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Distributed allocation of subcarrier, power and bit-level in multicell orthogonal frequency-division multiple-access networks

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Abstract: The downlink of multicell orthogonal frequency-division multiple-access (OFDMA) networks is studied, and the adaptive allocation of spectrum, power and rate is addressed. The authors consider networks with adaptive frequency reuse and discrete-level rates. Initially, the joint allocation problem is formulated as a centralised non-linear mixed-integer program (MIP), which is computationally intractable to solve optimally for practical problem sizes. Then, the capability of the receivers is exploited to estimate the subcarrier channel gains and the joint allocation problem is accordingly decomposed into subproblems, each of which is solved by a different base station with linear complexity. In the proposed iterative algorithm, the base stations perform rate and receiver allocation per subcarrier, with concurrent iterations. A filtering method is introduced to further decrease the algorithm complexity. Furthermore, for benchmarking purposes, the authors transform the original non-linear MIP to a linear MIP and find the optimal solution by means of standard branch-and-cut solvers. The merit of the proposed algorithm is demonstrated with numerical comparisons of its performance against the solutions of the linear MIP and the iterative waterfilling algorithm.

1 Introduction

Multicarrier transmission in the form of orthogonal frequency-division multiple access (OFDMA) has emerged as a promising technique towards high data transmission in the next generation wireless networks [1]. OFDMA mitigates the frequency selectivity of the broadband channel by dividing the bandwidth into a set of non-interfering narrowband subcarriers. Owing to independent subcarrier channel gains for different users, it is possible to dynamically assign subcarriers to users with adaptive power allocation. To fully realise the advantages of OFDMA, resource allocation schemes for the single-cell downlink have been extensively studied. Minimising the transmit power and maximising the overall throughput have been formulated as non-linear mixed integer programs (MIPs) in [2, 3], respectively. These problems have been tackled by integer relaxation and solved using convex optimisation techniques. Assuming uniform power allocation in [4], subcarriers have been assigned dynamically, and then a greedy power allocation algorithm has been used for bit loading. The key idea has been to load bits on subcarriers successively, that is, one bit per iteration, and at each iteration to select the subcarrier with the least additional required power. In [5], assuming a fixed rate for all subcarriers assigned to each user, the problem has been formulated as an integer programming problem, and solved suboptimally using linear programming.

Employing OFDMA in the context of multicell networks is a promising technique towards ubiquitous and high data rate transmission in the next generation networks [6]. Different from the single-cell case, the resource allocation in multicell networks needs to take advantage of spectrum sharing among adjacent cells to enhance the aggregate capacity. As a consequence of frequency reuse, the generated intercell interference couples the resource allocation in different cells and therefore the allocation is more challenging. Hence, the single-cell schemes cannot be directly applied to multicell networks as they do not take into account the intercell interference.

We consider the joint optimisation of resource allocation in the downlink of multicell OFDMA networks. From optimisation viewpoint, this allocation is a non-linear MIP, which is NP-hard to solve in general [7]. Significant research work has been conducted to reduce the computational complexity either in centralised or distributed manner. Centralised algorithms with partitioning reuse factor, which assign various reuse factor values to different groups of subcarriers, have been proposed in [8]. Using centralised formulations, authors in [9, 10] have proposed semi-distributed algorithms to provide partitioning reuse factor. A central unit, for example, a radio network controller, takes multicell multiuser diversity into account to determine the set of subcarriers to be used by each cell. This approach has also been adopted in [11, 12] but for fixed reuse factor. Coordinated suboptimal strategies among base stations (BSs) have also been investigated in [13, 14].

