Robust H2 and H∞ controller design for DC position motor control under uncertainties

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***Abstract*— DC motors are often used in a variety of industrial applications. These motors are easily controllable and their position can be adjusted with low cost. However, one of the problems that hampers the motor control, despite the proper initial design, is the change of control system parameters. Indeed, each control system should be robust for a pre-defined range of internal and external changes. In this paper, an attempt is made to discuss the mathematical model of the motor and the associated nonlinear factors. Afterwards, a robust control scheme, which considers torque as an external perturbation signal and nonlinear dynamics in the motor, is proposed to control position of DC. To this end, H2 and H∞ methods are combined.**

***Index Terms-* DC motors, position, robust control, uncertainty.**

NOMENCLATURE

L	Armature inductance
I	Armature current
k_e	Motor electrical constant
R	Armature resistance
V	Input voltage, taken as control input
J	Motor inertia
ω	Rotor rotation speed
k_d	mechanical dumping constant
Γ_d	Disturbance torque
X	Motor state variable vector
U	Control input
F(x), G(x)	Smooth vector field inR^3
i_a	Armature current
F	Friction
T_L	Load torque
T_{wu}	Transfer matrix from w to u
W_1 and W_2	Suitable weight functions
S	Sensitivity function

I. INTRODUCTION

Dc motors are extensively used in control systems due to easy position or speed control and reliability for a wide range of operating conditions. Among the uses of DC motors are their use in robots, electric drills, saws, launch and so on. DC motors with separate excitation play a valuable role because of the ability to independently control the flux and torque from a speed and position control point of view[1]. DC motors are commonly modeled as linear systems and then linear control approaches are implemented but due to changes of the load and motor dynamics and nonlinearities introduced by the armature reaction most of them have unsatisfactory performance. The change of under control system parameters is one of the issues of motor controllers design and the solution is to use robust controls.

One of the controls that are applied to the motors is position control of DC motors which will be discussed in this paper.

The purpose of the controller design is to increase the stability and reduce the time required to achieve the desired state against disturbances in a process. The proposed methods for controlling the position of DC motors are generally divided into three categories:

1. Classic methods such as using PI controllers and PID[2, 3]
2. Modern methods such as robust, adaptive and optimal methods[4]
3. Intelligent methods such as applying fuzzy theory and neural control[5]

As mentioned system's robustness is stability and performance against perturbation. Robust control is actually control in the presence of uncertainties.

The purpose of [6]is to design a DC motor position controller using PID. The parameters of the proposed controller are determined by genetic algorithm and compare the obtained results with the controller design by Ziegler and Nichols method.

In [7]SMC is used to solve time-varying parameter problems in robust control of DC motor position due to lower settling time and better stability than classical methods, and also fuzzy logic is used for more SMC flexibility.

In [8] To deal with uncertainties Super- Twisting Extended State Observer- Super Twisting Algorithm (STESO-STA) controller for Dc position motor control is presented which avoid an over-estimation of control gains. In this paper for tracking disturbances in finite time a disturbance observer is designed.

In [9] for fixed field-DC Motor Speed Control a robust controller is designed which in comparison to PID controller offset error, peak overshoot, settling time and disturbance rejection capability is improved.

In [10]for compensating cogging torque and model uncertainty an adaptive robust triple-step control method is presented. In this paper the disturbance, load torque, motor and model errors are considered as model uncertainty. The robust stability is analyzed via a Lyapunov method.

The main contribution of this paper is presenting a robust control strategy for position tracking of a DC motor with unknown system nonlinearities. The proposed control approach can compensate for changes which occur online to the system's dynamics.

The rest of the paper is organized as follows. Section II introduced DC motor modeling, definition of optimization problem of H_∞ and H_2 and how to select the weighting functions. In Section III simulation and results are presented and illustrate the effectiveness of the proposed controller finally a concluding discussion is given in section IV.

