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–Brief Paper–

ON THE DESIGN OF EVENT-TRIGGERED SUBOPTIMAL CONTROLLERS FOR NONLINEAR SYSTEMS

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ABSTRACT

In this paper, two suboptimal event-triggered control techniques are proposed for both the regulation and the tracking problems in a broad class of nonlinear networked control systems. The proposed techniques are based on the state-dependent Riccati equation (SDRE) methodology. In the case of the regulation problem, the asymptotic stability of the origin of the closed-loop system under the proposed event-triggered control law is investigated. In addition, for the tracking problem, it is proved that the tracking error between the system output and its desired trajectory converges asymptotically to zero under some mild conditions. It is shown that the proposed methods can considerably reduce the information exchange between the controller and the actuator. Due to the implementation procedures of the proposed techniques, no Zeno behavior is occurred. Three numerical simulations are provided to demonstrate the design procedure and the flexibility of the proposed event-triggered control techniques.

Key Words: Nonlinear networked control system, suboptimal event-triggered controller, state-dependent Riccati equation (SDRE) technique.

I. INTRODUCTION

In the traditional time-triggered control scheme, the control law is sent from the controller to the actuator at every sampling instant. However, in modern control systems, required signals may be transmitted from one place to another using a communication network since its components (the plant, the controllers, the sensors, and the actuators) might be located in different places. Therefore, many algorithms have been developed recently to handle important challenges in the field of networked control systems (NCS) such as packet dropout, network-induced delays, and disorder caused by limited network bandwidth [1–7]. The event-based aperiodic sampling and control appears to relieve the computational burden and to decrease the network resource utilization. The main idea of this recent approach is to sample and send the data only when required [2]. Just like a time-triggered control scheme, the stability of the system is the most important issue in an event-triggered control system. By selecting the necessary samples which must be sent from the sensors to the controller and/or from the controller to the actuator, an event-triggered mechanism should be designed in such a way that the stability of the closed-loop system is guaranteed.

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Performance of the system is another essential issue which should be taken into account in an event-triggered control design procedure. Hence, the problem of the optimal regulator design for networked control systems has recently gained extensive progresses, see [8–10]. Due to the practical importance of the optimal trajectory tracking problem, an event-triggered technique for a class of nonlinear dynamical systems has been proposed in [11]. However, one can note that less attention has been given to the development of the event-triggered control algorithms for nonlinear optimal control problems.

The state-dependent Riccati equation (SDRE) techniques were developed to solve many different control engineering problems in some broad classes of nonlinear systems [12–16]. The key idea behind an SDRE technique is in representing the nonlinear system dynamics as a state-dependent linear system which is called the pseudo linearization, extended linearization, and/or state-dependent coefficient (SDC) matrix representation [17]. The SDRE based methods were successfully applied to many different problems such as drug administration in cancer treatment [18] and optimal spacecraft attitude control [19]. Some reported reasons for this popularity are as follows [20]: (i) the SDRE techniques are based on simple concepts directly inherited from the well-established linear theories; (ii) they preserve the nonlinearities of the system without neglecting any nonlinear terms; (iii) by selecting two weighting matrices, the overall system performance can be directly affected with predictable outcomes; (iv) the SDRE techniques have robustness properties with respect to parametric uncertainties, unmodeled dynamics, and system disturbances; (v) these techniques provide the possibility of dealing with high dimensional systems. Two comprehensive surveys of the SDRE techniques and the related theories are [17] and [21].

This paper focuses on the design of optimal (suboptimal) regulators and tracking controllers for a wide class of nonlinear networked control systems. To benefit from the above mentioned properties of the SDRE techniques, two new SDRE based event-triggered methods are proposed to solve the optimal regulation and the optimal trajectory tracking problems. Toward this end, two general nonlinear optimal control problems (an infinite-time quadratic regulation problem and a discounted infinite-time trajectory tracking problem) are defined and their suboptimal solutions in the SDRE framework are reviewed. Using these solutions, two event-triggered control strategies are proposed to apply in a broad class of nonlinear networked control systems. It is proved that the proposed event-triggered regulator stabilizes the origin of the closed-loop system if some necessary conditions on the SDC representation of the system are held. In the trajectory tracking problem, it is proved that the tracking error between the system output and its desired trajectory converges asymptotically to zero under some mild conditions. Paying attention to the implementation procedures of the proposed techniques, no Zeno behavior can be occurred.

The rest of the paper is organized as follows. In Section II, two optimal control problems, an optimal regulation problem and an optimal trajectory tracking problem, are defined for a broad class of nonlinear dynamical systems. In Section III, using the SDRE techniques, two event-triggered methods are proposed to drive suboptimal solutions of the defined nonlinear optimal control problems. In Section IV, simulation results of applying the proposed methods to three nonlinear systems (jet engine compressor, power converter-based DC microgrid, and the Vander Pol's oscillator) are presented. Finally, Section V concludes the paper.

Throughout this paper, the following notation will be used. $\mathbb{R}$ stands for the set of all real numbers. The symbol $\mathbb{R}^+$ denotes the set of all positive real numbers greater than 0. $\mathbb{R}^n$ is the Euclidean space of all n-dimensional real vectors. $\mathbb{R}^{n\times m}$ is the space of all $n\times m$ real matrices. I_n represents the $n \times n$ identity matrix. A matrix $P \in \mathbb{R}^{n\times n}$ is said to be positive definite (positive-semidefinite), if for any nonzero vector $x \in \mathbb{R}^n$, it satisfies $x^{\mathrm{T}}Px > 0$ ($x^{\mathrm{T}}Px \geq 0$). The set $\mathbb{Z}^+ = \{0, 1, 2, \ldots\}$ contains the nonnegative integers.

