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Distributed Resource Optimization in Multicell OFDMA Networks

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***Abstract*—We consider the joint allocation of receiver, bit, and power to subcarriers in the downlink of multicell orthogonal frequency-division multiple-access (OFDMA) networks. Assuming that the cells share the entire bandwidth and that the rates are discrete, we formulate the joint allocation problem as a nonlinear mixed integer program (MIP), which however has exponential worst-case complexity. We capitalize on the capability of the receivers to measure the interference-plus-noise on every subcarrier and decompose the joint problem into a set of smaller-scale linear MIPs solved by individual base stations. Accordingly, we propose a distributed algorithm with linear complexity, in which the base stations participate in the problem solution in a round-robin manner. Simulation results demonstrate the effectiveness of the proposed algorithm in comparison with the iterative waterfilling algorithm and the successive optimal solution, by means of standard branch-and-cut solvers, of the individual MIPs.**

I. INTRODUCTION

Multicarrier transmission in the form of orthogonal frequency-division multiple access (OFDMA) has emerged as a promising technique towards high data transmission in the next generation wireless networks [1]. OFDMA mitigates the frequency selectivity of the broadband channel by dividing the bandwidth into a set of non-interfering narrowband subcarriers. Owing to independent subcarrier channel gains for different users, it is possible to dynamically assign subcarriers to users with adaptive power allocation. To fully realize the advantages of OFDMA, resource allocation schemes for the *single-cell* downlink have been extensively studied [2]–[5].

Employing OFDMA in the context of *multicell* networks is the promising technique towards ubiquitous and high data rate transmission in the next generation networks [6]. We study the resource allocation problem in the downlink of multicell OFDMA networks. Differently to the single-cell case, the resource allocation in multicell networks needs to take advantage of spectrum sharing among adjacent cells to enhance the aggregate capacity. As a consequence of frequency reuse, the generated intercell interference couples the resource allocation in different cells and therefore the allocation is more challenging. Hence, the single-cell schemes cannot be directly applied to multicell OFDMA networks, since they do not take into account the intercell interference. Also, the need of practical OFDMA resource allocation schemes necessitates optimization models that can be efficiently solved in a distributive manner.

From an optimization viewpoint, jointly optimizing resource allocation across an OFDMA network is a nonlinear MIP, which is NP-hard to solve in general [7]. Significant research work has been conducted to reduce the complexity either in a centralized or distributed manner. The search for decentralized solutions motivated significant work within the framework of non-cooperative game theory [8]–[10]. However, due to the selfish behavior of the transmitters as game players, co-channel interference degrades the network performance significantly. Alternatively, price or tax-based algorithms have been used to charge the transmitters for their transmit power or the number of allocated subcarriers.

In this paper, we assume that the transmission rate on each subcarrier is chosen from a finite set of discrete levels. We first formulate the sum-rate maximization in the downlink of multicell OFDMA networks as a nonlinear MIP. The discrete bit levels make the formulation more efficient for practical implementations. Moreover, the formulation takes multicell multiuser diversity into account to establish an adaptive reuse factor on subcarriers. Using the fact that each receiver is able to measure the interference-plus-noise on every subcarrier, we then decompose the joint resource allocation to individual linear MIP problems, one for each BS. Based on the proposed solution, we propose a low-complexity distributed subcarrier, power, and bit level (DSPB) allocation algorithm, which adapts to the variable channel gains.

The paper is organized as follows. The system model and problem formulation are given in Section II. The solution to the resource allocation problem of a single BS is presented in Section III. The distributed algorithm is proposed in Section IV. Numerical results are given in Section V and the paper is concluded in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider downlink transmission in a multicell OFDMA network with a set $\mathcal{L} \triangleq \{i : i = 1, \ldots, L\}$ of BSs and a set $\mathcal{K} \triangleq \{k : k = 1, \ldots, K\}$ of receivers, where every BS is assumed to serve the same number of receivers, i.e. K/L. The ith BS serves the receivers within the set $\mathcal{K}_i \triangleq \{(i -$

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$1)K/L+1,\ldots,iK/L\}$. The network bandwidth is shared by all BSs and it is divided into a set $\mathcal{N} \triangleq \{n : n = 1,\ldots,N\}$ of orthogonal subcarriers. The channel of each subcarrier is flat, since its bandwidth is chosen small enough compared to the coherence bandwidth. The number of bits loaded on each subcarrier is chosen from a finite set $\mathcal{Q} \triangleq \{q : q = 1,\ldots,Q\}$.