II. PROBLEM FORMULATION

A. The DC motor model

The system considered in this paper is a separately excited DC motor. Fig. 1 shows The DC motor model of the separately excited as follows:

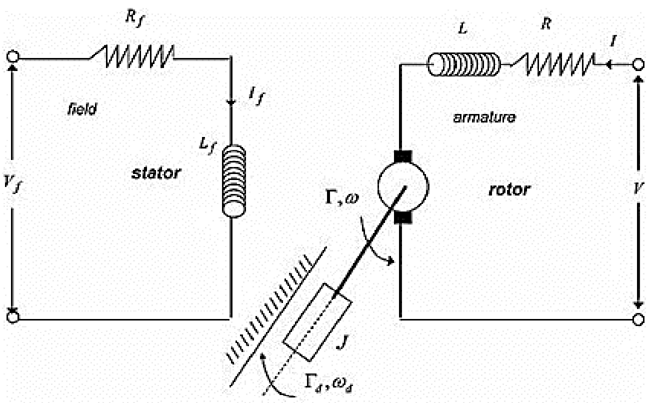

Figure 1. The DC motor model of the separately excited [11]

Kirchhoff's law yields the equation of the motor and the mechanics of rotation The DC motor model obtained as follow:

$$L\dot{I} = -k_e\omega - RI + V \tag{1}$$

$$J\dot{\omega} = k_e I - k_d\omega - \Gamma_d \tag{2}$$

B. Position tracking

For designing the tracking controller dynamic model given in [12] is used. Usually in the DC motor model the effect of armature reaction neglected or assume that compensating windings remove this effect so it is considered to be linear._Introducing the armature reaction leads to a nonlinear system and in that case a nonlinear model may be appropriate._The state space form of the model can be written as:

$$\dot{x} = f(x) + g(x)\mathrm{u} \tag{3}$$

Where the motor state variable vector x denote as:

$$x = [x_1, x_2, x_3]^T = [\theta, \dot{\theta}, i_\alpha] \tag{4}$$

Where θ is the rotor position (radians), $\dot{\theta}$ the rotor speed (rad/s) and u $=V_t$ is the input.

The functions f(x) and g(x) are defined as follows:

$$f(x) = \begin{pmatrix} x_2 \\ k_1x_2 + k_2x_3 + k_3x_3^2 + k_4T_1 \\ k_5x_2 + k_6x_2x_3 + k_7x_3 \end{pmatrix} \tag{5}$$

$$g(x) = \begin{pmatrix} 0 \\ 0 \\ k_8 \end{pmatrix}$$

Where

$$k_1 = -\frac{F}{J}, k_2 = \frac{A}{J}, k_3 = \frac{B}{J}, k_4 = -\frac{1}{J}, k_5 = -\frac{A}{L},$$

$$k_6 = -\frac{B}{L}, k_7 = -\frac{R}{L}, k_8 = -\frac{1}{L}$$

The state-space equation of the DC motor with choosing motor position to be the system output can be rewritten as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= k_1x_2 + k_2x_3 + k_3x_3^2 + k_4T_1 \\ \dot{x}_3 &= k_5x_2 + k_6x_2x_3 + k_7x_3 + k_8u \\ y &= x_1 \end{aligned} \tag{6}$$

Using the input-output linearization technique, the input–output relation can be written as:

$$\begin{aligned} \ddot{x}_2 &= k_1\dot{x}_2 + k_2\dot{x}_3 + 2k_3x_3\dot{x}_3 \\ \ddot{x}_2 &= k_1\dot{x}_2 + k_2\dot{x}_3 + 2k_3k_5x_2x_3 + 2k_3k_6x_2x_3^2 \\ &\quad + 2k_3k_7x_3^2 + 2k_3k_8x_3u \end{aligned} \tag{7}$$

Hence input–output relation can be rewritten as

$$\widetilde{x_2} = \tilde{f}(x) + \tilde{g}(x) \tag{8}$$

C. Robust controller design

To design a controller for tracking the DC position motor, the following model is considered [13]:

$$\begin{gathered}\dot{x}_1 = x_2 \\ \dot{x}_2 = k_1 x_2 + k_2 x_3 + \underline{k_3 x_3^2} + \underline{k_4 T_L} \\ \dot{x}_3 = k_5 x_2 + \underline{k_6 x_2 x_3} + k_7 x_3 + k_8 u \\ y = x_1\end{gathered} \tag{9}$$