II. SYSTEM DESCRIPTION AND PROBLEM STATEMENTS

Consider the following nonlinear dynamical system:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + b(x(t))u(t), \quad x(0) = x_0, \\ y(t) &= h(x(t)), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^q$ is the system output, and x_0 is the initial condition. $f(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$, $b(x(t)) : \mathbb{R}^n \to \mathbb{R}^m$, and $h(x(t)) : \mathbb{R}^n \to \mathbb{R}^q$ are assumed to be smooth functions, $f(0) = h(0) = 0$, and $b(x(t)) \neq 0$ for all $x(t) \in \mathbb{R}^n$.

For the above nonlinear system, two control problems are defined below. Then, in Section III, two event-triggered techniques will be developed based on the SDRE solutions of these problems.

2.1 Optimal regulation problem

Consider the nonlinear dynamical system (1). The optimal regulation problem seeks to find a feedback control law $u(t)$, $t \geq 0$ such that the system state $x(t)$ is set to zero as t tends to infinity and the following infinite-time horizon cost function is minimized:

$$J(x_0, u(t)) = \frac{1}{2}\int_0^\infty \left(x^{\mathrm{T}}(t)Qx(t)+u^{\mathrm{T}}(t)Ru(t)\right)\mathrm{d}t, \tag{2}$$

where $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ are respectively positive-semidefinite and positive-definite symmetric matrices.

2.2 Optimal trajectory tracking problem

Consider the nonlinear dynamical system (1). The optimal trajectory tracking problem is to find the control input $u(t)$, $t \geq 0$ such that the system output $y(t)$ tracks the desired trajectory $y_{\mathrm{d}}(t)$ as t tends to infinity and the following discounted infinite-time horizon cost function is minimized:

$$\begin{aligned} J(x_0, u(t), y_{\mathrm{d}}(t)) = \frac{1}{2}\int_0^\infty & e^{-2\gamma t}\big((y(t) - y_{\mathrm{d}}(t))^{\mathrm{T}}Q_1 \\ & (y(t) - y_{\mathrm{d}}(t)) + u^{\mathrm{T}}(t)Ru(t)\big)\mathrm{d}t \end{aligned} \tag{3}$$

where $\gamma > 0$ is the discount factor, $Q_1 \in \mathbb{R}^{q\times q}$ and $R \in \mathbb{R}^{m\times m}$ are respectively positive-semidefinite and positive-definite symmetric matrices. It is assumed that the desired trajectory has the nonlinear dynamics:

$$\begin{aligned}\dot{x}_{\mathrm{d}}(t) &= f_{\mathrm{d}}(x_{\mathrm{d}}(t)), \quad x_{\mathrm{d}}(0) = x_{\mathrm{d0}}, \\ y_{\mathrm{d}}(t) &= h_{\mathrm{d}}(x_{\mathrm{d}}(t)),\end{aligned} \tag{4}$$

where $x_{\mathrm{d}}(t) \in \mathbb{R}^{n_{\mathrm{d}}}$ and $y_{\mathrm{d}}(t) \in \mathbb{R}^{q}$ are the state and the output of the desired trajectory system (4). $f_{\mathrm{d}}(x_{\mathrm{d}}(t)) : \mathbb{R}^{n_{\mathrm{d}}} \to \mathbb{R}^{n_{\mathrm{d}}}$ and $h_{\mathrm{d}}(x_{\mathrm{d}}(t)) : \mathbb{R}^{n_{\mathrm{d}}} \to \mathbb{R}^{q}$ are two smooth functions and $f_{\mathrm{d}}(0) = h_{\mathrm{d}}(0) = 0$.

III. EVENT-TRIGGERED CONTROLLER DESIGN METHODOLOGY

In this section, two techniques are proposed to reduce the communication rate between the controllers and the actuators in a nonlinear networked control system. These techniques are based on the SDRE solutions of the optimal control problems described in Section II.

3.1 Event-triggered SDRE regulator

In order to use the SDRE technique to solve the optimal regulation problem, defined in Subsection 2.1, the function $f(x(t))$ should be rewritten in its SDC representation as $f(x(t)) = A(x(t))x(t)$, where $A(x(t)) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$. Then, it is possible to show that the feedback control law $u(t) = -R^{-1}b^{\mathrm{T}}(x(t))P(x(t))x(t)$ leads to a closed-loop system with suboptimal performance, where $P(x(t))$ is the unique symmetric, positive-definite solution of the following state-dependent Riccati equation [17]:

$$\begin{aligned}&A^{\mathrm{T}}(x(t))P(x(t)) + P(x(t))A(x(t)) - \\ &P(x(t))b(x(t))R^{-1}b^{\mathrm{T}}(x(t))P(x(t)) + Q = 0.\end{aligned} \tag{5}$$

Note that (5) has a unique symmetric, positive-definite solution if the triple $(A(x(t)), b(x(t)), Q^{1/2})$ is point-wise stabilizable and point-wise detectable for all $x(t)$ in a region containing the origin [17].

Finding the solution of the state-dependent Riccati equation (5) is the central component in the design procedure of the SDRE regulator. This equation can be solved using a sampled-data method, represented in [21]. In this method, the SDRE (5) is solved at the sampling instant iT ($i \in \mathbb{Z}^+$) in order to find $P(x(iT))$, where T is the sampling time. Next, the control law $u(x(iT))$ is calculated using $u(x(iT)) = -R^{-1}b^{\mathrm{T}}(x(iT))P(x(iT))x(iT)$ and it is applied to the system in the time interval $[iT, (i+1)T[$. These calculations are periodically repeated and the control input is updated at the next sampling time $(i+1)T$.