We define the binary allocation variables $x_k^{n,q}$, where $x_k^{n,q} = 1$ if subcarrier n is assigned to receiver k with rate q and $x_k^{n,q} = 0$ otherwise. To avoid intracell interference, each subcarrier can be used by *at most* one receiver per cell. Hence, for BS_i and subcarrier n we have the constraint

$$\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} \le 1. \tag{1}$$

The sum in the left-hand side of (1) equals to zero when BS_i does not allocate any receiver to subcarrier n.

Let $G_{i,k}^n$ denote the gain of the channel between BS_i and receiver k on the nth subcarrier. The signal-to-interference-plus-noise ratio (SINR) of receiver k, served by BS_i on subcarrier n with transmit power p_i^n, is

$$\gamma_k^n \triangleq \frac{G_{i,k}^n p_i^n}{I_k^n(p_{-i}^n)}. \tag{2}$$

In (2), the interference generated by simultaneous transmissions throughout the network on subcarrier n plus the AWGN noise variance σ_k^2 is denoted

$$I_k^n(p_{-i}^n) \triangleq \sum_{j=1, j\ne i}^{L} G_{j,k}^n p_j^n + \sigma_k^2, \tag{3}$$

where $p_{-i}^n \triangleq [p_1^n,\ldots,p_{i-1}^n,p_{i+1}^n,\ldots,p_L^n]$ is the vector of all interfering transmit powers.

Assuming Gaussian signaling, let T_q denote the threshold that the SINR should reach to load q bits, i.e., $\log_2(1+\gamma) = q \Leftrightarrow \gamma = 2^q - 1 \triangleq T_q$. When BS_i decides to serve receiver $k \in \mathcal{K}_i$ with q bits on subcarrier n, i.e. $x_k^{n,q} = 1$, then, due to (2), in order to have $\gamma_k^n = T_q$ the required transmit power is $p_i^n = I_k^n(p_{-i}^n)T_q/G_{i,k}^n$. Due to (1), this power is given, for an arbitrary subcarrier and bit allocation, by

$$p_i^n = \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} I_k^n(p_{-i}^n)T_q/G_{i,k}^n. \tag{4}$$

Moreover, we assume that the total transmit power of every BS cannot exceed the maximum budget P, i.e. $\sum_{n=1}^{N} p_i^n \le P$.

The objective is to maximize the achievable sum-rate in the network, i.e., the sum of bit rates of all subcarriers over all cells, subject to the aforementioned constraints. Consequently, the joint resource allocation problem is stated as

$$\max_{\mathbf{X},\mathbf{P}} \sum_{i=1}^{L}\sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} q x_k^{n,q} \tag{5}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} \le 1 \quad \forall i\in\mathcal{L}, \forall n\in\mathcal{N}, \tag{6a}$$

$$p_i^n = \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} I_k^n(p_{-i}^n)T_q/G_{i,k}^n \quad \forall i\in\mathcal{L}, \forall n\in\mathcal{N}, \tag{6b}$$

$$\sum_{n=1}^{N} p_i^n \le P \quad \forall i\in\mathcal{L}. \tag{6c}$$

Problem (5)–(6) is a MIP with KNQ binary allocation variables $\mathbf{X} = \{x_k^{n,q} \in \{0,1\}\}_{k\in\mathcal{K}}^{n\in\mathcal{N},\ q\in\mathcal{Q}}$ and LN continuous power variables $\mathbf{P} = \{p_i^n \in \mathbb{R}_+\}_{i\in\mathcal{L}}^{n\in\mathcal{N}}$. This problem is NP-hard in general [11]. The formulation is nonlinear due to the right-hand side of (6b) which involves, due to (3), bilinear products of the optimization variables. Finding the optimal solution requires an exhaustive search with worst-case complexity exponential in the total number of variables. The complexity is prohibitive for modern broadband networks which have hundreds of subcarriers. This motivates the low-complexity distributed approach that we are proposing in Section IV.

