligned} \tag{11}$$

The characteristic polynomial of closed-loop QBC, considering (8) and (9) can be written as

$$\begin{aligned} K(s) &= s^5 + a_3 s^4 + (a_2 + b_2 K_p) s^3 \\ &+ (a_1 + b_2 K_i + b_1 K_p) s^2 + (a_0 + b_1 K_i + b_0 K_p) s \\ &+ b_0 K_i \end{aligned} \tag{12}$$

As has mentioned, the problem of checking the Hurwitz stability of the family for the present QBC can be reduced to that of checking the Hurwitz stability of three polynomials (10). This procedure after some manipulations results a set of nine inequalities (see Appendix), which are satisfied for some values of K_p and K_i. These values are graphically shown in Fig. 3.

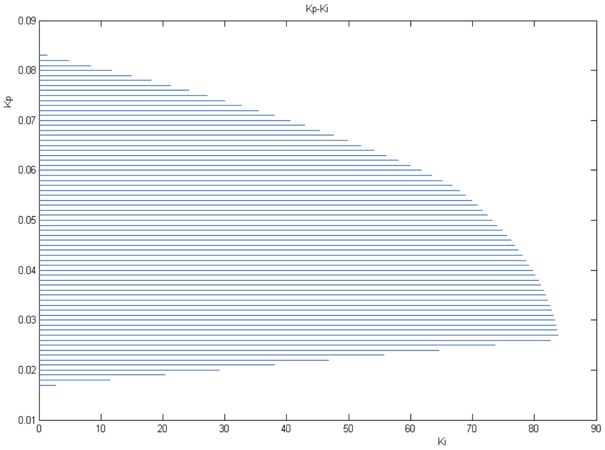


Fig. 3. Acceptable values of K_p and K_i to make a stabilizing PI controller.

The basic geometry associated with the zero exclusion condition [7, 9], is fully demonstrated in Fig. 4 for $0 < \omega < 50\,kHz$, while the PI parameters have been fixed at $K_p = 0.02, K_i = 2$.

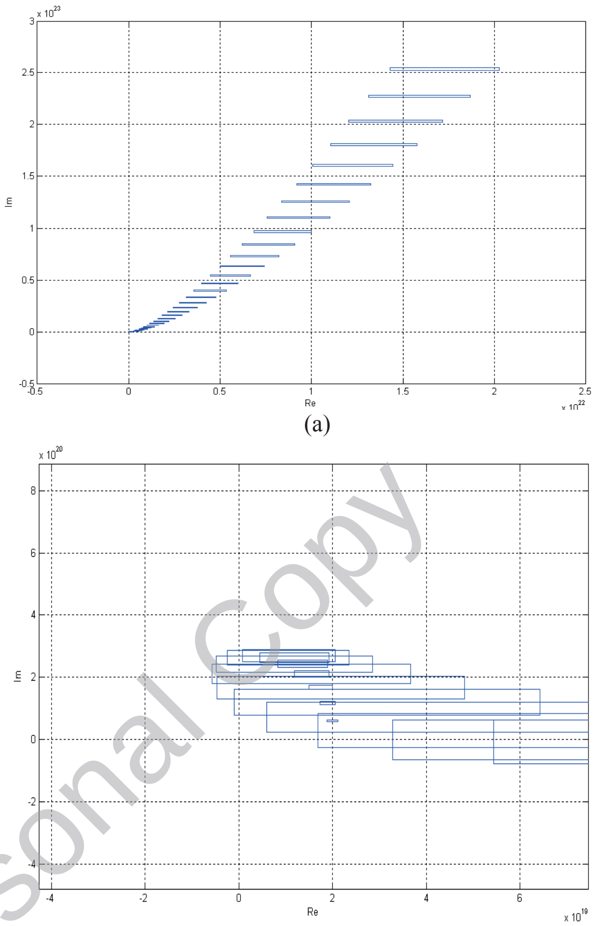
C. PI Tuning: D-Stability Approach

To ensure both stability robustness and specified performance robustness, it is important to guarantee that the roots of the characteristic equations for a linear time-invariant system under parameter perturbations remain in a specific region.

The locations of the roots of the characteristic equations for linear systems determine some performance specifications such as transient stability, damping and the speed of the time response. These specifications can be assured by the placement of the roots of the characteristic equations in an appropriate region (D) in the roots plane. *D-stability* investigates the robustness problem of the characteristic equation's roots clustering in specific regions for dynamical systems with parameter perturbations.

Locating the roots inside the left-half plane (LHP) guarantees the stability of the system, placing the roots inside the left-sectors in the LHP guarantees a minimum damping ratio for the roots, and clustering the roots inside the shifted LHP guarantees a maximum settling-time for the time response of the system [7]. To have the combined effects of the left-sector and the shifted LHP, one may choose a special region in the LHP as the root assignment region.

Various performance specifications for a QBC can be achieved by the placement of the closed loop roots of its characteristic polynomial in appropriate regions. The main aim is to ensure that the roots of the family of characteristic polynomials lie inside D. However, a set of feedback compensators rather than a unique feedback usually satisfies this requirement. One may select a compensator among all D-stabilizing compensators, following introducing an additional objective.



(b)

Fig. 4. a) Motion of the Kharitonove rectangle for $0 < \omega < 50\,kHz$; b) a magnified view around zero.

