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Output Voltage Control of Inverters Using SDRE Tracking and LQT Controllers

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***Abstract*—This paper presents novel nonlinear and linear control methods for output voltage control of a three-phase inverter. For implementation of these optimal controllers, the nonlinear and linear models of the system including LC filter have been derived. Moreover, analysis of the system under the State-Dependent Riccati Equation (SDRE) and Linear Quadratic Tracking (LQT) as the nonlinear and linear optimal controllers has been done and comparison between performance of them is investigated in MATLAB environment.**

***Index Terms*—Inverter, LC filter, SDRE tracking, LQT.**

I. Introduction

Some of the distributed generators like wind power generation for storing of the generated output power should be connected to a dc battery bank. This stored dc power can be converted to ac power by inverters. Usually, the LC filter is used to maintain the dynamic performance of the system, robustness against load fluctuations and voltage variations as well as generating of three-phase sinusoidal voltage with minimum harmonic [1]. It should be mentioned that for having low distortion, the output impedance of inverter should be minimized, *i.e.*, the inductance and the capacitance must have minimum and maximum amounts, respectively [2], [3].

By considering issues rising from high penetration of inverters in distributed energy resources, control of these equipments can be a major challenge. Thus, investigating different types control methods of inverters has been an important problem. In most of applications, proportional–integral–derivative (PID) controllers have been employed in a double loop structure which the inner and outer loops are used for transient response control and voltage regulating, respectively [4]. In this method, the control signal is applied to the inverter using the pulse width modulation (PWM) technique.

By coming digital signal processers (DSPs), the advanced controllers such as sliding mode [5], robust H_∞ [6], [7] model predictive [8], active disturbance rejection [9] and feedback linearization [10] were used to control of inverters. As well as, references [11] and [12] have employed a dead-beat controller for output regulating of an inverter. Despite of having fast dynamic response, high bandwidth and good tracking of sinusoidal signals, it is sensitive to the parameters and measurement noise. Although the mentioned controllers have shown good results with a global stable behavior, they need to complex computations.

Resonance damping of LC filter is an important problem since it can lead to transient distortions and steady-state harmonics as well as affect the overall system stability. Therefore, present of some new methods is necessary to overcome the mentioned problems. In line with these objectives and output voltage control of the three-phase inverter, this paper presents two new nonlinear State-Dependent Riccati Equation (SDRE) tracking and Linear Quadratic Tracking (LQT) controllers for nonlinear and linear models of inverter alongside LC filter.

The rest of this paper is organized as follows: Section II provides the nonlinear modeling of system. Section III presents the SDRE tracking and the LQT theories. Numerical simulations have been brought in Section IV and Section V concludes the paper.

II. System Modeling

Fig. 1 shows a three-phase inverter with output LC filters. Conventionally, to control this circuit, a double loops controller is employed where each loop has a PI controller but they have some limitations in transient modes. For implementation of nonlinear controller, the nonlinear model of this inverter with LC filter should be derived. By considering Fig. 1, the inverter circuit equations are expressed as follows:

$$L\dot{\bar{i}}_{in}(t) = \bar{v}_{in}(t) - \bar{v}_C(t) \quad (1)$$

$$C\dot{\bar{v}}_C(t) = \bar{i}_{in}(t) - \bar{i}_O(t) \quad (2)$$

$$\bar{v}_C = \begin{bmatrix} v_{CA} & v_{CB} & v_{CC} \end{bmatrix}^T, \quad \bar{v}_i = \begin{bmatrix} v_A & v_B & v_C \end{bmatrix}^T$$
$$\bar{i}_O = \begin{bmatrix} i_{OA} & i_{OB} & i_{OC} \end{bmatrix}^T, \quad \bar{i}_{in} = \begin{bmatrix} i_A & i_B & i_C \end{bmatrix}^T.$$

where $\bar{v}_C$, $\bar{v}_{in}$, $\bar{i}_{in}$ and $\bar{i}_O$ are the voltage of capacitor, the output voltage of inverter, the output current of inverter and load current vectors, respectively. C and L are the components of the LC filter. By describing (1) and (2) in $d-q$ axis, the following equations is achieved.

