Industrial Boiler-turbine-generator Process Control Using State Dependent Riccati Equation Technique

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Abstract—In this paper, to control the drum pressure, the generated power, and the fluid density in the industrial boiler-turbine-generator unit a tracking controller is designed. Due to the nonlinear model of this process and some constraints in its inputs, the proposed controller is designed based on the State Dependent Riccati Equation (SDRE) method which is a powerful approach in this area. In this system, that is a multiple inputs, multiple outputs (MIMO) system, three control valves (CV) - the fuel CV, the feed water CV, and the steam flow CV - are used as the actuators of the system in order to control three outputs of the system - the drum pressure, the generated power, and the fluid density - respectively. To evaluate the designed controller, it is applied to the system and the performance of the system is assessed for several operating points. The simulation results demonstrate that by means of the designed controller the system has a very good performance.

Keywords: Industrial boiler-turbine-generator, Nonlinear model, State-dependent Riccati equation (SDRE), Tracking controller.

I. Introduction

The boiler-turbine-generator is the main process of producing electricity from fossil fuels in thermal power plants which can be considered as a three-input-three-output process. Several dynamic models of this process have been developed by researchers in last years. In [1]–[3], the dynamic models based on data logs and parameter estimation are presented. In [4], the system identification has been used to obtain the model of the system. In [5], the existing complex nonlinear models has been simplified.

Many control techniques have been used to improve the performance of boiler-turbine-generator unit such as robust control, generic algorithm based control, fuzzy control, adaptive methods and nonlinear control. In [6], a fuzzy $H_\infty$ nonlinear state feedback tracking control has been presented. In this paper, a term of linear matrix inequalities is used to control of unit exactly. The model predictive control methods have been also applied to control the unit. In comparison with the $H_\infty$ controller, these approaches have improved the system performance, while the control effort has decreased [7]. In [8], a fuzzy model of a boiler-turbine-generator system based on a new fuzzy c-regression model clustering algorithm has been proposed. A fuzzy based control systems for thermal power plants has been presented in [9], [10]. In [11], a neuro-fuzzy modeling and a PI control of a steam-boiler system have been employed. In [12], a method based on the back stepping nonlinear adaptive control has been proposed. A linear control method has been used in [13]. In this system, the linear controllers can be useful, but in a limited range of variations. Also, the robust controllers based on the $H_\infty$ loop-shaping techniques have a good performance, however, they encounter some problems in dealing with the control constrains [14]. In [15], the boiler-turbine unit has been modeled by a linear uncertain system and a robust decentralized controller has been designed for this linearized model using the equivalent subsystem method.

To design a controller for the real system, there are some challenges. Firstly, the practical systems in the real world are generally nonlinear. If the parameters of the system do not widely change around the operating point, the linearized model of the system can be used to design a linear controller. In contrast, in wide variations in the parameters, using the nonlinear design methods is inevitable. The second challenge is the limitation of the control signals, which can drive the system to instability. In this way, presenting a method for nonlinear systems in such a way that these restrictions to be considered in the steps of the design is so useful for practical systems.

In this paper, therefore, the SDRE method is used to control a multi variable nonlinear model of a boiler-turbine-generator unit. The SDRE technique is a powerful approach to design the nonlinear controllers which is able to address the saturation of the control signals. Due to these abilities, this method is used to solve a lot of nonlinear control problems by researchers [16]–[18]. The simulation results show that the objectives are achieved, and closed loop system is able to move from one operating point to another smoothly. Also, they demonstrate that the constraints on the actuators are satisfied by limiting the control signals without any effects on the system performance.

The rest of this paper is organized as follows. Section II discusses the system description. Section III is a review of the SDRE method. The simulation results are presented in Section IV. Finally, the conclusion appears in Sections V.
II. System Description