The above described implementation technique needs to send messages from the SDRE regulator to the actuator at each sampling instant and therefore, the utilized network is always used to update the computed control input. To reduce the communication rate between the SDRE regulator and the actuator, a new event-triggered regulator is proposed. In this method, a triggering condition is monitored at every sampling instant iT and when violated, the new control signal is sent from the controller to the actuator through the channel.

Theorem 1 below represents the main results of this proposed event-triggered algorithm, called the event-triggered SDRE regulator.

Theorem 1. Consider the nonlinear dynamical system (1). Assume $f(x(t)) = A(x(t))x(t)$ and the triple $(A(x(t)), b(x(t)), Q^{1/2})$ is point-wise stabilizable and point-wise detectable in a bounded open set $\Omega \in \mathbb{R}^n$ containing the origin. Assume further that a basic sample and hold strategy is applied when no control input is transmitted (*i.e.* $u(t) = u(t_k)$, $t \in [t_k, t_{k+1}[$) and the system is under the following control law with $u(t_0) = -K(x(t_0))x(t_0)$:

$$u(t) = \begin{cases} u(t_k), & \mu(x(t)) < 0 \\ -K(x(t))x(t), & \mu(x(t)) \geq 0 \end{cases}, \tag{6}$$

where $t_0 = 0$ is the initial time, $t_k \in \mathbb{R}^+$ is the instants at which the control input has to be transmitted from the controller to the actuator ($k \in \mathbb{Z}^+$), $K(x(t)) = R^{-1}b^{\mathrm{T}}(x(t))P(x(t))$ is the feedback gain. The matrix-valued function $P(x(t))$ is the unique positive-definite symmetric solution of the state-dependent Riccati equation (5) at time t, and $\mu(x(t))$ is defined as follows:

$$\mu(x(t)) = \begin{bmatrix} x^{\mathrm{T}}(t) & e^{\mathrm{T}}(t)\end{bmatrix} \Psi(x(t)) \begin{bmatrix} x(t) \\ e(t)\end{bmatrix}. \tag{7}$$

In the above equality, $e(t) = b(x(t))K(x(t))x(t) - b(x(t))K(x(t_k))x(t_k))$ and $\Psi(x(t))$ is given by:

$$\Psi(x(t)) = \begin{bmatrix} \Psi_{1,1}(x(t)) & P(x(t)) \\ P^{\mathrm{T}}(x(t)) & 0 \end{bmatrix}, \tag{8}$$

where $\Psi_{1,1}(x(t)) = (\sigma(x(t)) - 1)(Q + K^{\mathrm{T}}(x(t))R\,K(x(t)))$ and $\sigma(x(t)) : \mathbb{R}^n \to [0, 1]$ is called the triggering function. Then, the origin of the system (1) is locally asymptotically stable.

Proof. Consider the Lyapunov function $V(x(t)) = x^{\mathrm{T}}(t)P(x(t))x(t)$. For sufficiently small values of $x(t)$, the derivative of $V(x(t))$ along the trajectory $\dot{x}(t) = A(x(t))x(t) - b(x(t))K(x(t))x(t) = A_{\mathrm{cl}}(x(t))x(t)$ is given as [17]

$$\dot{V}(x(t)) = -x^{\mathrm{T}}(t)\big(Q + K^{\mathrm{T}}(x(t))RK(x(t))\big)x(t),$$

and the following Lyapunov equation is held:

$$A_{\mathrm{cl}}^{\mathrm{T}}(x(t))P(x(t)) + P(x(t))A_{\mathrm{cl}}(x(t)) = \\ -(Q + K^{\mathrm{T}}(x(t))RK(x(t))). \tag{9}$$

The origin of the system (1) is locally asymptotically stable if the following weaker inequality is satisfied:

$$\dot{V}(x(t)) \le -\sigma(x(t))x^{\mathrm{T}}(t)\big(Q + K^{\mathrm{T}}(x(t))RK(x(t))\big)x(t). \tag{10}$$

Let t_k shows the instants at which the control input is computed and transmitted through the communication channel to the actuator ($\mu(x(t_k)) \ge 0$). In the time interval $[t_k, t_{k+1}[$, the closed-loop system dynamics is as follows:

$$\dot{x}(t) = A(x(t))x(t) - b(x(t))K(x(t_k))x(t_k). \tag{11}$$

By defining $e(t) = b(x(t))K(x(t))x(t) - b(x(t)) K(x(t_k))x(t_k)$, (11) can be rewritten as follows:

$$\dot{x}(t) = A_{\mathrm{cl}}(x(t))x(t) + e(t).$$

On the other hand, for sufficiently small values of $x(t)$, it is possible to approximate $P(x(t))$ with its value at the origin and therefore, the derivative of $V(x(t))$ is as follows:

$$\dot{V}(x(t)) = x^{\mathrm{T}}(t)\big(A_{\mathrm{cl}}^{\mathrm{T}}(x(t))P(x(t)) \\ + P(x(t))A_{\mathrm{cl}}(x(t))\big)x(t) + 2x^{\mathrm{T}}(t)P(x(t))e(t).$$

Now, using the Lyapunov equation (9), inequality (10), and the above equality, the triggering times are obtained when the following inequality is violated:

$$\begin{bmatrix} x^{\mathrm{T}}(t) & e^{\mathrm{T}}(t) \end{bmatrix} \Psi(x(t)) \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} < 0,$$

where $\Psi(x(t))$ is defined by (8). This completes the proof.