III. Single-Cell Resource Allocation

The most significant challenge in the solution of problem (5)–(6) is due to the interference-plus-noise terms $\{I_k^n(p_{-i}^n)\}$ in (6b) that couple the resource allocation performed in different cells. However, the fact that each receiver is able to sense and measure the interference-plus-noise on subcarriers motivates us to decompose the global problem into subproblems solved by individual BSs. In other words, BS_i takes as input the values $\{I_k^n\}_{k\in\mathcal{K}_i}^{n\in\mathcal{N}}$ collecting them from the receivers in its cell, when the other BSs have already performed the resource allocation. Hence, the coupling among the resource allocation problems in different cells is eliminated. Consequently, the joint problem (5)–(6) decouples into L sub-problems, each solved separately by a different BS. The problem corresponding to BS_i is

$$\max_{\mathbf{X}_i,\mathbf{P}_i} \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} q x_k^{n,q} \tag{7}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} \le 1 \quad \forall n\in\mathcal{N}, \tag{8a}$$

$$p_i^n = \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} I_k^n T_q/G_{i,k}^n \quad \forall n\in\mathcal{N}, \tag{8b}$$

$$\sum_{n=1}^{N} p_i^n \le P. \tag{8c}$$

Problem (7)–(8) is a MIP with KNQ/L binary variables $\mathbf{X}_i = \{x_k^{n,q} \in \{0,1\}\}_{k\in\mathcal{K}_i}^{n\in\mathcal{N},\ q\in\mathcal{Q}}$ and N continuous variables $\mathbf{P}_i = \{p_i^n \in \mathbb{R}_+\}^{n\in\mathcal{N}}$. Not only this problem has L times smaller dimension than the joint one, but also it is linear, since the constraints (8b) have now, for given $\{I_k^n\}$, become linear. There exist several solvers, implementing branch-and-cut techniques, that find the optimal solution of linear MIP problems frequently avoiding exhaustive search. However, the worst-case complexity of these techniques still increases exponentially with the number of variables and becomes impractical for large problem sizes, as experienced in a previous work [12]. This motivates us to investigate low-complexity solutions to (7)–(8). Due to the binary variables, this problem is nonconvex. Existing solutions to this problem of single-cell resource allocation typically exploits the relaxation of binary variables so that the problem can be solved using convex linear programming [13]. The disadvantage is that rounding off the variables into binary ones takes the solution far from the optimal solution. Herein, we take advantage of the seminal contribution on multicarrier systems in [14], which has shown that, using dual optimization, the duality gap decreases as the number of subcarriers increases. The large number of subcarriers in practical OFDMA networks therefore motivates us to solve (7)–(8) in the dual domain.

In the following, we focus on the resource allocation problem in the ith cell, assuming that the allocation has been already performed in the other cells, i.e. for some given $\{I_k^n\}_{k\in\mathcal{K}_i}^{n\in\mathcal{N}}$. Inspecting (8b), we observe that the transmit powers $\mathbf{P}_i$ depend entirely on the variables $\mathbf{X}_i$, provided that they also meet the bound (8c). Hence, substituting (8b) into (8c), we can rewrite problem (7)–(8), with respect to only the optimization variables $\mathbf{X}_i$, as

$$\max_{\mathbf{X}_i} \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} q x_k^{n,q} \tag{9}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} \leq 1 \qquad \forall n\in\mathcal{N}, \tag{10a}$$

$$\sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} I_k^n T_q / G_{i,k}^n \leq P. \tag{10b}$$

This enables us to solve the MIP (7)–(8) in two steps. First, we solve the linear binary problem (9)–(10) to determine the subcarrier and bit level allocation, and then plug the solution into (8b) to compute the transmit powers.