$$\dot{i}_{inq} = \frac{1}{L}v_{inq} - \frac{1}{L}v_{Cq} - wi_{ind} \quad (3)$$

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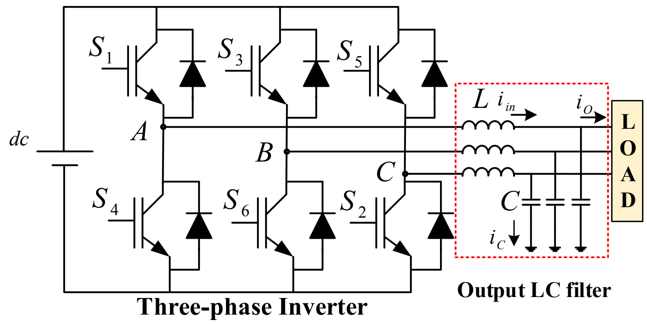

Fig. 1. Three-phase inverter with LC filters.

$$\dot{i}_{ind} = \frac{1}{L}v_{ind} - \frac{1}{L}v_{Cd} + wi_{inq} \tag{4}$$

$$\dot{v}_{Cq} = \frac{1}{C}i_{inq} - \frac{1}{C}i_{Oq} - wv_{Cd} \tag{5}$$

$$\dot{v}_{Cd} = \frac{1}{C}i_{ind} - \frac{1}{C}i_{Od} + wv_{Cq} \tag{6}$$

where v_{Cd}, v_{Cq}, v_{ind} and v_{inq} are voltages of the capacitor and output voltages of the inverter in $d-q$ axis, respectively. i_{ind}, i_{inq}, i_{Od} and i_{Oq} are input and output currents of the inverter in $d-q$ axis, respectively. w is the angular frequency of the output voltage. Assume the power balance has the following form:

$$v_{dc}i_{dc} = \frac{3}{2}\left(v_{Cq}i_{Oq} + v_{Cd}i_{Od}\right) \tag{7}$$

where v_{dc} and i_{dc} are the voltage and current of dc link, respectively. It is should be noted that for a balanced three-phase inverter, v_{Cd} can be set equal to zero. In the feedforward mode, the terms of wi_{ind} , wi_{inq} , wv_{Cd} and wv_{Cq} are eliminated. As a result, (5) can be written as

$$i_{Oq} = i_{inq} - C\dot{v}_{Cq} \tag{8}$$

Therefore, Substituting (8) in (7) leads to

$$\dot{v}_{Cq} = \frac{1}{C}i_{inq} - \frac{2v_{dc}i_{dc}}{3v_{Cq}C} \tag{9}$$

A nonlinear model can be obtained from (5) and (9) as follows:

$$\begin{bmatrix} \dot{i}_{inq} \\ \dot{v}_{Cq} \end{bmatrix} = \begin{bmatrix} -\frac{v_{cq}}{L} \\ \frac{i_{inq}}{C} - \frac{2v_{dc}i_{dc}}{3v_{Cq}C} \end{bmatrix} + \begin{bmatrix} \frac{v_{inq}}{L} \\ 0 \end{bmatrix} \tag{10}$$

where one can write (10) in the general form $\dot{x} = f(x) + B(x)u$ as follows [13]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{x_2}{L} \\ \frac{x_1}{C} - \frac{2v_{dc}i_{dc}}{3x_2C} \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u \tag{11}$$

where $x_1 = i_{inq}, x_2 = v_{Cq}$ and control input is $u = v_{inq}$. On the other hand, the general form of output can be stated as $y = C(x)$ where $y = v_{Cq} = x_2$.

III. Controllers Design

In this section, the SDRE tracking and the LQT theories, their design approach and relations are shortly reviewed .

A. SDRE tracking control design

This type of the control provides an effective algorithm for those systems which have nonlinear states in their models. In this strategy, the nonlinear dynamics have factorized into the state vector then it is multiplied by a matrix-valued which depends on its states [14], [15]. This algorithm includes minimizing of a semi-quadratic performance index and can state the nonlinear system such as a nonunique linear-like form by having the state-dependent coefficient (SDC) matrices. An algebraic Riccati equation (ARE), which is given by the SDC matrices, should be solved online to give the suboptimal control law [15].