Remark 1. To implement the proposed event-triggered regulator, the time-triggered SDRE implementation technique, represented in [21], is extended. To this end, the positive-definite solution of the sampled-data algebraic Riccati equation

$$A^{\mathrm{T}}(x(iT))P(x(iT)) + P(x(iT))A(x(iT)) - \\ P(x(iT))b(x(iT))R^{-1}b^{\mathrm{T}}(x(iT))P(x(iT)) + Q = 0$$

is computed periodically with the sample time T. Then, the control action $u(x(iT)) = -R^{-1}b^{\mathrm{T}}(x(iT)) P(x(iT))x(iT)$ is obtained at the current state $x(iT)$. To determine whether this computed control must be sent to the actuator or not, the obtained event-triggering condition (7) is checked at this current sampling instant $t = iT$. If $\mu(x(iT)) \ge 0$, the computed control input $u(iT)$ is sent to the actuator and t_{k+1} is set to iT. Therefore, different from the methods where the triggering condition is continuously evaluated, the inter-event times of the proposed event-triggered method are always greater than or equal to the sampling time T. As a result, the proposed event-triggered SDRE regulator overcomes the challenging problem of the minimum inter-event time and the Zeno free execution of the control updating instants is always guaranteed. From a practical point of view, this implementation procedure is so important since the event-triggering condition is periodically evaluated.

Remark 2. According to [21], it is possible to apply the SDRE technique to the following class of nonlinear dynamical systems:

$$\dot{x}(t) = f(x(t)) + g(x(t), u(t)), \tag{12}$$

where $f(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ and $g(x(t), u(t)) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$. Toward this end, an integral control is first defined as follows:

$$\dot{u}(t) = Cu(t) + D\tilde{u}(t), \tag{13}$$

where $\tilde{u}(t) \in \mathbb{R}^m$ is an auxiliary input, $C \in \mathbb{R}^{m\times m}$ and $D \in \mathbb{R}^{m\times m}$ are two arbitrary user-defined matrices. Next, (12) and (13) can be augmented with each other in order to have an affine nonlinear system in the form (1) as follows [21]:

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix}}_{\dot{X}(t)} = \underbrace{\begin{bmatrix} f(x(t)) + g(x(t), u(t)) \\ Cu(t) \end{bmatrix}}_{F(X(t))} + \underbrace{\begin{bmatrix} 0 \\ D \end{bmatrix}}_{B(X(t))} \tilde{u}(t),$$

where $X(t) = [x^{\mathrm{T}}(t) \quad u^{\mathrm{T}}(t)]^{\mathrm{T}}$. Using this technique, the event-triggered SDRE regulator can be also applied to the nonaffine nonlinear system (12).

3.2 Event-triggered SDRE tracking controller

In this section, using the proposed SDRE technique in [16,22], an event-triggered tracking controller is developed to solve the optimal trajectory tracking problem defined in Subsection 2.2. The main results of this proposed event-triggered algorithm, called the event-triggered SDRE tracking controller, are represented in the following theorem.

Theorem 2. Consider the nonlinear optimal tracking problem defined in Subsection 2.2. Assume that the triple $(A(e^{\gamma t}X(t)), B(e^{\gamma t}X(t)), Q^{1/2}(e^{\gamma t}X(t)))$ is point-wise stabilizable and point-wise detectable in a bounded open set

$\Omega \in \mathbb{R}^{n+n_\mathrm{d}}$, where

$$A(e^{\gamma t}X(t)) = \left(-\gamma I + \begin{bmatrix} F(x(t)) & 0 \\ 0 & F_\mathrm{d}(x_\mathrm{d}(t)) \end{bmatrix} \right),$$
$$B(e^{\gamma t}X(t)) = \begin{bmatrix} b(x(t)) \\ 0 \end{bmatrix},$$
$$Q(e^{\gamma t}X(t)) = \left[H(x(t)) \ -H_\mathrm{d}(x_\mathrm{d}(t)) \right]^\mathrm{T} Q_1 \left[H(x(t)) \ -H_\mathrm{d}(x_\mathrm{d}(t)) \right].$$

$F(x(t)) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $H(x(t)) : \mathbb{R}^n \to \mathbb{R}^{q\times n}$, $F_\mathrm{d}(x_\mathrm{d}(t)) : \mathbb{R}^{n_\mathrm{d}} \to \mathbb{R}^{n_\mathrm{d}\times n_\mathrm{d}}$, and $H_\mathrm{d}(x_\mathrm{d}(t)) : \mathbb{R}^{n_\mathrm{d}} \to \mathbb{R}^{q\times n_\mathrm{d}}$ are the SDC representations of $f(x(t))$, $h(x(t))$, $f_\mathrm{d}(x_\mathrm{d}(t))$, and $h_\mathrm{d}(x_\mathrm{d}(t))$, respectively. Then, the following control law with $u(t_0) = -K(e^{\gamma t_0}X(t_0))X(t_0)$ leads to a closed-loop system such that $\bar{e}(t) = \exp(-\gamma t)(y(t) - y_\mathrm{d}(t))$ asymptotically tends to zero:

$$u(t) = \begin{cases} u(t_k), & \mu(X(t)) < 0 \\ -K(e^{\gamma t}X(t))X(t), & \mu(X(t)) \geq 0 \end{cases}, \tag{14}$$

