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Article · April 2012

DOI: 10.1109/ICTEL.2012.6221287

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# A Game-Theoretic Approach to Spectrum Sharing in Multihop OFDMA Networks

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**Abstract**—We address the problem of subcarrier sharing with discrete rate allocation in the downlink of multihop OFDMA networks. The co-channel interference makes the problem computationally intractable. We therefore model the problem as a non-cooperative game among the transmitting nodes with smaller scale problems. Each problem is solved using dual decomposition. Moreover, to mitigate the degradation effect of co-channel interference in this game, we additionally propose a price-based game. This game charges the transmitting nodes for their power on subcarriers. Numerical results demonstrate that price-based game outperforms the non-cooperative game as a result of distributed interference avoidance. In addition, these games with subcarrier sharing achieve higher sum-rate compared with resource allocation schemes with no subcarrier sharing.

**Index Terms**—Game theory, multicarrier, multihop, OFDMA, optimization.

## I. INTRODUCTION

Next generation wireless networks are expected to provide ubiquitous and high data rate transmission. The integration of multicarrier transmission in the form of orthogonal frequency division multiple access (OFDMA) and multihop transmission is the promising technique toward this ambition. OFDMA mitigates the frequency selectivity of the broadband channel by dividing the bandwidth into a set of non-interfering narrow-band subcarriers. On the other hand, multihop transmission extends the coverage area of the network by overcoming the high path loss. This technology is cost efficient compared with increasing the number of base stations (BSs) [1].

To realize the advantages of multihop OFDMA networks, it is necessary to derive efficient resource allocation schemes for dynamic subcarrier assignment and adaptive power allocation. Because of the multihop spatial diversity and the demand of high data rate, it is beneficial to use aggressive frequency reuse, i.e., all the OFDMA subcarriers are shared among the serving nodes [2]. The co-channel interference caused by the subcarrier sharing makes the resource allocation to users more coupled and difficult to manage.

In [3]–[5], suboptimal centralized algorithms in multihop OFDMA networks are proposed with and without subcarrier sharing, respectively. These algorithms suffer from the huge amount of signalling as a result of feeding back the channel state information (CSI) throughout the network to a central controller and forwarding the scheduling decisions. One alternative approach is to extend the conventional single-

hop scheduling schemes to multihop networks. This approach partitions the users as well as the resources into clusters around the serving nodes and performs the resource allocation accordingly [6], [7]. However, this approach fails to manage the interference caused by the co-channel transmissions, in the case of subcarrier sharing.

Game theory is a mathematical tool for modeling the interactions among self-interested rational players [8]. Each player in the game aims to maximize its own pay-off function in a distributed fashion. The game settles down in a Nash equilibrium (NE), if one exists. Because of the selfish behavior of the players, the NE is not necessarily efficient from the social point of view. It has been increasing interest in employing the game theory for power control and frequency assignment in wireless networks. In non-cooperative games, network nodes myopically transmit on the subcarriers. The co-channel interference generated in these games degrades the network performance significantly. Therefore, price or tax-based algorithms, which charges the network nodes for their transmission power on subcarriers are highly interested. A tax-based algorithm to subcarrier sharing among the clusterheads in wireless mesh networks (WMNs) is proposed in [9]. Under the assumption of uniform power allocation to subcarriers and continuous rate allocation, it is shown that the proposed algorithm attains a NE. In [10], the frequency selection and power allocation in WMNs, where each node can only transmit on one carrier at a time, is performed by a price-based algorithm. It is shown that this algorithm performs better than the non-cooperative game but not as well as a negotiation-based cooperative algorithm.