The process of a boiler-turbine-generator is described as follow. A boiler consists of several components namely a water pump, drums, tubes, an economizer, a super heater, and a burner. The economizer main duty is the principle of heat transfer. As, the heat transfer usually takes place from high temperature to low temperature, and in the case of boilers, flue gases or exhaust from the boiler outlet are at high temperature and the feed water that needs to be preheated is at low temperature. So, the heat transfer from the gases to the feed water, and in this way, the feed water temperature increase (preheat). successively, in the water-tube boiler, the preheated water is fed into the steam drum by means of the water pump. After it, the water is heated and change to a mix of the steam and water in tubes, and this mixture enters the steam drum. Next, the steam is separated from the water and flows into the super heater. Afterwards, the steam is more heated, and is then fed into the steam turbine. A steam turbine is powered by the energy of the hot steam. Like a wind turbine, it has spinning blades that turn when the high-pressure steam blows past them, and in this way, the rotor of a synchronous generator which is coupled to the steam turbine commence to turn. The synchronous generator is an electrical machine which converts the mechanical power from a turbine into the AC electrical power at a particular voltage and frequency. Consequently, in order to generate more active power, it is necessary to increase steam inlet to the turbine. Therefore, it requires to rise the amount of fuel and water entering to the boiler so, more fossil fuel has to be burned.

The model used in this paper is a model of a steam power station with 160 MW rated power in Malmo, Sweden presented in [1]. The model is a third order nonlinear dynamics stated by Aström and Bell [2]. This model is a version of second order nonlinear dynamics that involves pressure and power dynamics [2]. The nonlinear dynamics of the system are in the form

\[
\dot{x} = f(x, u) \\
y = g(x, u)
\]

where \( f \) and \( g \) are the state and the output nonlinear equations, respectively. The nonlinear dynamics of the steam-boiler-generator unit is given by

\[
\begin{align*}
\dot{x}_1 &= -0.0018u_2x_1^{9/8} + 0.9u_1 - 0.15u_3 \\
\dot{x}_2 &= (0.073u_2 - 0.016)x_1^{9/8} - 0.1x_2 \\
\dot{x}_3 &= (141u_3 - (1.1u_2 - 0.19)x_1)/85
\end{align*}
\]

where the state variables \( x_1, x_2, \) and \( x_3 \) are the drum pressure \((\text{kg/cm}^2)\), the generated power \((\text{MW})\), and the fluid density \((\text{kg/cm}^3)\), respectively. The inputs \( u_1, u_2, \) and \( u_3 \) are the valve position of the fuel CV, the steam flow CV, and the feed water CV, respectively. The output equations are as follows:

\[
\begin{align*}
y_1 &= x_1 \\
y_2 &= x_2 \\
y_3 &= x_3.
\end{align*}
\]

It is noted worthy that in some papers, the drum level is considered as the third output variable, whereas it is obvious that by controlling all the state variables, the drum level is also controlled.

Due to actuator saturation limitations, control inputs are limited to \([0, 1]\) or

\[
0 \leq u_i \leq 1
\]

III. Review of the SDRE

The state-dependent Riccati equation (SDRE) method has been provided at 1962 to solve the regulation problem of the nonlinear systems and has been known as an effective regular conclusion tool and easy to implement. Provide dynamics of a nonlinear system as the state dependent linear system, called pseudo linearization or extended linearization, is the main idea of the SDRE method. In different papers, many useful properties has been mentioned about this method including: a) the intuitive and simple concepts directly adopt the well-established linear quadratic regulator design at every nonzero state; b) the design could directly affect system performance with reasonable outcome, by tuning the state and control weighting functions, which formulates a specific design-oriented performance index. For example, the designer may modulate the weighting of the system state to speed up the response, although at the expense of increased control effort; c) the scheme possesses an extra design degree of freedom, arising from the non-unique state-dependent coefficient (SDC) matrix representation for the nonlinear drift term, which could be utilized to improve the overall system performance; d) the approach preserves the essential system nonlinear because they do not truncate any nonlinear term, which is of great importance from practical viewpoints, particularly when the system dynamics is complicated; and e) ability of considering the saturated input signal and stable against system uncertainty. SDRE method has extended based on the integrate approach to tracking the reference signal [18].