The underlying sentences in (9) are considered as uncertainties and unknown sentences. T_L is an external perturbation and the other two sentences are nonlinear sentences in motor dynamics. Block diagram of the control scheme.is shown in Figure 2:

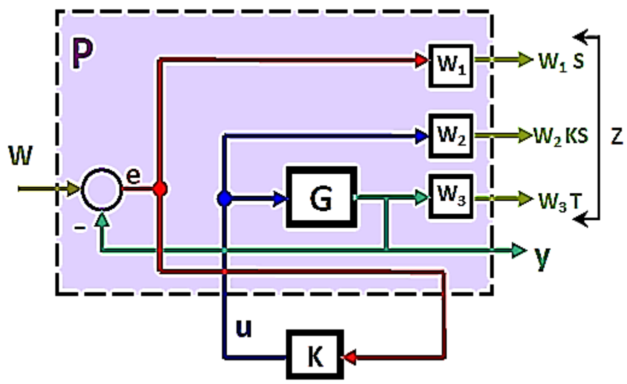

Figure 2. Block diagram of the control scheme

The weight function W1 and W2 have the task of proper tracking and reducing control effort, respectively. The weight function W3 is also used to reduce the effect of the perturbation signal.

D. Standard H_∞ AND H_2 problem

The standard problem of H_∞ and H_2 use a specific representation of the system called standard representation. Figure 3 shows the standard representation of an inaccurate feedback control system. The components shown in this figure are as follows:

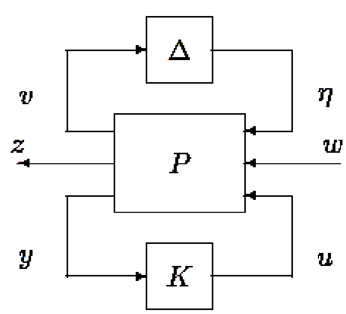

Figure 3. Standard representation of an inaccurate feedback control system[14].

Where P is the interconnection matrix, K is the controllerΔ is the set of all possible uncertainty these uncertainties can include system modeling errors, system parameter changes, system linearization errors, and so on. , w is a vector signal including noises, disturbances, and reference signals, z is a vector signal including all controlled signals and tracking errors, u is the control signal, and y is the measurement. The stability characteristics and optimal performance of the closed loop system are realized by considering (8).

$$\left\| \begin{matrix} W_1 S \\ W_2 T_{wu} \end{matrix} \right\|_\infty < 1 \tag{10}$$

Where

$$\begin{cases} S = (1 + GK)^{-1} \\ T_{wu} = K(1 + GK)^{-1} \end{cases} \tag{11}$$

The H_2 controller is stable if and only if it is internally stable and γ> 0 exists for:

$$\|T_{zw}(jw)\|_2 < \gamma \tag{12}$$

E. Weighting Functions

The most important step in the design of H controllers is to determine the weight functions that have not yet been comprehensively developed. Currently, only on the basis of the designer's experience and skill and with regard to specific control purposes such as closed loop system bandwidth, low frequency disturbances damping, and minimization of adverse effects of modeling errors and … preliminary weighting functions are obtained and by repeating several trial and error steps, testing stability and efficiency, initial functions modified and final weight functions are obtained [15]. In the design, weighting functions are selected as follows:

$$\begin{gathered} W_1 = \frac{s + 0.35}{3s + 0.0035} \\ W_2 = 2 \\ W_3 = \frac{s + 0.35}{0.01s + 350} \end{gathered} \tag{13}$$

These weight functions are selected by trial and error and are constant for both H_∞ and H_2control methods.

III. SIMULATION AND RESULTS

Fig. 4 shows a block diagram of DC motor closed loop system with controller K (S).