In this paper, we first model the subcarrier sharing and rate allocation in the downlink of multihop OFDMA networks as an optimization problem. Unlike the continuous rate in the majority of works in the literature, we consider a finite set of discrete modulation rates on each subcarrier in both the game and the price-based approach. To manage the conflict links in multihop networks, we secondly partition the network links into distinct sets, called independent sets, in such a way that the links in one set can simultaneously be active on every subcarrier. We allocate disjoint sets of subcarriers to independent sets based on the uniform power allocation. Given the aforementioned link and subcarrier sets, we thirdly consider the problem as a non-cooperative game among the serving

nodes to mitigate the high complexity. Due to the degradation effect of co-channel interference, we finally propose a price-based subcarrier sharing approach to improve the performance of the game. This approach charges the serving nodes for their transmission power so as to render the subcarrier sharing efficient.

The rest of this paper is organized as follows. The system model and problem formulation are described in Section II. Algorithms for independent set construction and subcarrier distribution are proposed in Section III. In Sections IV and V, non-cooperative game and price-based subcarrier sharing are presented, respectively. Performance evaluation is given in Section VI and the paper is concluded in Section VII.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider an OFDMA network with a set  $\mathcal{K} \triangleq \{i : i = 1, \dots, K\}$  of nodes and a set of  $\mathcal{L} \triangleq \{ij\}$  of links. Every link  $ij$  is the link from transmitting node  $i \in I(j)$  to the receiving node  $j \in O(i)$ .  $I(j)$  and  $O(i)$  are the sets of transmitting and receiving nodes of the links ingoing to and outgoing from the nodes  $j$  and  $i$ , respectively. Moreover,  $\Omega \triangleq \{n : n = 1, \dots, N\}$  is the set of subcarriers, which are assigned to the links. The total transmission power at each node  $i$  is allocated to the assigned subcarriers to the outgoing links of this node. We consider a finite set  $\mathcal{Q} \triangleq \{1, \dots, Q\}$  of modulation rates on each subcarrier.

We define  $x_{ij}^{n,q}$  as a binary variable, where  $x_{ij}^{n,q} = 1$  if subcarrier  $n$  is assigned to link  $ij$  with modulation rate  $q$ , otherwise  $x_{ij}^{n,q} = 0$ . We assume that nodes in the network are not able to both transmit and receive on a given subcarrier at the same time. This constraint is written as

$$\sum_{j \in O(i)} \sum_q x_{ij}^{n,q} + \sum_{j \in I(i)} \sum_q x_{ji}^{n,q} \leq 1 \quad \forall i, n \quad (1)$$

which means that a given subcarrier  $n$  can simultaneously be used only on one link ingoing to or outgoing from node  $i$ .

Let  $G_{ij}^n$  be the channel gain of subcarrier  $n$  on link  $ij$  and  $p_i^n$  is the transmission power of node  $i$  on this subcarrier. Under the assumption that each subcarrier can be reused throughout the network, signal-to-interference-plus-noise ratio (SINR) of subcarrier  $n$  on link  $ij$  is given by

$$\gamma_{ij}^n \triangleq \frac{G_{ij}^n p_i^n}{I_{ij}^n(p_{-i}^n)} \quad (2)$$

where

$$I_{ij}^n(p_{-i}^n) \triangleq \sum_{k=1, k \neq i}^K G_{kj}^n p_k^n + \sigma^2.$$

Moreover,  $\sigma^2$  is the noise power and  $p_{-i}^n \triangleq [p_1^n, \dots, p_{i-1}^n, p_{i+1}^n, \dots, p_K^n]$  is the vector of all interfering transmission powers. Let  $T_q$  be the SINR to transmit with modulation rate  $q$ , i.e.,  $q = \log_2(1 + T_q)$ . To be able to transmit with rate  $q$  on subcarrier  $n$  over link  $ij$ , we need  $\gamma_{ij}^n = T_q$  or equivalently  $p_i^n = I_{ij}^n(p_{-i}^n) T_q / G_{ij}^n$ . Due to (1),

that each can only be assigned to at most one outgoing link from  $i$ , we come up with

$$p_i^n = \sum_{j \in O(i)} \sum_{q=1}^Q x_{ij}^{n,q} I_{ij}^n(p_{-i}^n) T_q / G_{ij}^n. \quad (3)$$