A. Tracking controller through the SDRE method

Consider a general presentation for the nonlinear dynamical system as follows:

\[
\begin{align*}
\dot{x}(t) &= f(x(t)) + b(x(t))u(t), \quad x(0) = x_0, \\
y(t) &= h(x(t))
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, \) and \( y(t) \in \mathbb{R}^p \) are the state vector, the control input, and the system output respectively. Assume that \( f : \mathbb{R}^n \to \mathbb{R}^n, b : \mathbb{R}^n \to \mathbb{R}^{n \times m}, \) and \( h : \mathbb{R}^n \to \mathbb{R}^p \) are smooth functions and for all \( x \in \mathbb{R}^n \)
we have \( f(0) = h(0) = 0 \) and \( b(x) \neq 0 \) The desired trajectory - which the output of the system (6) should follow that - can be presented as the following general nonlinear dynamics:

\[
\begin{align*}
\dot{x}_d(t) &= f_d(x_d(t)), \quad x_d(0) = x_{d0}, \\
y_d(t) &= h_d(x_d(t)),
\end{align*}
\]  

(5)

where \( x_d(t) \in \mathbb{R}^{n_d} \) is the state and \( y_d(t) \in \mathbb{R}^p \) is the output of this system (7) and functions \( f_d : \mathbb{R}^{n_d} \to \mathbb{R}^{n_d} \) and \( h_d : \mathbb{R}^{n_d} \to \mathbb{R}^p \) are considered as smooth function and \( f_d(0) = h_d(0) = 0 \). It is not worthy that many applicable desired trajectories namely steps, sinusoidal signals, etc can be generated by (7). Therefore, we want to find the control input \( u(t) \), \( t \geq 0 \) in such a way that the system output \( y(t) \), \( t \geq 0 \) follows the desired trajectory \( y_d(t) \), \( t \geq 0 \) and simultaneously the following discounted cost function is minimized:

\[
J(x_0, u(t), y_d(t)) = \int_0^\infty e^{-\gamma t} \left( (y(t) - y_d(t))^T Q_1 (y(t) - y_d(t)) + \dot{u}^T(t) R \dot{u}(t) \right) dt
\]  

(6)

where \( \gamma > 0 \) is the discount factor. \( Q_1 \) is a positive-semidefinite symmetric matrix, and \( R \) is a positive-definite symmetric matrix with appropriate dimensions. Solving the above problem by means of the Bellman’s principle of optimality needs to solve an Hamilton-Jacobi-Bellman (HJB) equation which is too difficult or in some cases impossible to be analytically solved. Hence, finding approximate solutions of the problem is our desired. To achieve this goal consider the following SDC forms:

\[
\begin{align*}
f(x(t)) &= F(x(t))x(t), \\
f_d(x_d(t)) &= F_d(x_d(t))x_d(t), \\
h(x(t)) &= H(x(t))x(t), \\
h_d(x_d(t)) &= H_d(x_d(t))x_d(t),
\end{align*}
\]  

(7)

where \( F : \mathbb{R}^n \to \mathbb{R}^{n \times n} \), \( H : \mathbb{R}^n \to \mathbb{R}^{p \times n} \), \( F_d : \mathbb{R}^{n_d} \to \mathbb{R}^{n_d \times n_d} \) and \( H_d : \mathbb{R}^{n_d} \to \mathbb{R}^{p \times n_d} \) are four matrix-valued functions.

In order to convert the above tracker problem to a standard regulation problem, consider \( X(t) = e^{-\gamma t} \left[ x^T(t) \quad x_d^T(t) \right]^T \in \mathbb{R}^{n+n_d} \) and \( U(t) = e^{-\gamma t} u(t) \) so, the cost function (6) can be rewritten as follows:

\[
J(X_0, U(t)) = \int_0^\infty \left( X^T(t)Q(e^{-\gamma t}X(t))X(t) + U^T(t)R \dot{U}(t) \right) dt,
\]  

(8)

where

\[
Q(e^{-\gamma t}X(t)) = \begin{bmatrix} H(x(t)) & -H_d(x_d(t)) \\ H_d(x_d(t)) & -H_d(x_d(t)) \end{bmatrix}^T Q_1 \begin{bmatrix} H(x(t)) & -H_d(x_d(t)) \end{bmatrix}.
\]

Hence, the nonlinear dynamics of \( X(t) \) is obtained as:

\[
\dot{X}(t) = -\gamma X(t) + e^{-\gamma t} \left[ \dot{x}^T(t) \quad \dot{x_d}^T(t) \right]^T.
\]

Now, using the equations of \( \dot{x}(t) \) and \( \dot{x}_d(t) \) from (4) and (5), respectively, we can write a pseudo linearized dynamics for the augmented system as:

\[
\dot{X}(t) = \left( -\gamma I + \begin{bmatrix} F(x(t)) & 0 \\ 0 & F_d(x_d(t)) \end{bmatrix} \right) X(t) + \begin{bmatrix} b(x(t)) \\ 0 \end{bmatrix} U(t) = A(e^{-\gamma t}X(t))X(t) + B(e^{-\gamma t}X(t))U(t),
\]