The solution to (9)–(10) would be straightforward if we decouple the power budget constraint in (10b) and perform the optimization per subcarrier. This motivates the incorporation of (10b) into the objective function and form a Lagrangian function as

$$L_i(\mathbf{X}_i,\lambda_i) = \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} q x_k^{n,q} - \lambda_i\left(\sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} I_k^n T_q / G_{i,k}^n - P\right) \tag{11}$$

and the corresponding dual function as

$$D_i(\lambda_i) = \sup_{\mathbf{X}_i}\left\{L_i(\mathbf{X}_i,\lambda_i) : \text{(10a)}\right\}, \tag{12}$$

where λ_i is the Lagrange multiplier. This multiplier is obtained in the dual domain for a given $\mathbf{X}_i$ by solving the corresponding dual problem

$$\min_{\lambda_i\geq 0} \; D_i(\lambda_i). \tag{13}$$

This problem can be solved by the subgradient method, i.e., beginning with an initial $\lambda_i(0)$, given $\lambda_i(t)$ at iteration t, we obtain $\mathbf{P}_i$ from $\mathbf{X}_i$ using (8b). We then update the Lagrange multiplier as

$$\lambda_i(t+1) = \left[\lambda_i(t) - \alpha\left(P - \sum_{n=1}^{N} p_i^n\right)\right]^+, \tag{14}$$

where $P - \sum_{n=1}^{N} p_i^n$ is the subgradient of $D_i(\lambda_i)$ with respect to λ_i and α is a step size that should be small enough to ensure the convergence [15]. The aforementioned approach therefore enables individual BSs to contribute the solution of the original problem separately.

To evaluate $D_i(\lambda_i)$ for a given λ_i in (12), BS_i substitutes $L_i(\mathbf{X}_i,\lambda_i)$ in (12) with (11) and forms an optimization problem represented by

$$\max_{\mathbf{X}_i} \sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} f_k^{n,q} \tag{15}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} \leq 1 \qquad \forall n\in\mathcal{N}, \tag{16}$$

where $f_k^{n,q} \triangleq q - \lambda_i I_k^n T_q / G_{i,k}^n$. Due to (16), each subcarrier can be used by at most one receiver, with a single bit rate, within cell i. This statement along with the decomposable form of (15)–(16) enables separate allocation for each individual subcarrier. The solution is therefore obtained by assigning each subcarrier n to receiver $k_n \in \mathcal{K}_i$ with bit rate q_n as

$$(k_n, q_n) = \underset{(k,q):k\in\mathcal{K}_i, q\in\mathcal{Q}}{\arg\max} \; f_k^{n,q} \tag{17}$$

provided that $f_{k_n}^{n,q_n} > 0$. In other words, for each subcarrier in cell i, we go over the QK/L possible receiver-bit assignments and select the one giving the largest positive value. Hence, $x_k^{n,q} = 1$ if $k = k_n$ and $q = q_n$, otherwise $x_k^{n,q} = 0$. Due to (8b), the transmit power is $p_i^n = I_k^n T_q / G_{i,k}^n$ in the former case and $p_i^n = 0$ in the latter case. However when $f_{k_n}^{n,q_n} \leq 0$, then $x_k^{n,q} = 0$ for all $k \in \mathcal{K}_i$, $q \in \mathcal{Q}$, and accordingly $p_i^n = 0$.

IV. Distributed Resource Allocation Algorithm

Given the solution for the allocation problem of each BS, presented in Section III, in the sequel we propose a distributed subcarrier, power, and bit level (DSPB) allocation algorithm for the downlink of multicell OFDMA networks. The DSPB algorithm is based on the iterative update of the Lagrange multiplier in (14). We assume that there is a network coordinator, which synchronizes the BSs so that they know their order in the algorithm. This coordinator, also, terminates the algorithm upon satisfaction of the convergence condition. During the algorithm iterations, the channel gains are assumed to be constant. In addition, there is a mechanism to feedback, for all subcarriers, the channel gains and perceived interference from all receivers in each cell to the corresponding BS.