Moreover, the total transmission power at every node  $i$  should not exceed  $P$ , i.e.,

$$\sum_{n=1}^N p_i^n \leq P. \quad (4)$$

The resource allocation problem, which aims to maximize the network sum-rate subject to the aforementioned constraints, is presented in  $P_1$ , i.e.,

$$\max_{\mathbf{X}, \mathbf{P}} \sum_{i=1}^K \sum_{j \in O(i)} \sum_{n=1}^N \sum_{q=1}^Q q x_{ij}^{n,q} \quad (5)$$

$$\text{s.t.} \quad \sum_{j \in O(i)} \sum_{q=1}^Q x_{ij}^{n,q} + \sum_{j \in I(i)} \sum_{q=1}^Q x_{ji}^{n,q} \leq 1 \quad \forall i, n, \quad (6a)$$

$$p_i^n = \sum_{j \in O(i)} \sum_{q=1}^Q x_{ij}^{n,q} I_{ij}^n(p_{-i}^n) T_q / G_{ij}^n \quad \forall i, n, \quad (6b)$$

$$\sum_{n=1}^N p_i^n \leq P \quad \forall i. \quad (6c)$$

where  $\mathbf{X} = \{x_{ij}^{n,q}\}$  and  $\mathbf{P} = \{p_i^n\}$  are vectors of optimization variables.

Problem (5)–(6) is a mixed integer programming problem with high complexity [11]. Complexity exponentially grows with the number of nodes, subcarriers, and outgoing links. Therefore, we deploy distributed decision making approaches such as game theory in subsequent sections to solve the problem. Although the outcome achieved is expected to be less efficient than a possible centralized optimization, these approaches are favorable in terms of computational complexity, and scalability.

## III. INDEPENDENT SET CONSTRUCTION

Because of constraint (6a), some links which are referred to as conflicting links [12] can not simultaneously be active on one subcarrier. In general, a conflict arises between two links when they have a node in common. We consider the conflict between the outgoing links of each transmitting node in Section IV as a constraint in the optimization problem. We hereby assume that links  $ij$  and  $pq$  conflict if either  $i = q$  or  $j = p$  or  $j = q$ . We address conflicting links by defining an *independent set* (IS) as follows.

*Definition 1:* One IS is the set of links such that no two links mutually conflict with each other, i.e., all the links in each IS can simultaneously be active on every subcarrier.

Determining the possible ISs of a given network is beyond the scope of this paper. We hereby propose a heuristic approach in Algorithm 1 to partition the network links into distinct ISs. In this algorithm,  $e$  is the index of ISs and  $E$

is the number of generated ISs since the beginning of the algorithm. Moreover,  $IsConflict(e, ij)$  is equal to 1 if there is at least one conflicting link in  $IS_e$  with link  $ij$ , otherwise it is equal to 0. This algorithm first aims to insert given link  $ij$  into an existing IS,  $IS_e$ , in which there is not a conflicting link with  $ij$ . Otherwise, it generates a new IS.

Once network links have been grouped into ISs, subcarriers are also required to be included into distinct sets; each subcarrier set to be shared within an IS. The reason is that each subcarrier can not be used simultaneously by two ISs. We propose Algorithm 2 to divide the subcarriers into disjoint sets corresponding to ISs. In this algorithm,  $\Omega_e$  denotes the set of subcarriers assigned to  $IS_e$  and  $|\Omega_e|$  is the cardinality of  $\Omega_e$ . In step 4, provided that subcarrier  $n$  is assigned to  $IS_e$ , we allocate the same transmission power to this subcarrier on every  $ij \in IS_e$ . Notice that  $p_i^n$  in step 4 is an auxiliary power allocation, not the final and desired one. Corresponding to each subcarrier, achieved sum-rate in different ISs are computed in step 5. In step 7 and 8, subcarrier  $n$  is assigned to  $IS_{e^*}$  for which assigning this subcarrier achieves the highest sum-rate.