where $t_0 = 0$ is the initial time, $t_k \in \mathbb{R}^+$ is the times at which the control input has to be transmitted from the controller to the actuator, $K(e^{\gamma t}X(t)) = -R^{-1}B^\mathrm{T}(e^{\gamma t}X(t))\ P(e^{\gamma t}X(t))$ is the state-dependent gain. $P(e^{\gamma t}X(t))$ is the unique positive-definite symmetric solution of the state-dependent Riccati equation

$$A^\mathrm{T}(e^{\gamma t}X(t))P(e^{\gamma t}X(t)) + P(e^{\gamma t}X(t))A(e^{\gamma t}X(t))$$
$$- P(e^{\gamma t}X(t))B(e^{\gamma t}X(t))R^{-1}B^\mathrm{T}(e^{\gamma t}X(t))P(e^{\gamma t}X(t))$$
$$+ Q(e^{\gamma t}X(t)) = 0,$$

and $\mu(e^{\gamma t}X(t))$ is given by:

$$\mu(e^{\gamma t}X(t)) = \left[X^\mathrm{T}(t) \quad E^\mathrm{T}(t)\right]\Psi(e^{\gamma t}X(t)) \begin{bmatrix} X(t) \\ E(t) \end{bmatrix}.$$

In the above, $E(t) = B(e^{\gamma t}X(t))K(e^{\gamma t}X(t))\ X(t) - B(e^{\gamma t}X(t))K(e^{\gamma t_k}X(t_k))X(t_k))$ and the matrix-valued function $\Psi(e^{\gamma t}X(t))$ is as follows:

$$\Psi(e^{\gamma t}X(t)) = \begin{bmatrix} \Psi_{1,1}(e^{\gamma t}X(t)) & P(e^{\gamma t}X(t)) \\ P^\mathrm{T}(e^{\gamma t}X(t)) & 0 \end{bmatrix},$$

where $\Psi_{1,1}(e^{\gamma t}X(t)) = (\sigma(X(t)) - 1)(Q(e^{\gamma t}X(t)) + K^\mathrm{T}(e^{\gamma t}X(t))RK(e^{\gamma t}X(t)))$ and $\sigma(X(t)) : \mathbb{R}^n \to [0, 1]$.

Proof. By defining $U(t) = e^{-\gamma t}u(t)$, $X(t) = e^{-\gamma t}\left[x^\mathrm{T}(t)\ x_\mathrm{d}^\mathrm{T}(t)\right]^\mathrm{T}$, and using the SDC representations $f(x(t)) = F(x(t))x(t)$, $h(x(t)) = H(x(t))x(t)$, $f_\mathrm{d}(x_\mathrm{d}(t)) = F_\mathrm{d}(x_\mathrm{d}(t))x_\mathrm{d}(t)$, and $h_\mathrm{d}(x_\mathrm{d}(t)) = H_\mathrm{d}(x_\mathrm{d}(t))x_\mathrm{d}(t)$, the optimal tracking problem defined in Subsection 2.2 is converted to an optimal regulation one with the following infinite-time quadratic cost function [16,22]:

$$J(X_0, U(t)) = \frac{1}{2}\int_0^\infty \left(X^\mathrm{T}(t)Q(e^{\gamma t}X(t))X(t) + U^\mathrm{T}(t)RU(t)\right)\mathrm{d}t, \tag{15}$$

and the dynamics of the augmented state $X(t)$ is as follows [16,22]:

$$\dot{X}(t) = \left(-\gamma I + \begin{bmatrix} F(x(t)) & 0 \\ 0 & F_\mathrm{d}(x_\mathrm{d}(t)) \end{bmatrix}\right)X(t) + \begin{bmatrix} b(x(t)) \\ 0 \end{bmatrix}U(t) = A(e^{\gamma t}X(t))X(t) + B(e^{\gamma t}X(t))U(t). \tag{16}$$

Since the triple $\left(A(e^{\gamma t}X(t)), B(e^{\gamma t}X(t)), Q^{1/2}(e^{\gamma t}X(t))\right)$ is assumed to be point-wise stabilizable and point-wise detectable, it is possible to apply the event-triggered SDRE regulator, proposed in Subsection 3.1, to the latter optimal problem (15) and (16). Using the results of Theorem 1, one can conclude that the augmented state $X(t)$ asymptotically tends to zero under the control law (14). Therefore, the tracking error $\bar{e}(t) = \exp(-\gamma t)(y(t) - y_\mathrm{d}(t)) = \left[H(x(t)) \ - H_\mathrm{d}(x_\mathrm{d}(t))\right] X(t)$ tends to zero as t tends to infinity. The proof is completed. □

Based on the above theorem, using the proposed event-triggered tracking controller leads to a closed-loop system such that $\bar{e}(t) = \exp(-\gamma t)e(t)$ is guaranteed to tend to zero. Nevertheless, it is of our interest to make sure that the tracking error $e(t)$ asymptotically tends to zero. To achieve this aim, the discount factor γ must be selected small, *i.e.* $\gamma \to 0$. However, according to the results of Theorem 2 in [22], the discount factor γ must be selected in such a way that it is greater than the maximum value of the real parts of the eigenvalues of $F_\mathrm{d}(x_\mathrm{d}(t))$ for all $x_\mathrm{d}(t)$. Therefore, in some cases, we have to select larger values for the discount factor γ which causes error in the tracking. However, the observed error can be decreased by selecting larger values for the elements of the user-defined weighing matrix Q_1. It is worth mentioning that if the desired trajectory is generated by a stable or marginally stable system, the maximum value of the real parts of the eigenvalues of $F_\mathrm{d}(x_\mathrm{d}(t))$ are not positive and therefore, the discount factor γ can be any small positive numbers.

