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Localized Demand-Side Management in Electric Power Systems

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*Abstract***—Energy consumption scheduling to achieve a low power generation cost and a low peak-to-average ratio load is a critical component in the next generation of power systems, known as smart grids. Implementing such a component requires the knowledge of the whole power demand throughout the system. However, due to the diversity of power demands, this requirement is not always satisfied in practical scenarios. To address this inconsistency, the present paper addresses energy consumption scheduling in a multi-grid power system (consisting of a local grid with some neighbor inaccessible grids). The total cost minimization is formulated as an optimization problem. In addition to optimal solution, the performed formulation is provided with** *online* **stochastic iterations to capture the randomness of unknown load adaptively over time. Numerical results demonstrate the effectiveness of the proposed algorithm in following the results obtained from the optimal solution.**

*Index Terms***—Energy consumption scheduling, smart grid, power demand, stochastic load, optimization.**

I. INTRODUCTION

Reports on energy consumption reveals the increasing demand for electrical energy world wide [1]. This increasing demand along with the growing environmental concerns motivates the idea of establishing new power systems with flexible and intelligent approaches for smart demand side management. While many of these approaches are still under investigation, there already exist a number of practical applications in many countries across the world [2]. Demand side can managed by either reducing or shifting the consumption of energy. While the former can be efficient to some extent, the latter proposes the shift of high-load household consumptions to off-peak hours in order to reduce *peak-to-average ratio* (PAR). The high PAR might lead to degradation of power quality, voltage problems, and even potential damages to utility and consumer equipments.

With the advancement of smart metering technologies [3] and the increasing interest in smart grids with two-way communications capability [4], load management has been appeared in form of *energy consumption scheduling* (ECS) [5], [6]. In the ECS, the power charging time of household appliances is optimally scheduled so that demand-side load management could be managed efficiently. This results in reducing the risk of getting into a condition that may lead to blackout. As an incentive that subscribers follow ECS decisions, intelligent pricing schemes in the form of lower

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utility charges should be provided. Consequently, customers will be encouraged to shift their heavy loads to off-peak hours. This issue is not only useful to improve the overall system performance but also to pay less individually.

The proposed ECS algorithms in the literature mainly perform system-wide load management with the assumption of the knowledge of the whole system load demands a priori [7]–[9]. In other words, the system manager should be aware of the whole system demand exactly. Due to the diversity of power customers ranging from household to industrial domains, however, this case is not *mostly* valid. Alternatively, a manager who is aware of power demand in a local grid might be interested in the ECS within its own area. The fact that power price in the utility is a function of the whole system demand, the local manager needs to consider the loading impact of neighbor grids demand.

To investigate the mentioned difficulty, in this paper, we consider a source of energy shared within a power system consisting of a local grid (LG) and some neighbor grids (NGs). The LG includes a set of household appliances, which is scheduled by an ECS entity. It is assumed that the ECS is aware of appliances power demands within the LG, but dose not have the knowledge of power demands in the NGs. In other words, there is no possibility to access even to predict the demand of NGs. This motivates the authors to consider this load as a stochastic process varying over time.

One common objective of the ECS in the context of next generation power systems is to minimize the cost of power generation. We model this objective as an optimization problem and provide it with optimal solution. Moreover, we propose an adaptive solution to make decisions on the scheduling of the LG household facilities while taking into account the NG's stochastic loading.

The paper is organized as follows. The system model is described in Section II. The cost minimization formulation along with its solution are presented in Section III. Numerical results are given in Section IV and the paper is concluded in Section V.

II. SYSTEM MODEL

For the sake of energy consumption scheduling, a multi-gird power system as shown in Fig.1 is considered. This system consists of an LG controlled by an ECS and some NGs out of the control of ECS. The aim of the ECS is not to reduce the energy consumption, but instead to optimally manage and shift the LG load demand to reduce cost and PAR within the power system.

Assume that the LG consists of a set $\mathcal{N} \triangleq \{n : n =$ $1, \ldots, N$ of appliances to be scheduled during a time interval $\mathcal{T} \triangleq \{t : t = 1, \ldots, T\}$. The load demand of each appliance $n \in \mathcal{N}$ during this interval is assumed to be a known value E_n . On other hand, the load demand by the NGs is unknown. We consider it as a time varying random variable γ , but without any knowledge on its probability density function (PDF).

The load demand of the LG is scheduled by the ECS. Let l_n^t be the load provided for appliance $n \in \mathcal{N}$ during time slot t. The objective of the ECS is to determine load demands $\mathcal{L} \triangleq \{l_n^t\}_{n \in \mathcal{N}}^{t \in \mathcal{T}}$ to optimize a target performance measure, and at the same time to provide each appliance n with the required load E_n in average. Due to the random variation of NGs load demand, provided loads within the LG can be considered as functions of γ . Moreover, many home appliances have strict minimum and maximum power charging levels. As an example, the PHEVs can be charged only up to 3.3 kWh per hour [10]. This imposes the constraint that each l_n^t must be within l_n^{min} and l_n^{max} , minimum and maximum power levels, respectively.