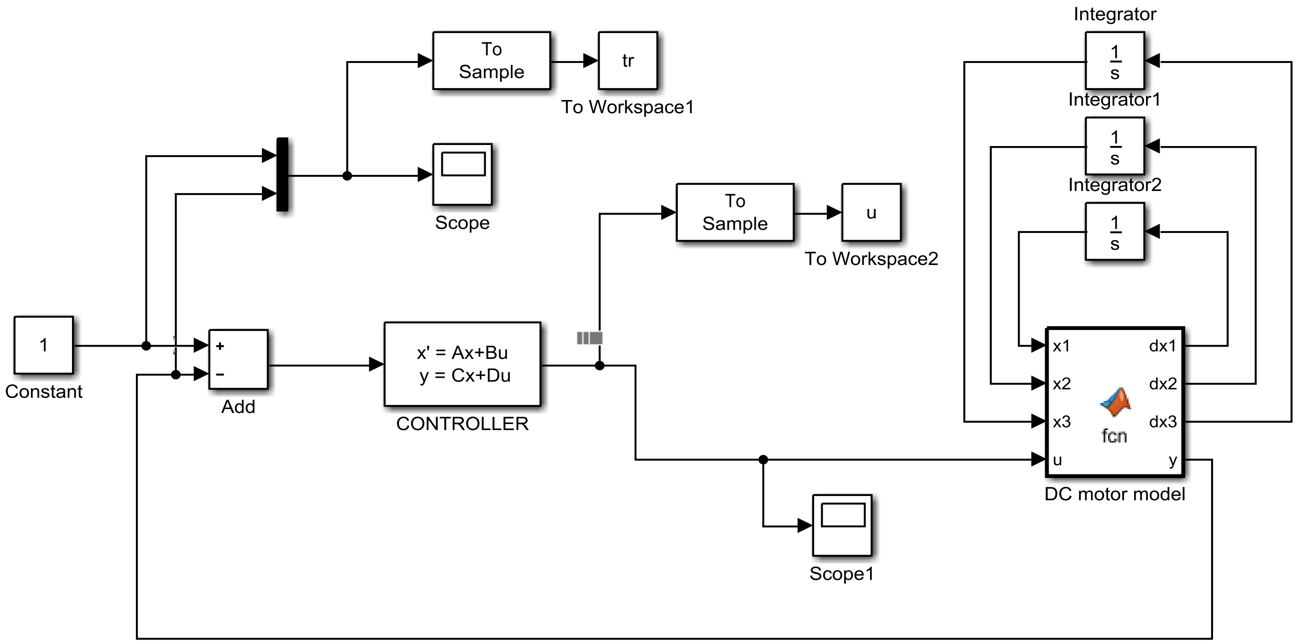

Figure 4. DC motor closed loop system with controller K (S).

The parameters of the simulated DC motor are shown in Table 1 [13].

TABLE I. The parameters of the simulated DC motor

Quantity	value
v_t	110
P_{out}	2.5 hp
n_{sync}	1800 rpm
A	0.57
R	1
J	0.093 $kg.m^2$
F	0.008$N.m.s/rad$
a_1	0.00028
L_a	46 Mh
B	-0.01

The controller designed based on H_∞ method is of 5th order follows:

$$G = \frac{-11.35s^4 - 367s^3 - 3484s^2 - 9258s + 77.84}{s^5 + 63.3s^4 + 1404s^3 + 1.349e04s^2 + 5.063e04s + 88.56}$$

Also for The controller designed based on H_2 method is of 5th order follows:

$$G = \frac{-0.1735s^4 - 5.607s^3 - 53.25s^2 - 141.7s - 0.076}{s^5 + 35.48s^4 + 413.9s^3 + 1866s^2 + 1249s + 2.181}$$

Fig. 5 and 6 show the system simulation results, with several uncertainties for the designed controller based on H_∞ and H_2 methods, respectively.