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#### Algorithm 1 ISs Construction

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1: set  $E = 1$  and  $IS_1 = \emptyset$ .
2: for all  $ij \in \mathcal{L}$  do
3:    $e = 1$ .
4:   while  $IsConflict(e, ij) = 1$  do
5:      $e = e + 1$ .
6:   end while
7:   if  $e \leq E$  then
8:      $IS_e = IS_e \cup \{ij\}$ .
9:   else
10:     $E = E + 1$ .
11:    set  $IS_E = \{ij\}$ .
12:   end if
13: end for

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#### Algorithm 2 Subcarrier assignment to ISs

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1: initialize  $\Omega_e = \emptyset$  for all  $e$ .
2: for all  $n$  do
3:   for all  $e$  do
4:     set  $p_i^n = P / (1 + |\Omega_e|) \forall ij \in IS_e$ .
5:     compute  $u_e^n = \sum_{ij \in IS_e} \log_2(1 + \gamma_{ij}^n)$ .
6:   end for
7:    $e^* = \arg \max_e (u_e^n)$ .
8:    $\Omega_{e^*} = \Omega_{e^*} \cup \{n\}$ .
9: end for
10: return  $\Omega_e$  for all  $e$ .

```

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## IV. NON-COOPERATIVE GAME

We model  $P_1$  as a non-cooperative game in Section IV-A and solve the game in Section IV-B.

### A. Game Formulation

In order to optimize power allocation and subcarrier assignment vectors, we construct a non-cooperative game  $NCG = \{\mathcal{K}, \{X_i\}_{i \in \mathcal{K}}, \{u_i\}_{i \in \mathcal{K}}\}$ , where  $\mathcal{K}$  is the set of transmitting nodes and  $X_i = \{x_{ij}^{n,q}\}$  is the strategy profile of node  $i$ . In accordance with the objective function in  $P_1$ ,  $u_i$ , the pay-off function of node  $i$  is

$$u_i(X_i) = \sum_{j \in o(i)} \sum_{n \in \Omega_{e_{ij}}} \sum_{q=1}^Q q x_{ij}^{n,q} \quad (7)$$

where  $e_{ij}$  is the index of IS containing link  $ij$ . The strategy space of this game is represented by the Cartesian product of individual nodes' strategy profiles, i.e.,  $\mathcal{S} = X_1 \times X_2 \dots \times X_K$ . Notice that  $X_{-i} = \mathcal{S} \setminus X_i$  denotes the set of strategy profiles for all users except for user  $i$ . The most common solution in game theory is NE, which is defined as follows.

*Definition 2:* A strategy profile  $X^* \in \mathcal{S}$  is a NE of NCG, if for all  $i \in \mathcal{K}$ ,

$$u_i(X_i^*, X_{-i}^*) \geq u_i(X_i, X_{-i}^*) \quad \forall X_i \in \mathcal{S}. \quad (8)$$

NE is the stable point of the game, where no one of the nodes can increase its own pay-off function by unilateral deviation. In other words, NE is a mutual best response from each node to the other nodes' strategies. In NCG, given  $X_{-i}$  as the strategy profile of node  $i$ 's opponents and defined ISs, this node maximizes its own pay-off function by solving problem

$$\max_{X_i} u_i \quad (9)$$

$$\text{s.t.} \quad \sum_{j \in o(i)} \sum_{q=1}^Q x_{ij}^{n,q} \leq 1 \quad \forall n \in \Omega_{e_i}, \quad (10a)$$