In the subsequent section, the ECS synthesis is reduced to develop an optimization formulation with the objective of cost minimization within the system.

Fig. 1. A multi-grid power system model

III. COST MINIMIZATION FORMULATION

Pricing of electricity can be used as a mechanism to encourage customers to follow a specified load scheduling. Various pricing schemes have been proposed by economists and regulatory agencies such as flat pricing, critical-peak pricing, time-of-use pricing, and real-time pricing. These schemes have been also used in communications and transportation networks [11]. Among them, real-time pricing is motivated to be used in next generation of power systems thanks to its environmental and economical gains [12], [13]. Following these results, in the present paper, an energy scheduling approach based on real-time pricing is proposed.

Let $l^t = \sum_{n=1}^{N} l_n^t + \gamma$ be the total amount of load generated at time t . The power generation cost at this time can be denoted as a differentiable and convex function $C(l^t)$. Consequently, the average cost minimization problem is

$$
\min_{\mathcal{L}} \frac{1}{T} \sum_{t=1}^{T} C\left(\sum_{n=1}^{N} l_n^t + \gamma(t)\right) \tag{1}
$$

$$
\text{s.t.} \quad \frac{1}{T} \sum_{t=1}^{T} l_n^t \ge E_n \qquad \qquad \forall n \in \mathcal{N} \tag{2a}
$$

$$
l_n^{\min} \le l_n^t \le l_n^{\max} \qquad \forall n \in \mathcal{N}, \forall t \in \mathcal{T}.
$$
 (2b)

Constraints (2a) satisfy the average load demands by the appliances, while constraints (2b) restrict the power levels within the upper and lower bounds. This problem is convex and can be solved using convex optimization techniques such as interior point method (IPM) [14]. This requires the availability of $\gamma(t)$ for all t at the beginning of time period \mathcal{T} . However, this assumption is not valid in practice as we do not aware of NGs load demand in advance. Alternatively, we consider γ as a random variable varying over time, but without any assumption on its PDF. Therefore, (1) – (2) can be rewritten as

$$
\min_{\mathcal{L}} \mathbb{E}_{\gamma} \left[C \left(\sum_{n=1}^{N} l_n + \gamma \right) \right]
$$
 (3)

s.t.
$$
\mathbb{E}_{\gamma}[l_n] \ge E_n
$$
 $\forall n \in \mathcal{N}$ (4a)
 $\min_{l \in \mathbb{N}} \mathbb{E}_{\gamma}[l_n] \ge \max_{l \in \mathbb{N}} \mathbb{E}_{\gamma}[l_n] \ge \max_{l \in \mathbb{N}} \mathbb{E}_{\gamma}[l_n] \ge \max_{l \in \mathbb{N}} \mathbb{E}_{\gamma}[l_n] = \mathbb{E}_{\gamma}[l_n]$

$$
l_n^{\min} \le l_n \le l_n^{\max} \qquad \forall n \in \mathcal{N}.
$$
 (4b)

where \mathbb{E}_{γ} denotes the expectation with respect to γ . The above problem is also a convex optimization. We are interested in solving this problem progressively over time, when γ is realized at each time instant t.

The most significant challenge in the solution of problem (3)–(4) is due to the expectations that couple the scheduling over time. The solution would be straightforward if one decouples the energy demand constraints over time slots. This motivates the incorporation of (4a) into the objective function and forms a Lagrangian function as

$$
L(\mathcal{L}, \Lambda) = \mathbb{E}_{\gamma} \left[C \left(\sum_{n=0}^{N} l_n + \gamma \right) \right]
$$

-
$$
\sum_{n=1}^{N} \lambda_n \left(\mathbb{E}_{\gamma} [l_n] - E_n \right)
$$
 (5)

and the corresponding dual function as

$$
D(\Lambda) = \inf_{\mathcal{L}} \{ L(\mathcal{L}, \Lambda) : (4b) \}
$$
 (6)

where $\Lambda = {\lambda_n \geq 0}_{n \in \mathcal{N}}$ is the set of Lagrange multipliers. The dual function provides a lower bound on the optimal solution of (3)–(4). The best lower bound is surely achieved by the corresponding dual problem as

$$
\max_{\Lambda \ge 0} D(\Lambda). \tag{7}
$$

Prior to solve the problem in the dual domain, we first need to evaluate $D(\Lambda)$. The $L(\mathcal{L}, \Lambda)$ can be rewritten as follows

$$
L(\mathcal{L}, \Lambda) = \mathbb{E}_{\gamma} \left[C \left(\sum_{n=1}^{N} l_n + \gamma \right) - \sum_{n=1}^{N} \lambda_n l_n \right]
$$
 (8)
+
$$
\sum_{n=1}^{N} \lambda_n E_n.
$$