Consider the deterministic, infinite-horizon nonlinear optimal regulation problem, whereas the system is nonlinear in states and affine in the input. This system can be represented as $\dot{x} = f(x) + B(x)u$, with the initial value $x(0) = x_0$ where $x \in \Re^n$ is the state vector. This system can be stated as a linear-like SDC form so that a continuous nonlinear matrix-valued $A : \Re^n \mapsto \Re^{n\times n}$ always be existed as

$$f(x) = A(x)x \tag{12}$$

where $A(x)$ is founded by algebraic factorization and is clearly nonunique. So, by SDRE method, $\dot{x} = f(x) + g(x)u$ can be regarded as the following form:

$$\begin{aligned} \dot{x} &= A(x)x + B(x)u \\ x(0) &= x_0 \end{aligned} \tag{13}$$

It can be seen that (13) has a linear-like structure with state dependent $A(x)$ and $B(x)$ matrices. In the SDRE strategy which uses extended linearization as the main concept of this startegy, the following infinite-time performance index of (14) should be minimized [15].

$$\begin{aligned} J = \int_0^\infty \big(x^T(t)Q(x)x(t) + u^T(t)R(x)u(t)\big)dt \\ Q(x) \geq 0, R(x) > 0. \end{aligned} \tag{14}$$

Under the specified condition, the state-feedback control law can be stated as

$$u(x) = -R^{-1}(x)B^T(x)P(x)x(t) \tag{15}$$

where $P(x) : \Re^n \mapsto \Re^{n\times n}$ is unique, symmetric and positive solution of the following algebraic SDRE,

$$\begin{aligned} &P(x)A(x) + A^T(x)P(x) - \cdots \\ &\quad P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0. \end{aligned} \tag{16}$$

Therefore, the closed loop system under the SDRE control law (15) is as follows.

$$\dot{x}(t) = \left[A(x) - B(x)R^{-1}(x)B^T(x)P(x)x(t)\right]x(t). \tag{17}$$

It should be mentioned that for having local asymptotic stability and symmetric positive-definite answer $\left\{A(x), B(x), Q^{1/2}(x)\right\}$ must be point-wise stabilizable, detectable and controllable. It is worth noting that Q is symmetric, positive semidefinite and R should be symmetric positive definite matrix. Although the tuning method of these matrices has not a straightforward rule and depends on problem condition, the following notes can be useful to good selection of them.

1) *Q choosing*: The larger values of the matrix will be led to faster disturbance rejection and more control signal effort. For imposing constraints on the states, the corresponding entry of them in Q should be altered. The trade-off between overshoot and settling time must be done by this matrix.
2) *R choosing*: An Increase in values of this matrix can decrease feedback gain values which make the system slower.

Regarding to these notes, the following SDC form is considered:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & \frac{2}{3C}\frac{v_{dc}i_{dc}}{x_2^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u \quad (18)$$

The output voltage $v_{Cq} = x_2$ should be set to 60V where this tracking problem is solved using SDRE tracking method.

In a SDRE tracking problem, the dynamic of the desired output should be augmented to the main system. For achieving to this goal, the dynamic of the desired output can be regarded as

$$\begin{aligned} \dot{x}_d &= F(x_d)\, x_d \\ y_d &= H(x_d)\, x_d. \end{aligned} \quad (19)$$

The considered cost function is as follows:

$$J = \int_0^{\infty} \left(X^T Q_1 X + U^T R U\right) \mathrm{d}t \quad (20)$$

where $X = e^{-\gamma t}\begin{bmatrix} x & x_d \end{bmatrix}^T$, $U = e^{-\gamma t}u$, and

$$Q_1 = \begin{bmatrix} C(x) & -H(x) \end{bmatrix}^T Q \begin{bmatrix} C(x) & -H(x) \end{bmatrix}. \quad (21)$$

One can write the final augmented form of state-space model as follows:

$$\dot{X} = (\Psi(x) - \gamma I)\, X + \Phi(x) U \quad (22)$$

where,

$$\Psi(x) = \begin{bmatrix} A(x) & 0 \\ 0 & F(x_d) \end{bmatrix}, \quad \Phi(x) = \begin{bmatrix} B(x) \\ 0 \end{bmatrix} \quad (23)$$

Regarding to the SDRE tracking dynamic, the control input in (22) can be considered as

$$U = -R^{-1}\Phi^T(x)P(x)X \quad (24)$$

where $P(x)$ is obtained from the following state-dependent Riccati equation.

$$\begin{aligned} &\Psi^T(x)P(x) + P(x)\Psi(x) - \cdots \\ &\quad P(x)\Phi(x)R^{-1}(x)\Phi^T(x)P(x) + Q_1(x) = 0. \end{aligned} \quad (25)$$

As well as, according to (24) and mentioned relations for X and U, one can write

$$e^{-\gamma t}u = -R^{-1}\Phi^T(x)P(x)e^{-\gamma t}\begin{bmatrix} x \\ x_d \end{bmatrix} \quad (26)$$

Further,

$$u = -R^{-1}\Phi^T(x)P(x)\begin{bmatrix} x \\ x_d \end{bmatrix}. \quad (27)$$

More details can be found in [14].