$$\sum_{n=1}^N p_i^n \leq P. \quad (10b)$$

where  $\Omega_{e_i} = \bigcup_{j \in o(i)} \Omega_{e_{ij}}$  is the set of subcarriers that can be assigned to the outgoing links of node  $i$ . We eliminated the second term in the left hand side of the inequality (10a) compared to (6a). Because of the independent sets defined in Section III, if  $n \in \Omega_{e_i}$  then  $\sum_{j \in I(i)} \sum_q x_{ji}^{n,q} = 0$ . Moreover, with a given power from other nodes,  $\{p_i^n\}$  are function of  $X_i$  as in (3).

### B. Game Solution

We form the Lagrangian function

$$L(X_i, \lambda_i) = u_i - \lambda_i \left( \sum_{n=1}^N p_i^n - P \right) \quad (11)$$

where  $\lambda_i$  is the Lagrange multiplier. Accordingly, dual function is given as

$$D(\lambda_i) = \sup_{X_i} L(X_i, \lambda_i) = \max_{X_i} \left( u_i - \lambda_i \sum_{n=1}^N p_i^n \right) + \lambda_i P. \quad (12)$$

Evaluating dual function for a given  $\lambda_i$ , we obtain the optimization problem as follows:

$$\max_{X_i} u_i - \lambda_i \left( \sum_{n=1}^N p_i^n - P \right) \quad (13)$$

$$\text{s.t. (10a).} \quad (14a)$$

Substituting  $u_i$  and  $p_i^n$  by their equivalents in (7) and (3), we solve (13)–(14) by assigning subcarrier  $n \in \Omega_{e_{ij}}$  to link  $ij_n$  with modulation rate  $q_n$  as

$$(j_n, q_n) = \arg \max_{(j,q):n \in \Omega_{e_{ij}}} (q - \lambda_i I_{ij}^n(p_{-i}^n) T_q / G_{ij}^n). \quad (15)$$

In other words,  $x_{ij}^{n,q} = 1$  if  $(j, q) = (j_n, q_n)$ , otherwise  $x_{ij}^{n,q} = 0$ . Accordingly, transmission power on subcarriers can be obtained in (3).

Moreover, the Lagrange multiplier is obtained in the dual domain by solving the dual problem

$$\min_{\lambda_i \geq 0} D(\lambda_i). \quad (16)$$

For a given  $\{p_i^n\}$ , the dual problem is solved by the subgradient method as

$$\lambda_i(t+1) = \left[ \lambda_i(t) - \alpha \left( P - \sum_{n=1}^N p_i^n \right) \right]^+ \quad (17)$$

where  $\left( P - \sum_{n=1}^N p_i^n \right)$  is the subgradient of the dual function with respect to  $\lambda_i$  and  $\alpha$  is the step size that should be small enough to ensure the convergence [13].

A non-cooperative game solution is of importance if it attains a NE. In the aforementioned solution, a given node  $i$  obtains its best response with the respect of opponents strategies in the feasible region of  $P_2$ . Due to the binary variables in  $X_i$ , this region is not convex. On the other hand, existing fixed point theorems such as Kakutani [14], which are used for the proof of NE existence, are based on the convexity of the best response feasible region. Consequently, the existence of NE in NCG can not be established based on these theorems. We investigate the existence of NE via simulations in Section VI. Obtained results show that the game does not converge to a NE.