(9)

where \( I \) and 0 are the identity and zero matrices with appropriate dimensions, respectively. In [18], the solution of this problem has been presented as follows:

\[
U(t) = -R^{-1}B^T(e^{-\gamma t}X(t))P(e^{-\gamma t}X(t))X(t)
\]

(10)

where the matrix \( P(e^{-\gamma t}X(t)) \) is the unique symmetric positive-definite solution of the following state-dependent algebraic Riccati equation

\[
A^T(e^{-\gamma t}X(t))P(e^{-\gamma t}X(t)) + P(e^{-\gamma t}X(t))A(e^{-\gamma t}X(t))
\]

\[
- P(e^{-\gamma t}X(t))B(e^{-\gamma t}X(t))R^{-1}B^T(e^{-\gamma t}X(t))P(e^{-\gamma t}X(t)) = 0.
\]

(11)

It should be mentioned the SDRE equation (11) has a unique symmetric positive-definite solution \( P(e^{-\gamma t}X(t)) \) if the triple \((A(e^{-\gamma t}X(t)), B(e^{-\gamma t}X(t)), Q^{1/2}(e^{-\gamma t}X(t)))\) is point-wise stabilizable and detectable.

As mentioned, one of the constraints of the real system is the inputs saturation. As the SDC method has this ability to address the input restriction in its design steps [19], it is one of the interesting technique to deal with this constraint. To attain this aim, the following dynamics for the control input \( \dot{u}(t) \) is considered by defining an auxiliary input \( \ddot{u}(t) \):

\[
\ddot{u}(t) = \ddot{u}(t).
\]

(12)

Therefore, to consider the inputs limitation, (4), (5), and (12) are augmented. In this way, an SDC representation can be defined for the new state \( X(t) = e^{-\gamma t} \left[ x^T(t) \quad x_d^T(t) \quad u^T(t) \right]^T \):

\[
\dot{X}(t) = \left( -\gamma I + \begin{bmatrix} F(x(t)) & 0 & b(x(t))S(u(t)) \\ 0 & F_d(x_d(t)) & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) X(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ddot{U}(t),
\]

(13)

where \( \ddot{U}(t) = e^{-\gamma t} \ddot{u}(t) \) and \( S(u(t)) \) is as follows:

\[
S(u(t)) = \begin{cases} \text{sat}(u(t), u_a), & u(t) \neq 0; \\ 1, & u(t) = 0, \end{cases}
\]

and in this equation, function \( \text{sat}(u(t), u_a) \) is described as:

\[
\text{sat}(u(t), u_a) = \begin{cases} -u_a, & u(t) \leq -u_a; \\ u(t), & |u(t)| < u_a; \\ u_a, & u(t) \geq u_a; \end{cases}
\]
IV. Simulation Results

Due to proper performance of the boiler-turbine-generator unit, the control system should satisfy some requirements according to the varying operating conditions and load demands. The closed-loop system must be able to move from one operating point to another smoothly. In addition, several constraints on the actuators must be satisfied by the control signals. These constraints are the saturation rate for the CVs of the fuel, the steam and the feed water. Table I gives some typical operating points of the Bell and Astrom model [1].

Table I: Typical operating points of the Bell and Astrom model [1]

<table>
<thead>
<tr>
<th>States &amp; Inputs</th>
<th>Operating points</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>( 15.6 )</td>
<td>86.4</td>
<td>99.2</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( 15.27 )</td>
<td>36.65</td>
<td>50.52</td>
<td>66.65</td>
<td></td>
</tr>
<tr>
<td>( x_3 )</td>
<td>( 299.6 )</td>
<td>342.4</td>
<td>385.2</td>
<td>428</td>
<td></td>
</tr>
<tr>
<td>( u_1 )</td>
<td>0.156</td>
<td>0.209</td>
<td>0.274</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>( u_2 )</td>
<td>0.483</td>
<td>0.552</td>
<td>0.621</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>( u_3 )</td>
<td>0.183</td>
<td>0.256</td>
<td>0.342</td>
<td>0.433</td>
<td></td>
</tr>
</tbody>
</table>

In this section, we apply the SDRE tracking controller to the system (1) in order to track several set points. As mentioned, the outputs of the system are drum pressure (kg/cm²), the generated power (MW), and the fluid density (kg/cm³). That is why, the most important output variable is the generator power output. When changing the operating point, the output power must be changed without fluctuation or oscillation. The input and output variables of the process are shown in Fig. 1.