B. LQT control designing

The LQT algorithm minimizes a performance index that it is very similar to well-known LQR problem. It worth noting that the characteristics of LQT are better than LQR when a trajectory should be tracked with high precise [16]. As it is known from its name, the LQT algorithm is a type of linear controllers which uses linear state-space or linearized models. For linear time invariant (LTI) systems, the LQT performance index is defined as,

$$\begin{aligned} J_x = &\tfrac{1}{2}\left[x^T(t_f)\, H(t_f)\, x(t_f) + e^T(t_f)\, F(t_f)\, e(t_f)\right] + \\ &\tfrac{1}{2}\int_{t_0}^{t_f}\left[x^T(t)\, Q_x x(t) + e^T Q_e e + u^T(t)\, R u(t)\right]\mathrm{d}t \end{aligned} \quad (28)$$

where Q_x, Q_e, F and H are the weighting matrices of states, tracking error, final states and steady-state tracking error, respectively. Moreover,

$$\vartheta(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \quad M = \begin{bmatrix} H & 0 \\ 0 & F \end{bmatrix}, \quad Q_\omega = \begin{bmatrix} Q_x & 0 \\ 0 & Q_e \end{bmatrix}$$

So, the described cost function in (28) can be stated as follows:

$$\begin{aligned} J_x = &\tfrac{1}{2}\vartheta^T(t_f)\, M(t_f)\, \vartheta(t_f) + \cdots \\ &\tfrac{1}{2}\int_{t_0}^{t_f}\left[\vartheta^T(t)\, Q_\omega \vartheta^T(t) + u^T(t)\, R u(t)\right]\mathrm{d}t \end{aligned} \quad (29)$$

where t_f is known and finite. The optimal control signal for minimizing J_x can be stated as,

$$u^*(t) = -R^{-1}B^T P(t)\vartheta^*(t) = -K(t)\vartheta^*(t) \quad (30)$$

where *P(t)* is reachable by solving the following Riccati equation.

$$\dot{P}(t) = -A^T P(t) - P(t)A + P(t)BR^{-1}B^T P(t) - Q \quad (31)$$

It should be mentioned that the solution of differential equation (31) is not easy due to existing of steady-state and transient terms. Hence, to make simplify of calculations, the finite time problem for LQT can be regarded as an infinite time problem which in this regard, *P(t)* is approximated to $\bar{P}$ with a high approximation. Therefore, LQT is changed into a linear quadratic with infinite horizon and the final terms in (29) is removed. So, the new cost function can be written as,

$$J_x = \frac{1}{2}\int_{t_0}^{\infty}\left[\vartheta^T(t)\, Q_\omega \vartheta^T(t) + u^T(t)\, R u(t)\right]\mathrm{d}t. \quad (32)$$

Using the infinite horizon LQT problem, the system must be fully controllable. Regarding these notes, the differential Riccati equation of (31) is changed to [17]

$$A^T\bar{P} + \bar{P}A - \bar{P}BR^{-1}B^T\bar{P} + Q = 0. \quad (33)$$

The optimal control loop is modified to

$$u^*(t) = -R^{-1}B^T\bar{P}\vartheta^*(t) = -\bar{K}\vartheta^*(t) \quad (34)$$

IV. Simulation Results

In this section, the performance results of the SDRE tracking and the LQT controllers have been shown. In this regard, the design parameters have been listed in Table I. As well as, by linearizing of (18) around its equilibrium points ($x_1^* = 5, x_2^* = 60$), the following state-space model is obtained which can be used for LQT controller design.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -250 \\ 10^4 & 833.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 250 \\ 0 \end{bmatrix} u \quad (35)$$

According to the Table I, $Q = 1850$ also (18) and (35) for SDRE tracking and LQT, Fig. 2 and Fig. 3 indicate the performance of these controllers and their control signals, respectively. Moreover, controllability and observability of the system have been investigated and it has been concluded that the utilized SDC form is fully point-wise controllable and observable.

For achieving a good tracking by these controllers, the output voltage has been arrived to 60V exactly by changing weighting matrices which can be observed from Fig. 2. Moreover, Fig. 3 shows that the control signal of the SDRE tracking is milder compared to the LQT in the transient states. Since, these controllers have the same results in low sampling times, the rest of results only are investigated for the SDRE tracking controller. Therefore, in what follows, three scenarios such as impact of Q and R weighting matrices on the system performance, uncertainty in parameters and comparison of the different SDC forms have been considered to examine the SDRE tracking controller.