IV. SIMULATION RESULTS

In this section, the proposed event-triggered SDRE techniques are applied to three physical nonlinear systems.

4.1 Jet engine compressor

The mathematical model of the jet engine compressor is as follows [23]:

$$\begin{aligned}\dot{x}_1(t) &= -x_2(t) - \frac{3}{2}x_1^2(t) - \frac{1}{2}x_1^3(t),\\ \dot{x}_2(t) &= x_1(t) - u(t),\end{aligned} \tag{17}$$

where $x_1(t)$ is the mass flow, $x_2(t)$ is the pressure rise, and $u(t)$ is the throttle mass flow. The problem is to find a feedback control law $u(t)$ such that the origin of the obtained closed-loop system is asymptotically stable. To solve this problem using the proposed event-triggered SDRE regulator, an SDC representation of (18) should be considered such that its corresponding pair $(A(x), b(x))$ is point-wise stabilizable. One can see that the following SDC has this property:

$$\dot{x}(t)=\begin{bmatrix} -\frac{3}{2}x_1(t)-\frac{1}{2}x_1^2(t) & 1\\ 1 & 0\end{bmatrix}x(t)+\begin{bmatrix}0\\-1\end{bmatrix}u(t). \tag{18}$$

By selecting $Q = I_2$, the point-wise detectability of the pair $(A(x), Q^{1/2})$ is satisfied. Therefore, the proposed event-triggered SDRE regulator stabilizes the origin of the system (18). Fig. 1 shows the obtained control input using the event-triggered SDRE regulator for the sampling time $T = 0.1$ s, the input weighting parameter $R = 0.1$, and the triggering function $\sigma(x(t)) = \max\{(|x_1(t)| + |x_2(t)|)/7, 1\}$. In this figure, the obtained control input using the SDRE technique, which is equivalent to the proposed event-triggered regulator with $\sigma = 1$, is also depicted. Fig. 2 shows the system state variables where it can be seen that the proposed event-triggered SDRE regulator stabilizes the origin of the system (18). It should be mentioned that the values of the corresponding cost functions J are 2.780 and 4.109 for $\sigma = 1$ and $\sigma((x(t)) = \max\{(|x_1(t)| + |x_2(t)|)/7, 1\}$, respectively. Fig. 3 depicts the corresponding inter-event intervals where each pulse represents the occurrence of an event that leads to a data transmission. In this figure, the magnitude of each pulse specifies the length of time period between that event and the previous one. Fig. 4 shows the triggering condition $\mu(x(t))$, where red circles depict times of sending the control input from the controller to the actuator through the channel. In this example, the proposed method reduces 76% of messages that need to be sent from the controller to the actuator which is equal to an average sampling interval of 0.416 s.

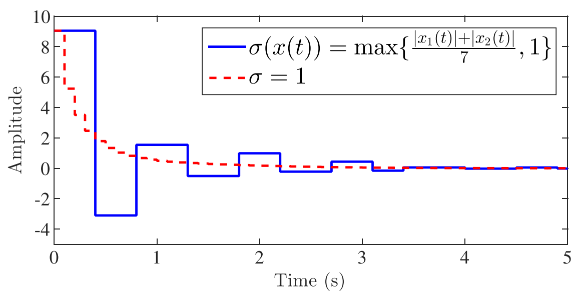

Fig. 1. Graphs of the throttle mass flow $u(t)$. [Color figure can be viewed at wileyonlinelibrary.com]

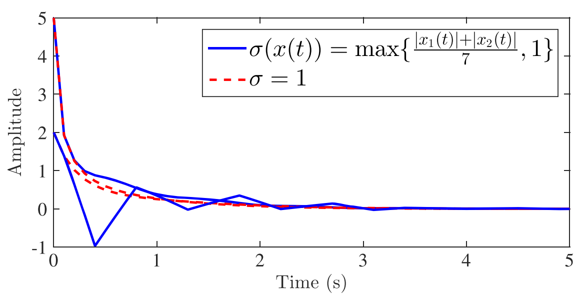

Fig. 2. Evolution of the system state variables $x_1(t)$ and $x_2(t)$. [Color figure can be viewed at wileyonlinelibrary.com]

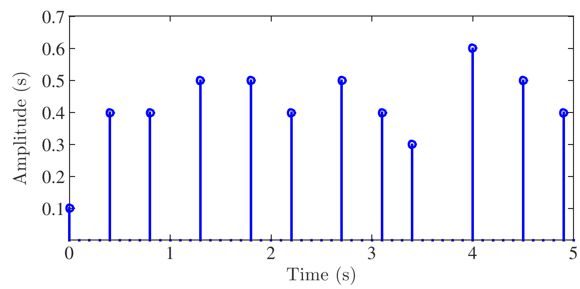

Fig. 3. Inter-event intervals corresponding to the event-triggered tracking controller for Example 1. [Color figure can be viewed at wileyonlinelibrary.com]

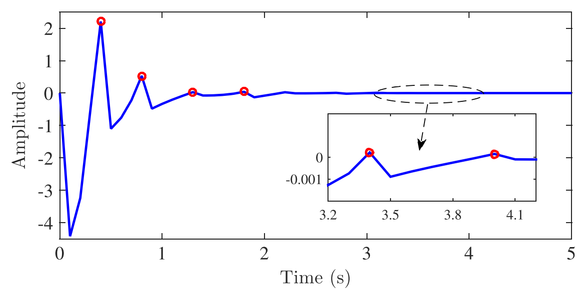

Fig. 4. Graph of $\mu(x(t))$ corresponding to Example 1: Red circles depict times of sending the control input from the SDRE controller to the actuator through the channel. [Color figure can be viewed at wileyonlinelibrary.com]