## V. PRICE-BASED SUBCARRIER SHARING

In order to overcome the lack of NE in NCG, we propose a price-based approach for subcarrier sharing in this section. The key motivation in this approach is to design a power pricing scheme to manage the interference throughout the network and achieve an efficient power allocation accordingly. The interference generated by node  $i$  on subcarrier  $k$  to the other nodes degrades their pay-off functions. In order to model this degradation, we define  $T_i^n = \sum_{k \neq i} \left| \frac{\partial u_k^n}{\partial p_i^n} \right|$  as the price of a unit allocated power to subcarrier  $n$  by node  $i$ , where  $u_k^n$  is the pay-off of node  $k$  on this subcarrier. Since pay-off functions in (7) are not continuous and hence non-differentiable, we

obtain the prices based on the differentiations taken over integer relaxed rates<sup>1</sup>. We mitigate the interference effect by modifying the pay-off function of each node to include the price of power allocation. Consequently, each node  $i$  in the price-based approach solves the problem as in the following:

$$\max_{X_i} u_i - \sum_{n=1}^N T_i^n p_i^n \quad (18)$$

$$\text{s.t. (10).} \quad (19a)$$

We assume that the nodes prefer not to transmit on a subcarrier whenever its pay-off value gets negative. Using the same method in Section IV-B, the solution is obtained by assigning subcarrier  $n$  to link  $ij_n$  with modulation level  $q_k$  as

$$(j_n, q_n) = \arg \max_{(j,q):n \in \Omega_{e_{ij}}} (q - (\lambda_i + T_i^n) I_{ij}^n(p_{-i}^n) T_q / G_{ij}^n). \quad (20)$$

Moreover, the Lagrangian multiplier  $\lambda_i$  are computed similar to NCG in (17).

Given the aforementioned solution, Algorithm 3 presents the price-based subcarrier sharing (PBSS) scheme. This algorithm initializes the transmission power such that all nodes transmit the same power on the subcarriers. At each iteration of the algorithm, chosen node  $i^*$  is allowed to update its own strategy profile and the transmission power as well. The prices on subcarriers corresponding to this node are computed in step 6. Given  $\mathcal{S}(t-1)$ , node  $i^*$  solves (18)–(19) so as to obtain its own transmission power on the subcarriers in step 7. In step 8, the transmission power of the other nodes is set to the same in the previous iteration so as to update the strategy vector  $\mathcal{S}(t)$ . The average pay-off achieved by each node so far is derived using an exponential moving average in step 9, where  $T$  is the number of iterations over which the utilities are averaged. The game continues upon the sum of the magnitudes of differential utilities in two successive iterations would be less than a small enough value  $\epsilon$ .