Algorithm 1 Distributed Subcarrier, Power, and Bit level allocation (DSPB)

1: Initialization: $t = 0$, $\lambda_i(0) = \lambda_{\text{init}} \quad \forall i \in \mathcal{L}$, $p_i^n = \delta P/N \quad \forall i \in \mathcal{L}, \forall n \in \mathcal{N}$
2: **while** $\sum_{i \in \mathcal{L}} |\sum_{n \in N} p_i^n - P| \geq \epsilon$ **do**
3: $\quad t = t + 1$
4: $\quad$ Network coordinator chooses BS_j in a round-robin order.
5: $\quad$ BS_j measures $\{I_k^n(p_{-j}^n)\}_{k \in \mathcal{K}_j}^{n \in \mathcal{N}}$.
6: $\quad$ BS_j determines $\mathbf{X}_j$ and $\mathbf{P}_j$ using (17) and (8b) respectively.
7: $\quad$ BS_j updates $\lambda_j(t)$ using (14).
8: **end while**

At first, every BS initializes the Lagrange multiplier and distributes uniformly a part of the power budget on all subcarriers (step 1, where $\delta < 1$). The network coordinator continues the iterations till the aggregate differential power in the network would be less than an accuracy threshold ϵ (step 2). This condition characterizes the satisfaction of the power constraints (8c). At each iteration, a BS is chosen in a round-robin manner to update its subcarrier, power, and bit level allocation subject to the measured interference from the other BSs (steps 4, 5, and 6). Using the new power settings, the chosen BS updates its Lagrange multiplier (step 7).

The DSPB algorithm takes advantage of two decomposition levels to overcome the exponential complexity of exhaustive search methods over the NQK binary variables. First, decoupling the original resource allocation problem (5)–(6) into subproblems, we decrease the exponential complexity to be linear in L. The linearity is due to the Lagrange multiplier update in (14). Second, the complexity $O((QK/L)^N)$ of subcarrier and rate allocation within each cell is decreased to $O(NQK/L)$ by dual decomposition in Section III, as we came up with an optimization per subcarrier. In overall, the complexity is $O(NQK)$, linear in the number of subcarriers, bit levels, and users. On the other hand, the algorithm burdens some signalling overhead. The network coordinator notifies the BSs of their order in the algorithm and finally terminates the algorithm. At the end of every iteration, the chosen BS has to send to the coordinator its updated aggregate transmit power.

V. Performance Evaluation

We consider downlink transmission in a network with four cells of radius $R = 1$ Km and 8 users. Every BS, located at the center of the corresponding cell, serves 2 users, randomly placed within the cell. The path loss (in dB) at a distance d from a BS is given by $L(d) = L(d_0) + 10\alpha \log_{10}(d/d_0)$, where for the reference point it is $d_0 = 50$ m, $L(50) = 0$, and the path loss exponent is $\alpha = 3.5$. The shadowing effect is modeled as an independent log-normal random variable with 8 dB standard deviation. The channel on each link is assumed to be Rayleigh fading, modelled by a six-tap impulse response with exponential power delay profile indicated by $ge^{-(l-1)}$, where $g = 1$ is the first path's average power gain and l is the path index. Moreover, the root-mean-square delay spread is 0.9 μs. The transmission budget of each BS is $P = 5$ W and the noise variance is assumed to be $\sigma_k^2 = -90$ dBm for all receivers. The bit level on each subcarrier is chosen from the set $\mathcal{Q} = \{1, 2, ..., 5\}$, so that the corresponding SINR thresholds are $T_q = \{1, 3, 7, 15, 31\}$, respectively.

Firstly, to investigate the performance of DSPB for a typical number of subcarriers, e.g. $N = 64$, we show in Fig. 1 the sum-rate achievement (in bits per OFDM symbol) of each cell versus the iteration number. It is seen that with the convergence of the transmit powers ($\epsilon = 0.1$), the cell sum rates attain their final values.

In the following, we compare, in the aforementioned setup, the performance of DSPB with the result obtained by solving the individual MIP (IMIP) (7)–(8) at individual BSs. The optimal solution in the primal domain of each IMIP is obtained calling the GNU linear programming kit (GLPK) [16]. In this scheme, similar to DSPB, beginning with uniform power allocation, the individual problems at BSs are solved optimally in a round-robin manner. As a lower bound, we also include the sum-rate values achieved from the iterative waterfilling algorithm (IWF) [17], [18] customized to OFDMA systems using joint subcarrier and power allocation as in [3] and [4]. Since the subcarrier rates in IWF are assumed to be continuous, we round off each achievable rate to the largest integer value not greater than that rate. We compare the overall sum-rate of the aforementioned schemes with different number of subcarriers, i.e. N. For each value of N, we obtain the sum rates for 50 realizations of the fading channel gains and show the average sum rates in Fig. 3. We observe that DSPB outperforms both IMIP and IWF schemes. The performance gap between DSPB and IWF becomes larger as the number of subcarriers increases. This is due to the degradation effect of the rounding operation in IWF which increases with the number of subcarriers.