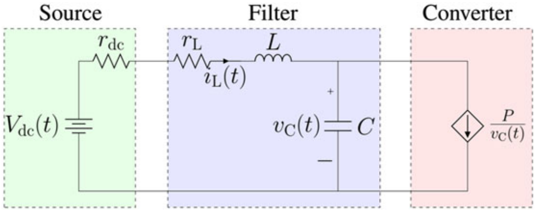

Fig. 5. Schematic diagram of the DC microgrid. [Color figure can be viewed at wileyonlinelibrary.com]

Remark 3. From (10) and the results of this example, one can conclude that selecting a triggering function $\sigma(x(t))$ with smaller values leads to more reduction of sending messages from the controller to the actuator. However, the price of this reduction is a decrease in the performance of the closed-loop system. Indeed, the triggering function $\sigma(x(t))$ can be used to make a trade-off between the communication rate reduction and the value of the corresponding cost function.

4.2 Power converter-based DC microgrid

A DC Microgrid composed of one DC source interfaced with an LC filter supporting a constant power load (CPL), as shown in Fig. 5, is considered. The dynamics of this system is as follows [24]:

$$\begin{aligned}\dot{x}_1(t) =& \frac{r}{L}x_1(t) - \frac{1}{L}x_2(t) + \frac{1}{L}V_{dc}(t),\\ \dot{x}_2(t) =& \frac{1}{C}x_1(t) - \frac{P}{Cx_2(t)},\end{aligned} \tag{19}$$

where $r = r_{dc} + r_L$, $x_1(t)$ and $x_2(t)$ are the inductor current and the capacitor voltage, respectively.

The utilized parameters of the above model are $r = 1.1\ \Omega$, $L = 39.5$ mH, $C = 500\ \mu$F, and $P = 300$ W. The control problem is to find $V_{dc}(t)$, $t \geq 0$ such that the capacitor voltage $x_2(t)$ is set to the constant desired value x_{2d}. To solve this problem using the proposed event-triggered SDRE tracking controller, the following SDC representation of the augmented state $X(t) = e^{-\gamma t}\begin{bmatrix}x_1(t) & x_2(t) & x_{2d}\end{bmatrix}^{\mathrm{T}}$ is used:

$$\dot{X}(t) = \begin{bmatrix}-\gamma + \frac{r}{L} & -\frac{1}{L} & 0\\ \frac{1}{C} & a_{22}(X(t)) & 0\\ 0 & 0 & -\gamma\end{bmatrix}X(t) + \begin{bmatrix}1\\0\\0\end{bmatrix}U(t), \tag{20}$$

where $a_{22}(X(t)) = -\gamma - \frac{P}{Cx_2^2(t)}$ and $U(t) = e^{-\gamma t}V_{dc}(t)$. Paying attention to Theorem 2, the SDC (20) should be point-wise stabilizable to apply the proposed event-triggered tracking controller. One can see that this property is satisfied. On the other hand, by selecting $Q_1 = 1000$, the point-wise detectability condition is also held. Therefore, it can be concluded that the event-triggered SDRE tracking controller can be applied to find a solution of the above trajectory tracking problem based on the SDC representation (20). Since the desired trajectory is generated by a stable system, the discount factor γ can be any positive real numbers. For the initial condition $\begin{bmatrix}x_1(0) & x_2(0)\end{bmatrix} = \begin{bmatrix}1.5 & 190\end{bmatrix}$, $\gamma = 0.01$, the weighting parameters $Q_1 = 1000$ and $R = 0.001$, and the sampling time $T = 50\ \mu$s, the obtained control input by applying the SDRE tracking controller, which equivalent to $\sigma = 1$, is depicted in Fig. 6. In this figure, the obtained control input using the event-triggered SDRE tracking controller for the triggering function $\sigma(x(t)) = \max\{(10 + |x_2(t) - x_{2d}|)/20, 1\}$ is also depicted. Fig. 7 shows the graphs of the corresponding capacitor voltage $x_2(t)$. In this example, the proposed event-triggered method leads to 56 triggering and the communication rate reduction between the controller and the actuator is 72%. Although this reduction is considerable, Fig. 7 shows that the tracking error between $x_2(t)$ and its desired value asymptotically tends to zero. The values of the discounted cost functions J are 23.46 and 29.08 for $\sigma = 1$ and $\sigma(x(t)) = \max\{(10 + |x_2(t) - x_{2d}|)/20, 1\}$, respectively.

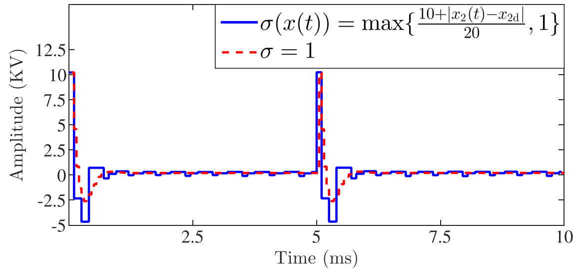

Fig. 6. Graphs of the control input $V_{dc}(t)$. [Color figure can be viewed at wileyonlinelibrary.com]