## VI. PERFORMANCE EVALUATION

In this section, we compare the performance of the proposed NCG and PBSS schemes in the downlink of OFDMA transmission. Since NCG does not attain a NE, we record the highest achieved sum-rate value during the game as its performance for comparison. There are 128 subcarriers occupying a one MHz frequency channel, which is assumed to be 6-tap Rayleigh fading with 0.9  $\mu s$  RMS delay spread. The

<sup>1</sup>Let subcarrier  $n$  be assigned to link  $kj$  at node  $k$ . Under the assumption of  $u_k^n = \log_2(1 + \gamma_{kj}^n)$ , we obtain

$$\frac{\partial u_k^n}{\partial p_i^n} = \left( \frac{G_{kj}^n p_k^n}{\ln 2} \right) \left( \frac{-G_{ij}^n}{\sum_{l \in \mathcal{L}} G_{lj}^n p_l^n + \sigma_2} \right).$$

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**Algorithm 3** PBSS scheme
 

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- 1: set  $t = 0$  and  $\bar{u}_i(0) = 0$  for all  $i$ .
  - 2: set  $p_i^n(0) = \frac{P}{|\Omega_{e_i}|}$  for all  $i$  and  $n \in \Omega_{e_i}$ .
  - 3: **while**  $\sum_i |\bar{u}_i(t) - \bar{u}_i(t-1)| \leq \epsilon$  **do**
  - 4:    $t = t + 1$ .
  - 5:   choose node  $i^*$  in a sequential order.
  - 6:   compute the price  $T_{i^*}^n$  for all  $n \in \Omega_{e_{i^*}}$ .
  - 7:   given  $P_i(t-1)$  for  $\forall i \neq i^*$ , obtain  $P_{i^*}(t)$ .
  - 8:    $P_i(t) = P_i(t-1)$  for all  $i \neq i^*$ .
  - 9:    $\bar{u}_i(t) = (1 - \frac{1}{T})\bar{u}_i(t-1) + (\frac{1}{T})u_i(t)$  for all  $i$ .
  - 10: **end while**
- 

exponential power delay profile is  $g_{ij}e^{-(l-1)}$ , where  $g_{ij}$  is the first path's average power gain of link  $ij$ , and  $l$  is the path index. Single-sided power spectral density of noise is assumed to be unity. Numerical results are obtained for the a typical multihop relay network shown in Fig.1 includes a base station, which sets end-to-end connections with end nodes via relay nodes. In this network, the arriving and outgoing links in nodes  $n_2, n_3, n_4, n_5$  and  $n_6$  conflict with each other. Using Algorithm 1, two ISs are constructed:  $IS_1 = \{l_1, l_2, l_3, l_8, l_9, l_{10}\}$  and  $IS_2 = \{l_4, l_5, l_6, l_7\}$ .

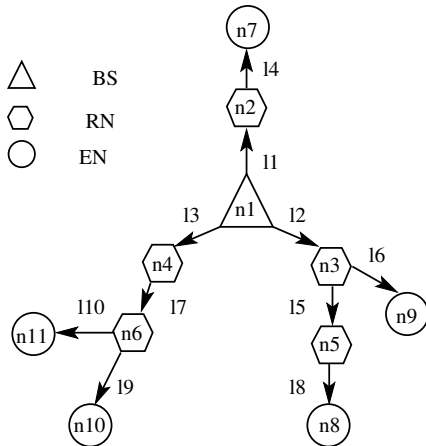


Fig. 1. Relay network

For each receiving node  $j$ , we assume that  $g_{ij} = g_S$  if node  $i$  is the corresponding transmitter of node  $j$ , otherwise  $g_{ij} = g_I$ , where  $g_S$  and  $g_I$  are the signal and interference gains, respectively. We perform the simulation for with  $g_S = 0dB$  and  $g_I = -3dB$ . For one realization of the channel, the variation of the pay-off values of four transmitting nodes in the network for PBSS and NCG schemes are illustrated in Fig.2(b) and Fig.2(a), respectively. In both cases, some nodes pay-off values converge, whereas the others oscillate between some values in the steady state. This oscillation is due to the subcarrier back and forth among the nodes resulted from the non-convex feasible region in  $P_2$ .

Moreover, the achieved sum-rate values with PBSS and NCG schemes are shown in Fig.3. Despite of the oscillation in the individual nodes pay-off values, the sum-rate values