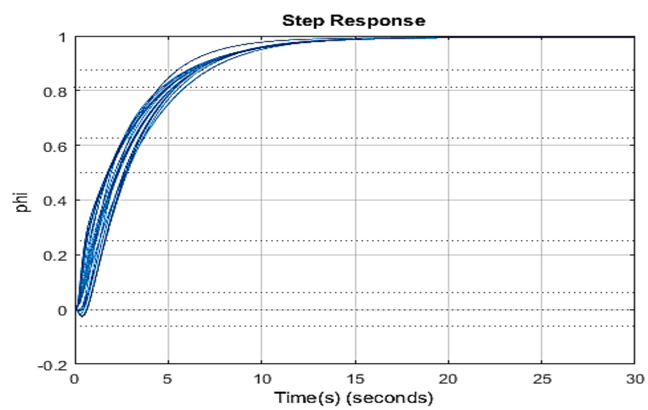

Figure 5. The system simulation results, with several uncertainties for the designed controller based on H_∞ methods.

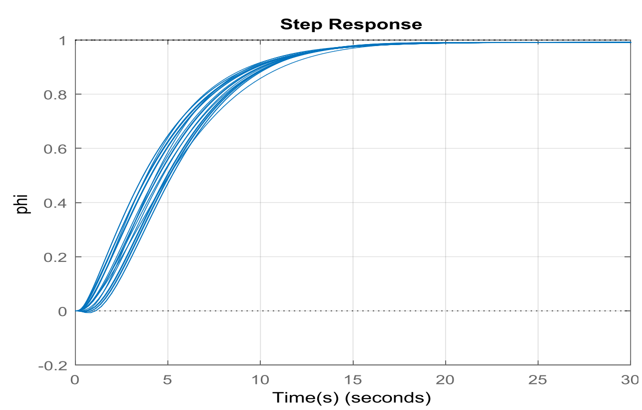

Figure 6. The system simulation results, with several uncertainties for the designed controller based on H_2 methods.

It is found that for all processes associated with uncertainty the system response is optimally converged and stable and results in system robustness.

Fig. 7 and 8 illustrate whether the criterion of the weighted functions H_∞ and H_2 are observed in the closed-loop system or not.

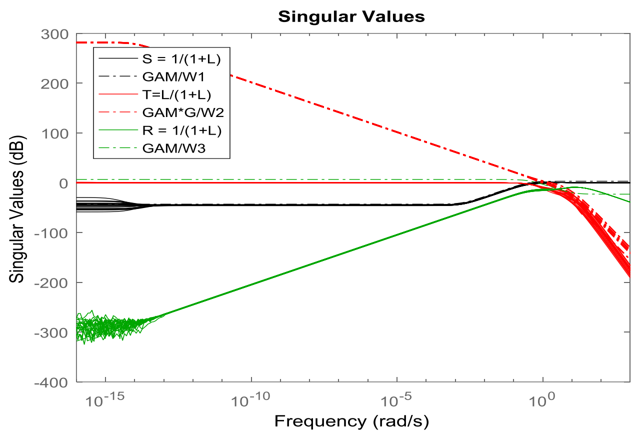

Figure 7. The criterion of the weighted functions H_∞

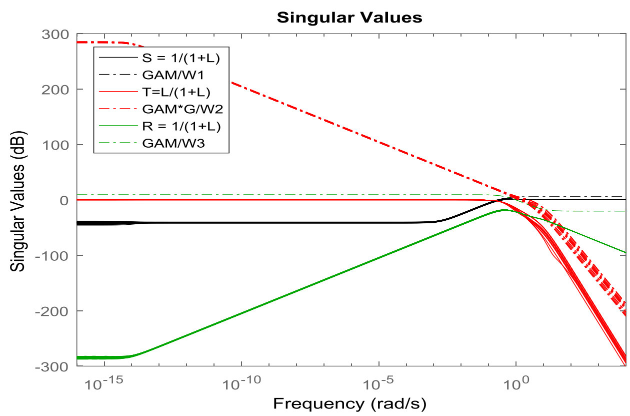

Figure 8. The criterion of the weighted functions H_2

It is observed that the criterion of weight functions are met in the closed loop system designed.Fig. 9 and 10 show the simulation results of outputs of H_∞ and H_2 .