TABLE I
Designing Parameters

P_{in}	v_{dc}	L	C	R	γ
450W	144V	4mH	100μF	185	0.02

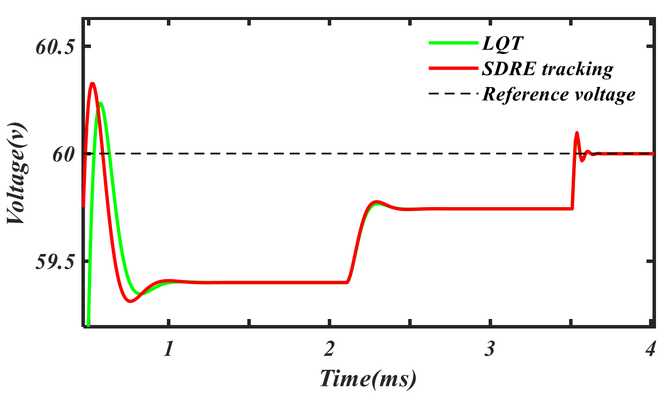

Fig. 2. Performance of system under SDRE tracking and LQT controllers

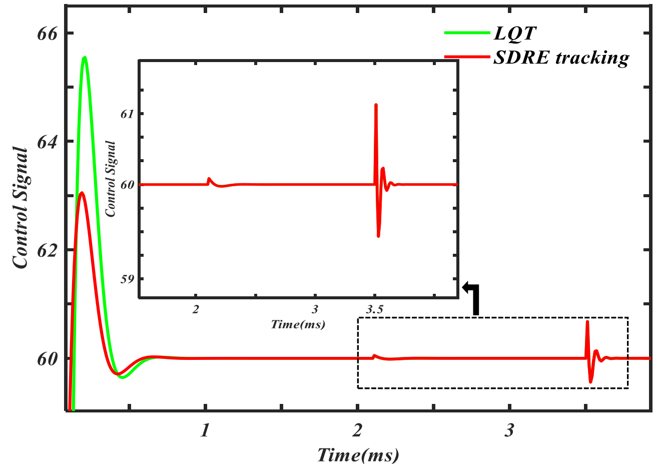

Fig. 3. Control signals

a) Impact of Q and R weighting matrices: In this subsection, the performance of system is surveyed for changes in weighting matrices. It is worth noting that if Q remains constant and R is increased, the steady-state tracking error is increased. On the other side, by keeping R constant and increasing of Q, the steady-state tracking error is decreased. So, for having good tracking of "$y_d = x_d$" by "$y = x_2$" without steady-state error, Q and R must be simultaneously increased and decreased , respectively. This matter has been shown in Fig. 4. It can be observed from this figure that by R decreasing and Q increasing, the settling time is improved but this will be achieved at the price of more control effort which has been shown in Fig. 5.

b) Robustness of the SDRE controller against uncertainty: Here, the impact of uncertainty in performance of the system under the SDRE tracking control is assessed. Toward this end, 15% uncertainty is considered for both of L and C, *i.e.*, $L \pm 0.15L, C \pm 0.15C$. The achieved results from Fig. 6 show that the SDRE tracking controller is robust against high amount of uncertainties in considered parameters and the performance of system is not affected by them.

c) Influence of different SDC forms in the SDRE controller performance: As mentioned earlier, the SDC form is not unique. So, in this scenario two different SDC forms separate from (18) are used to assessing of the SDRE tracking controller performance. These two SDC forms are stated as (36) and (37) that their results are compared with the stated SDC form (18) in Fig. 7. One can see from this figure that the results of different SDCs are similar to each other.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{L}x_2 & \frac{1}{L}(x_1 - 1) \\ \frac{1}{C} & \frac{2}{3C}\frac{v_{dc}i_{dc}}{x_2^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u \quad (36)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{L}x_2 & -\frac{1}{L}(x_1 + 1) \\ \frac{1}{C} & \frac{2}{3C}\frac{v_{dc}i_{dc}}{x_2^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u \quad (37)$$