Therefor, to evaluate $D(\Lambda)$ in (6) we solve

$$
\min_{\mathcal{L}} \mathbb{E}_{\gamma} \left[C \left(\sum_{n=1}^{N} l_n + \gamma \right) - \sum_{n=1}^{N} \lambda_n l_n \right] \tag{9}
$$

$$
s.t. \quad (4b). \tag{10a}
$$

For each value of γ , this problem is convex and can be solved using interior point method (IPM) to obtain ${l_n^*(\gamma)}_{n \in \mathcal{N}}$. Having obtained ${l_n^*(\gamma)}_{n \in \mathcal{N}}$, the dual problem in (7) can be solved using subgradient method. Here, λ_n can be interpreted as the marginal benefit of appliance *n*. Beginning with an initial $\lambda_n(0)$, given $\lambda_n(t)$ at time t, the $\{l^t_n$ * (γ) _{n∈N} can be obtained from (9)–(10). We then update the Lagrange multiplier as

$$
\lambda_n(t+1) = \lambda_n(t) + \alpha \left(E_n - \mathbb{E}_{\gamma} \left[t_n^{t*}(\gamma) \right] \right)^{+}
$$
 (11)

where $E_n - \mathbb{E}_{\gamma} \left[l_n^t \right]$ *(γ) is the subgradient of $D(\Lambda)$ with respect to λ_n , and α is a step size.

The above described solution can be summarized as an adaptive solution in Algorithm 1.

IV. PERFORMANCE EVALUATION

We consider an LG with $N = 10$ appliances scheduled by an ECS over an interval of length 6 hours, e.g from 12:00 PM to 6:00 AM. The scheduling is updated every 1 minute, i.e. T=360. Load demands are $E = [1:1:10]/T$ kWh per unit of time. Minimum and maximum power levels are l^{min} =[0:1:9] and l^{max} =[5:1:14], respectively. The load demand of the NGs is assumed to be a normal random variable with mean 100 kWh per unit of time and standard deviation σ, i.e. $\gamma \sim \mathcal{N}(100, \sigma)$. Moreover, the power generation cost function is considered to be quadratic, i.e. $C(.) = (.)^2$ in (1)– (2).

With the aim of comparison optimal and adaptive solutions of the cost minimization, we first generate a set of normal random variables as NGs load demand at the beginning of the simulation with $\sigma = 20$ kWh. The corresponding optimal solution in (1) – (2) is shown in Fig. 2. As observed, the optimal solution regulates LG load such that the system-wide total load becomes smooth. In fact, the LG load provides a diversity for the ECS to mitigate the stochastic demand of NGs. The adaptive cost minimization algorithm 1 is also applied to this scenario to determine the scheduling of the LG load. The provided load with this scheduling (adaptive load) is also shown in Fig. 2. Intuitively, after some initial time units, the behavior of this curve approximately converges to that of the optimal solution.

Fig. 2. System-wide optimal and adaptive load demands

In the following, we compare the optimal and adaptive solutions of cost minimization in terms of price per kWh and peak-to-average ratio (PAR). These performances versus the standard deviation of γ , i.e. σ , are shown in Fig. 3 and Fig. 4, respectively. As another scheduling scheme, the results of uniform solution are also included. In this solution, the demand of each appliance is *uniformly* distributed over the whole time interval, independent of the objective function. As shown, the price and PAR performances of adaptive algorithm outperform those of uniform strategy. This is reasonably expected as the adaptive algorithm takes advantage of the diversity in NGs demand to achieve a better performance.

From comparison of adaptive and optimal solution, it is investigated that the optimal solution achieves lower price. This is due to the fact that this solution takes into account the knowledge of NGs demand at the beginning of the interval. However, the adaptive algorithm makes a scheduling decision per time unit, when the power demand of the NGs is available in that time unit. Remarkably, the PAR of the adaptive algorithm is comparable to that of the optimal solution. This implies that the optimality of production cost does not necessarily implies the outperforming of the PAR. This observation motivates the performance evaluation of the PAR minimization in the following.

V. CONCLUSION

The unpredictable load demand throughout a grid avoids global and optimal energy consumption scheduling. Alterna-

Fig. 3. Price per kWh in cost minimization

Fig. 4. PAR in cost minimization

tively, we resort to local and suboptimal scheduling schemes that adaptively perform load scheduling. In this paper, a stochastic model of load scheduling in a local grid with the objective of cost minimization has been presented. It is shown that the optimal scheduling can be followed by an online iteration that captures the randomness of neighbor grids demand adaptively. This approach makes scheduling decisions progressively over time. Indeed, the proposed adaptive algorithm can provide an estimate of the optimal solution.

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