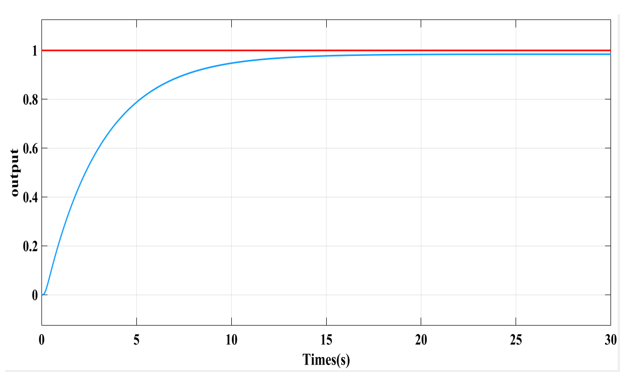

Figure 9. The simulation results of outputs of H_∞

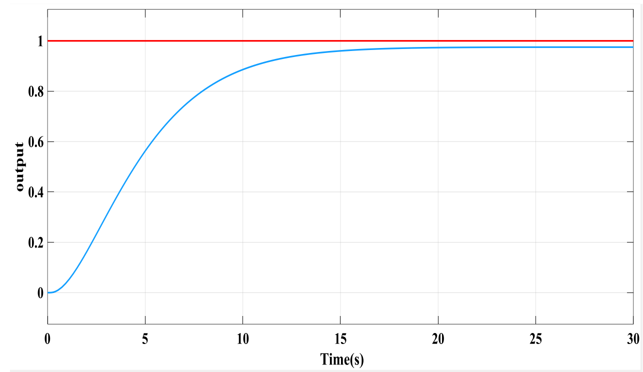

Figure 10. The simulation results of outputs of H_2

Fig. 11 and 12 show the outputs of the controllers H_∞ and H_2 .

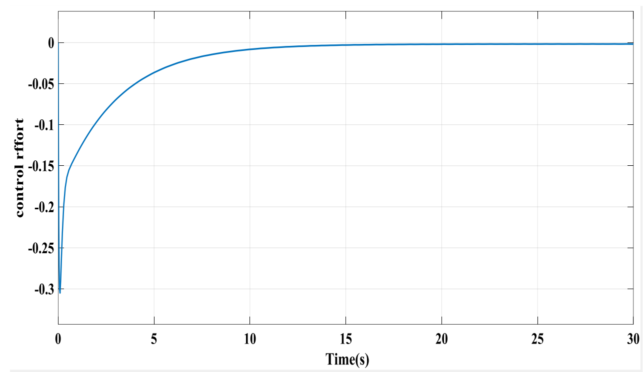

Figure 11. Show the outputs of the controllers H_∞

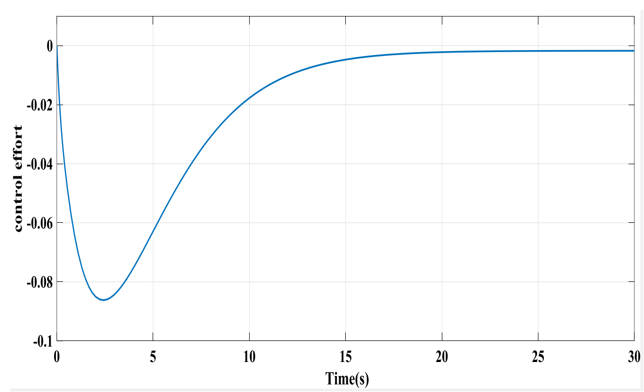

Figure 12. Show the outputs of the controllers H_2

IV. CONCLUSION

In this paper, two robust controllers for the DC motor position in the presence of uncertainties are proposed using H_∞ and H_2 methods. The controllers are in order of fifth and show acceptable performance.

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