Considering the assumed parameter perturbation (11), the characteristic equation of the closed-loop system formed by (12) in the feedback configuration, has a parametric uncertainty structure. Holding the robust performance in the presence of input voltage and load variation and decreasing the sensitivity to the disturbances are considered as main objectives to calculate D-stability margin of the present QBC system. For the example at hand, according to the required desirable performance, the D region for the characteristic polynomial roots is defined and the ultimate controller's gains are achieved as follows:

$$K_p = 0.07, K_i = 22 \tag{13}$$

IV. Experimental Results

The proposed PI controller is implemented using operational amplifiers. Figures 5 and 6, were obtained from experiments to demonstrate the effectiveness of the proposed design. Fig. 5 shows the waveform for the output voltage in the presence of a fast permanent variation in line voltage from 12 V to 17 V. This figure does not show a significant change in the level of output voltage.

Fig. 6 depicts the sensitivity of converter output voltage to a periodic step change of load between 3 and 6 ohms. From this figure, it is evident that the tracking error decays to near zero; that is, the output voltage converges to the reference one with a small amplitude variation.

These experimental results show that the proposed controller achieves robust performance in against of reference voltage and load changes.

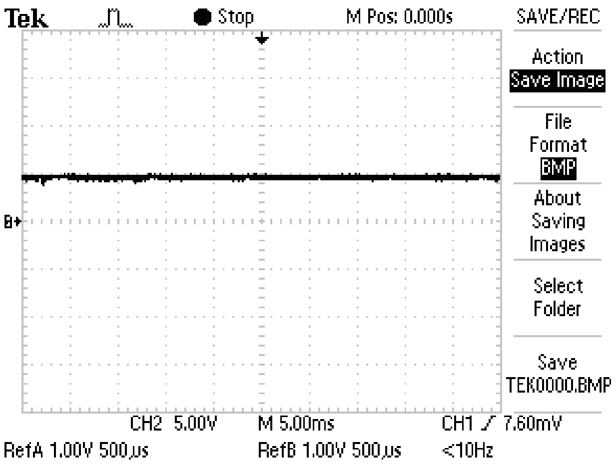


Fig. 5. Output voltage in the presence of a permanent variation in line voltage (from 12 V to 17 V).

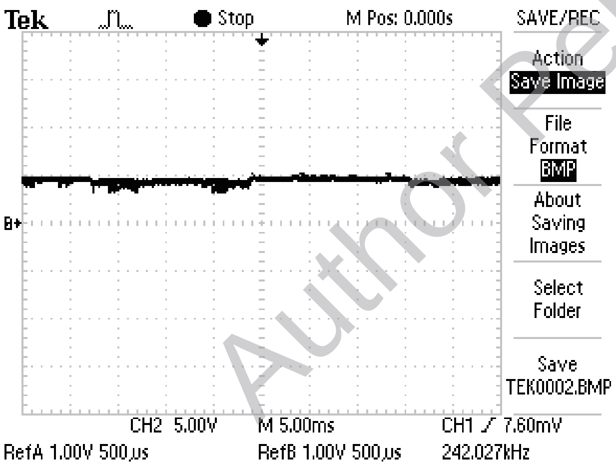


Fig. 6. Output voltage response to a periodic load step variation (between 3 and 6 ohms).

V. Conclusion

A simple robust PI controller based on Kharitonov's theorem is designed for a quadratic buck converter with a single switch. The proposed controller has been implemented for a 28 W QBC using operational amplifiers. The experimental results show robustness against perturbation in parameters, and a desirable performance over a wide range of line voltage and load variations.

VI. References

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VII. Appendix

The obtained inequalities from testing Hurwitz stability criteria for three polynomials given in (10) are as follows:

$$\begin{cases} a_2^{-} < a_3^{-}\, a_4^{+} \\ -(a_2^{-})^2 + a_2^{-}\, a_3^{-}\, a_4^{+} - (a_4^{+})^2\, a_1^{+} > -\, a_0^{+}\, a_4^{+} \\ -(a_4^{+})^3 (a_1^{+})^2 + 2a_0^{+}\, a_1^{+}\, (a_4^{+})^2 + a_0^{+}\, a_2^{-}\, a_3^{-}\, a_4^{+} + a_1^{+}\, a_2^{-}\, a_3^{-}\, (a_4^{+})^2 - a_1^{+}\, a_4^{+}\, (a_2^{-})^2 \\ > (a_0^{+})^2\, a_4^{+} + a_0^{+}\, (a_3^{-}\, a_4^{+})^2 \end{cases}$$

$$
\begin{cases}
a_2^{+} < a_3^{-} a_4^{-} \\
-(a_2^{+})^2 + a_2^{+} a_3^{-} a_4^{-} - (a_4^{-})^2 a_1^{+} > -a_0^{-} a_4^{-} \\
-(a_4^{-})^3 (a_1^{+})^2 + 2a_0^{-} a_1^{+} (a_4^{-})^2 + a_0^{-} a_2^{+} a_3^{-} a_4^{-} + a_1^{+} a_2^{+} a_3^{-} (a_4^{-})^2 - a_1^{+} a_4^{-} (a_2^{+})^2 \\
> (a_0^{-})^2 a_4^{-} + a_0^{-} (a_3^{-} a_4^{-})^2
\end{cases}
$$

$$
\begin{cases}
a_2^{-} < a_3^{+} a_4^{+} \\
-(a_2^{-})^2 + a_2^{-} a_3^{+} a_4^{+} - (a_4^{+})^2 a_1^{-} > -a_0^{+} a_4^{+} \\
-(a_4^{+})^3 (a_1^{-})^2 + 2a_0^{+} a_1^{-} (a_4^{+})^2 + a_0^{+} a_2^{-} a_3^{+} a_4^{+} + a_1^{-} a_2^{-} a_3^{+} (a_4^{+})^2 - a_1^{-} a_4^{+} (a_2^{-})^2 \\
> (a_0^{+})^2 a_4^{+} + a_0^{+} (a_3^{+} a_4^{+})^2
\end{cases}
$$