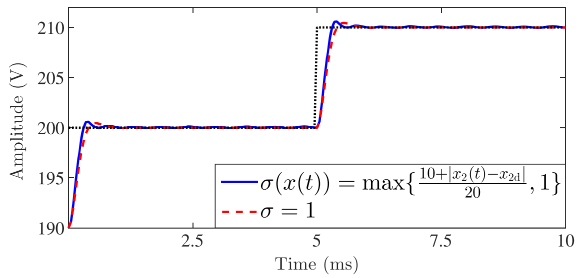

Fig. 7. Graphs of the capacitor voltage $x_2(t)$. [Color figure can be viewed at wileyonlinelibrary.com]

4.3 Vander Pol's oscillator

Consider the dynamical model of a controlled Vander Pol's oscillator as follows [16]

$$\dot{x}(t)=\begin{bmatrix} x_2(t) \\ -x_1(t)+\alpha(1-x_1^2(t))x_2(t) \end{bmatrix}+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t),\quad y(t)=x_1(t). \tag{21}$$

It is assumed that the control objective is to find $u(t)$ such that the system output $y(t)$ tracks the desired sinusoidal trajectory $y_d(t)=\sin(t)$. As a result, the following dynamics are considered for the desired trajectory:

$$\dot{x}_d(t)=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}x_d(t),\quad y_d(t)=\begin{bmatrix} 1 & 0 \end{bmatrix}x_d(t). \tag{22}$$

To apply the proposed event-triggered tracking controller for solving the above problem, the following SDC representation is used in our simulations:

$$\dot{X}(t)=\begin{bmatrix} -\gamma & 1 & 0 & 0 \\ -1 & a_{22}(X(t)) & 0 & 0 \\ 0 & 0 & -\gamma & 1 \\ 0 & 0 & -1 & \gamma \end{bmatrix}X(t)+\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}U(t), \tag{23}$$

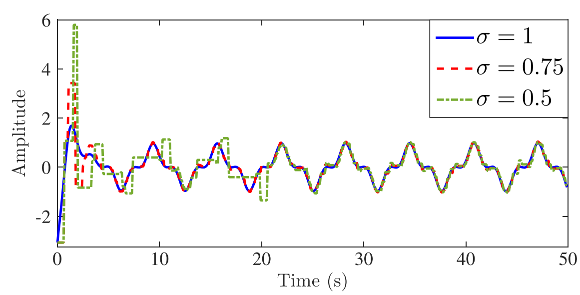

Fig. 8. Graphs of the control input $u(t)$. [Color figure can be viewed at wileyonlinelibrary.com]

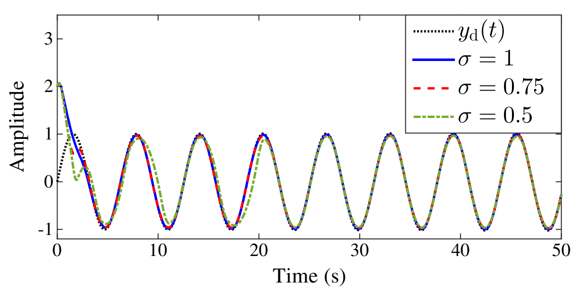

Fig. 9. Graphs of the system output $y(t)$. [Color figure can be viewed at wileyonlinelibrary.com]

Table I. Results of changing the triggering factor σ.

σ	Reduction percentage	Cost function (J)
1	0	38.21
0.75	58.40	41.92
0.5	75.20	46.15

where $X(t)=e^{-\gamma t}\left[x_1(t)\quad x_2(t)\quad x_d^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, $U(t)=e^{-\gamma t}u(t)$, and $a_{22}(X(t)) = \alpha(1-x_1^2(t))-\gamma$. To check the state-dependent stabilizability of the SDC representation (23), let us find its state-dependent controllability matrix

$$\Phi_c=\begin{bmatrix} 0 & 1 & \star & \star \\ 1 & \alpha(1-x_1^2(t))-\gamma & \star & \star \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where $\star$ is used to show the uncalculated elements. One can see that while the first two state variables are point-wise controllable, the states of the trajectory are not. Nevertheless, for any $\gamma > 0$, these states are stabilizable and therefore, the point-wise stabilizability of the SDC representation (23) is guaranteed. Note that the desired trajectory is generated by a marginally stable system and therefore, the discount factor γ can be any positive real numbers. In the following simulations, the tuning gains and the user-defined parameters are selected as $Q_1 = 20$, $R = 0.1$, $\gamma = 0.1$, and the sampling time $T = 0.1$ s. For $x_0 = [2, 1]^{\mathrm{T}}$, $\alpha = 0.9$, and three values of the triggering factor the obtained control inputs of applying the proposed event-triggered tracking controller are depicted in Fig. 8. Fig. 9 shows the evolution of the system outputs $y(t)$, which achieve appropriate tracking of the desired trajectory. The values of the reduction percentages of sending messages from the controller to the actuator and the corresponding cost function (3) are reported in Table I. As it is stated in Remark 3, the percentage of the reduction is decreased by increasing σ which leads to a larger value of the cost function.

V. CONCLUSIONS

By defining two optimal control problems (a quadratic regulation problem and a discounted trajectory tracking problem) and using the SDRE techniques, two event-triggered control methods have been proposed to reduce the information exchange between the controller and the actuator in a nonlinear networked control system. It has been proved that the origin of the closed-loop system under the proposed event-triggered regulator is asymptotically stable provided that the SDC representation of the nonlinear system is point-wise stabilizable and point-wise detectable. For the proposed event-triggered tracking controller, it has been proved that the tracking error between the system output and its desired trajectory asymptotically tends to zero under some conditions on the SDC representation and the discount factor. Numerical simulations have confirmed that the proposed event-triggered controllers are so effective and can be applied in a wide variety of applications.

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