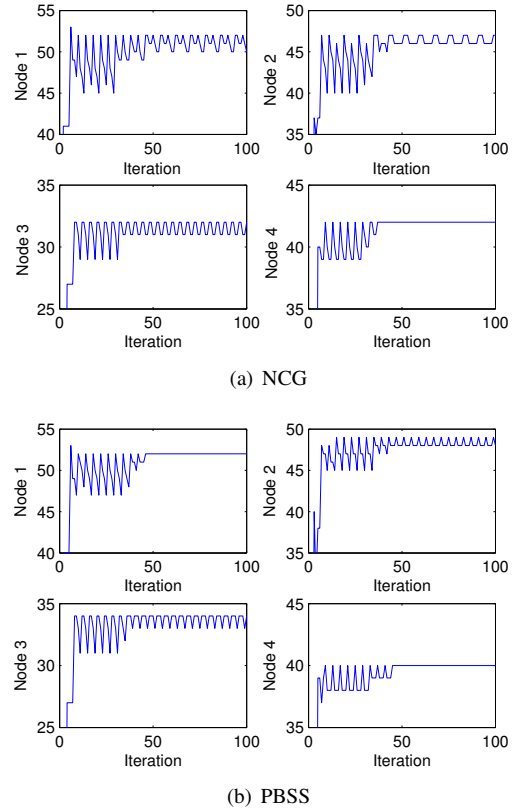


Fig. 2. Pay-off values over the game iteration

converge in this figure. In other words, subcarrier back and forth among the nodes in the steady state does not alter the performance from the network point of view. In addition, PBSS outperforms NCG in both networks because of the prices imposed on the transmission power.

In the following, we vary the interference to signal ratio,  $\frac{g_I}{g_S}$ , and perform the simulations over 500 realizations of the frequency selective fading channel. We also obtain the sum-rate value from the scheduling scheme in our work [6], refereed to as *No sharing* scheme in this paper. In this scheme, subcarriers are not shared, i.e., each subcarrier is used only once in the network. Achieved average sum-rate values are shown in Fig.4. As shown, subcarrier sharing generally outperforms the No sharing scheme. In addition, the sum-rate value decreases as the interference increases in both PBSS and NCG schemes. This is due to the fact that co-channel interference resulted from subcarrier sharing degrades link capacities and sum-rates accordingly. On the other hand, No sharing scheme does not suffer from co-channel interference because of not subcarrier reuse. In both networks, PBSS outperforms NCG and their performance gap increases as the interference gain increases. This is due to the fact that every node in NCG transmits power on subcarriers selfishly. Therefore, the degradation effect of this power on the other nodes grows with the increase of interference gain. On the other hand, the power prices on subcarriers imposed by the price-based approach prevents the

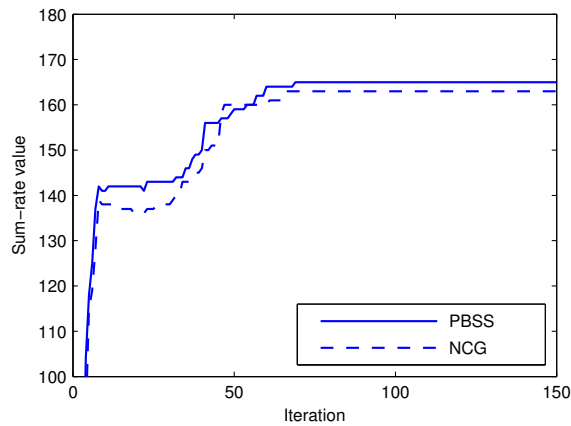


Fig. 3. Relay network:sum-rate value

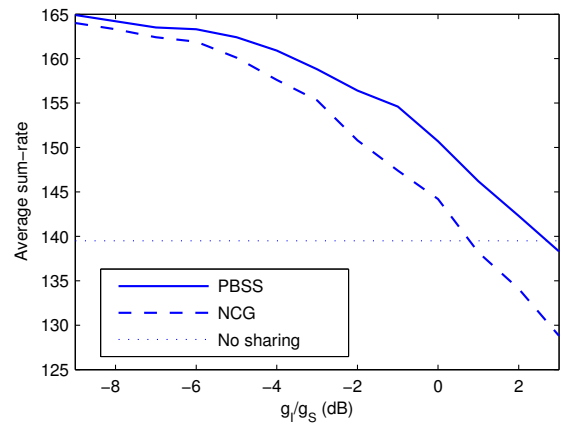


Fig. 4. Relay network: average sum-rate

nodes to increase their transmission power myopically. This issue results in an interference management scheme in the network. Because of this management, the performance of PBSS converges approximately to the same of No sharing scheme when  $g_I$  becomes larger than  $g_S$ , whereas the performance of NCG is inferior to that of No sharing scheme.

## VII. CONCLUSION

We addressed the subcarrier sharing in multihop OFDMA networks by a non-cooperative game and a game with power charging. While aggregate pay-off values are shown to converge, these games do not attain a NE. The possible explanation is due to subcarrier back and forth among competitive nodes in the steady state. Moreover, power charging game outperforms non-cooperative game as a result of mitigating the co-channel interference. In overall, subcarrier sharing improves the network sum-rate value, especially when the interference gain is low in the network.

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