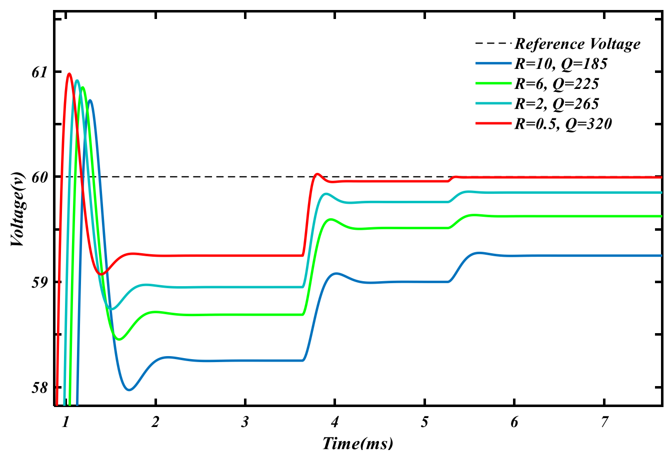

Fig. 4. Impact of weighting matrices in SDRE controller's performance

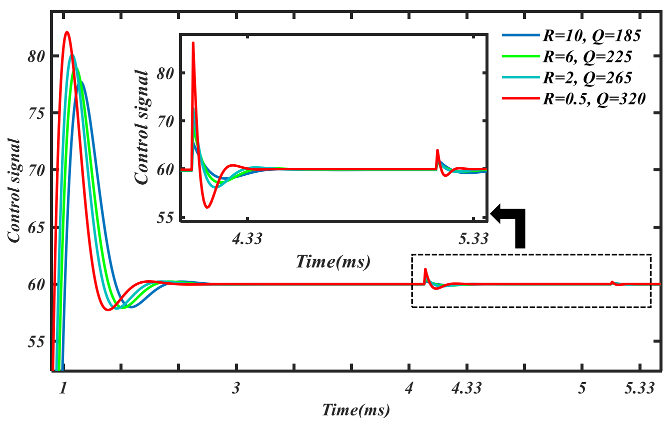

Fig. 5. Control signals of SDRE controller by different weighting matrices

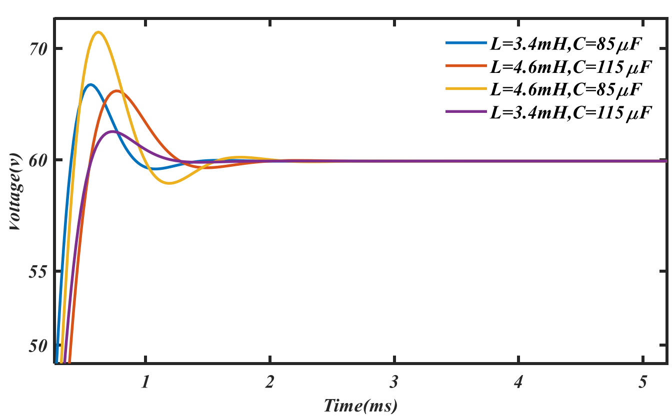

Fig. 6. Performance of SDRE controller in presence of uncertainty

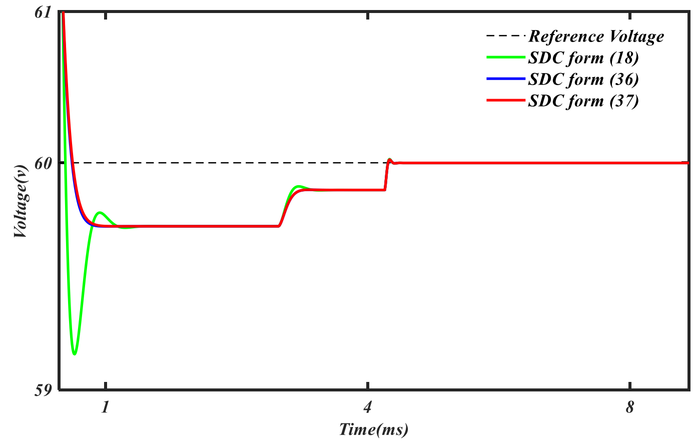

Fig. 7. performance of SDRE controller with different SDC forms

V. Conclusion

In this paper, the SDRE tracking and the LQT controllers were considered to set the output voltage of a three-phase PWM inverter to a desired amount. After extracting nonlinear model of the system and linearization its, the governing relations on the mentioned controllers were expressed. It was observed that for very small step sizes, the results of the SDRE tracking and the LQT are the same to each other. In addition, the performance of the SDRE tracking controller was evaluated under three scenarios. The results deduced that the SDRE tracking is robust against uncertainties of the model. As well as, it was shown that the SDRE tracking controller had the same results for the different SDC forms.

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