Fig. 1: Input-output variables of the boiler-turbine-generator process

In this system, the control inputs must be limited to the specified range \([0, 1]\). Hence, the input signals \( u_1, u_2 \) and \( u_3 \) are replaced with \( \text{Sat}(u_1, 0.5) + 0.5, \text{Sat}(u_2, 0.5) + 0.5 \) and \( \text{Sat}(u_3, 0.5) + 0.5 \), respectively and the value of \( u_6 \) is considered equal to 0.5.

To apply the SDRE tracking controller explained in the former section, the pseudo linearised form of the boiler-turbine-generator unit and the desired trajectories are required. According to (1), one of the infinite number of the boiler-turbine-generator pseudo linearised forms is presented as follows:

\[
\dot{x}(t) = A(x)x(t) + Bu(t) \\
y(t) = Cx(t),
\]

where

\[
A(x) = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & 0.1 & 0 \\ a_{31} & 0 & 0 \end{bmatrix}
\]

\[
B = \begin{bmatrix} 0.9 & 0 & 0.15 \\ 0 & 0 & 0 \\ 1.66 & 0 & 0 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

and

\[
a_{11} = -0.0018u_2x_1^{1/8} \\
a_{21} = (0.073u_2 - 0.016)x_1^{1/8} \\
a_{31} = -(1.1u_2 - 0.19)/85.
\]

The desired trajectory in this system is described as follows:

\[
\dot{x}_d(t) = f_d(x_d(t)) = 0, \quad x_d(0) = x_{d0},
\]

\[
y_d(t) = h_d(x_d(t)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x_d(t)
\]

\[
x_d(t) = [x_{d1}, x_{d2}, x_{d3}]^T,
\]

where \( x_{d1}, x_{d2}, \) and \( x_{d3} \) are determined based on the operating points.

One of the most important advantages of using the SDRE method is the possibility of changing response speed to include the limitations of the control signals. When there is a limitation on the speed of control signals variation, these limitations can be met by changing the weighting matrices \( Q \) and \( R \). The amount of these matrices are considered as follows:

\[
Q = \text{diag}[1.3396 \times 10^{-4}, 7.444 \times 10^{-4}, 8.5197 \times 10^{-5}, 0.01, 0.01, 4.44 \times 10^{-4}, 0.01, 0.01, 0.01]
\]

\[
R = \text{diag}[15000, 1500, 1500]
\]

The value of weighting matrices \( Q \) and \( R \) depend on the control goals and the properties of the system under study. Basically, choosing a large value for \( R \) led to stabilize the system with less energy. On the other hand, choosing a small value for \( R \) means you don’t want to penalize the control signal. For systems with constraint on the control signals, \( R \) should be large. If you choose a large value for \( Q \) means you try to stabilize the system with the least possible changes in the states and vice versa.

Fig. 2 shows the time responses of the system outputs after applying the SDRE controller. Tracking of operating points is done correctly and the system goes from one operating point to another without any problems. There are no steady state error or sudden change in outputs.
Fig. 2: Time response of output variables, using SDRE method for operating points

Fig. 3 shows the time responses of the input control signals after applying the proposed method. As it can be seen, by applying the SDRE controller, the limitations of the input signals are well respected. And the control signals have no unacceptable changes.

Fig. 3: Time response of input variables, using the SDRE approach for several operating points

V. Conclusion

The boiler-turbine-generator process is one of the main ways to generate electricity in the world. In this paper, due to the properties of the SDRE method, an SDRE tracking controller is designed for the industrial boiler-turbine-generator. At first, a multi variable nonlinear model of a boiler-turbine-generator unit with several input constraints has been used to describe the system. After reviewing the SDRE method, the SDRE tracking controller has been applied to the system and a set point tracking problem has been defined due to several operating points. The SDRE method is able to track all operating points with a good performance, while respecting the restrictions of the control signals did not cause the slightest disruption to the system performance.

References