The performance difference between IMIP and DSPB is due to the fact that, in IMIP, each BS adopts the optimal solution in (7)–(8) to maximize its own sum rate. This optimal strategy most likely generates a large interference and therefore degrades the performance of other BSs significantly. However, in DSPB, each BS assigns each subcarrier as in (17), where $f_k^{n,q}$ can be written as $f_k^{n,q} = (q - \lambda_i p_i^n)$. In other words, in addition to the achieved rate q, DSPB also takes the required transmit power p_i^n into account in subcarrier allocation via the Lagrange multiplier acting as power price. Apparently, DSPB tends to minimize the generated interference on the other cells and therefore they undergo small rate degradation at the last iteration.

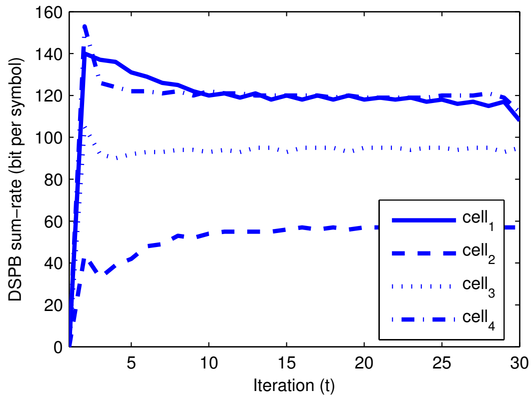

Fig. 1. Sum-rate variation in DSPB

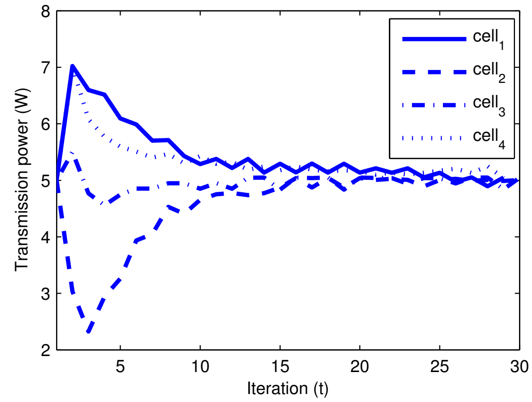

Fig. 2. Power variation in DSPB

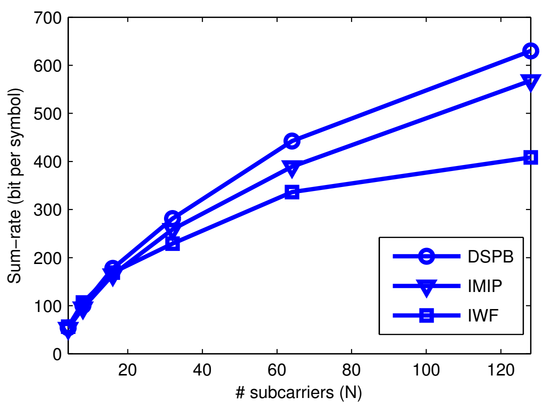

Fig. 3. Average sum-rate in DSPB, IMIP, and IWF

VI. Conclusion

We formulated the spectrum, power, and rate allocation problem that maximizes the sum rate of multicell OFDMA networks as a nonlinear MIP, which is computationally intractable. The capability of the receivers to measure the perceived interference-plus-noise enabled us to decouple the global problem into individual linear MIPs. These problems can be solved optimally, in a sequential manner by the BSs, using branch-and-cut techniques, albeit with worst-case complexity exponential to the number of variables. We proposed a distributed algorithm, in which the BSs participate in a round-robin manner to the solution of the whole problem, by performing the optimization per subcarrier and updating a Lagrange multiplier at each iteration. The complexity of this solution in every iteration is linear to the number of allocation variables of each BS. We demonstrated with numerical results the proposed algorithm outperforms the sequential solution of the individual MIPs and the IWF algorithm.

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