Distributed approaches are favourable in large-scale networks due to the concern of scalability, signalling overhead and complexity [15]. Distributed schemes with only one transmission format on all subcarriers have been proposed in [16, 17]. Such a choice allows great simplification of the problem, as the allocation problem reduces to only channel assignment. Moreover, the search for decentralised solutions has motivated significant work within the framework of non-cooperative game theory [18–20]. The selfish behaviour of transmitters as game players increases co-channel interference and degrades the network performance significantly. Alternatively, price or tax-based algorithms have been used to charge the transmitters for their transmit power or the number of allocated subcarriers. The necessity of attaining a Nash equilibrium in these games forces the transmit rates to be continuous. The iterative waterfilling (IWF) algorithm [21], originally proposed for digital subscriber lines, also models the interference channel as a non-cooperative game. Customising this approach for multicell networks, each BS iteratively maximises its own rate by performing single-cell power allocation with fixed intercell interference. It has been shown in [22] that the duality gap of multicarrier IWF with non-convex functions decreases as the number of carriers increases.

In this paper, we revisit the problem of resource allocation in the downlink of multicell OFDMA networks with a set of multiple discrete rates on the subcarriers. This is a generalised version of the problem, as in the literature it is mostly assumed that the rates are either continuous or discrete but with one or two bit levels. Capitalising on the structure of our non-linear MIP formulation, we devise a distributed algorithm between the network BSs. The algorithm breaks down the problem exponential complexity by allowing the BSs to collaborate in the problem solution. This collaboration is intended to mitigate the co-channel interference and accordingly provide an adaptive frequency reuse, instead of the fixed pattern in modern systems. Our distributed algorithm is different from those of [10, 23] in that, in our proposed algorithm, the individual BSs synchronously perform receiver, power and rate allocation per subcarrier at every iteration, taking into account the solution of the previous iteration. Moreover, different from [22] and the bulk of the game-theoretic literature, our problem has non-convex feasible region due to the discrete rates. On the one hand, this makes the problem more practical, but on the other hand more challenging in terms of complexity, signalling overhead and convergence. Finally, we recast our centralised formulation to overcome non-linearity and form an equivalent linear MIP, which enables us to find the optimal solution using off-the-shelf solvers, for the purpose of benchmarking.

The paper is organised as follows. The system model and joint resource allocation problem are given in Section 2. The problem decomposition is presented in Section 3. The distributed algorithm is proposed in Section 4. Simulation results are given in Section 5 and the paper is concluded in Section 6.

2 System model and problem formulation

We consider downlink transmission in a multicell OFDMA network with a set $\mathcal{L} \triangleq \{i: i = 1, \ldots, L\}$ of cells with radius R and a set $\mathcal{K} \triangleq \{k: k = 1, \ldots, K\}$ of receivers, as shown in Fig. 1. BS_i is located at the centre of cell i and serves the, randomly located within the cell, receivers in the set $\mathcal{K}_i \subseteq \mathcal{K}$ with cardinality $K_i = |\mathcal{K}_i|$. The network bandwidth is shared by all BSs and it is divided into a set $\mathcal{N} \triangleq \{n: n = 1, \ldots, N\}$ of orthogonal subcarriers. The channel of each subcarrier is frequency flat, since its bandwidth is chosen smaller than the coherence bandwidth. The rate allocated in each subcarrier is chosen from a finite set $\mathcal{Q} \triangleq \{q: q = 1, \ldots, Q\}$ of discrete bit levels. Non-linear and linear formulations of the joint resource allocation problem are proposed in the sequel subsections.

2.1 Non-linear MIP formulation

We denote the binary allocation variables $x_k^{n,q} \in \{0, 1\}$, where $x_k^{n,q} = 1$ if subcarrier n is allocated to receiver k with rate q, and $x_k^{n,q} = 0$ otherwise. To avoid intracell interference, each subcarrier can be used by at most one receiver per cell. Hence, for cell i and subcarrier n, we have the constraint

$$\sum_{k \in \mathcal{K}_i} \sum_{q=1}^{Q} x_k^{n,q} \le 1 \qquad (1)$$

The sum in the left-hand side of (1) is zero when BS_i chooses not to allocate subcarrier n to any receiver in the cell and one otherwise.

Let $G_{i,k}^n$ denotes the gain of the channel between BS_i and receiver k on subcarrier n. The signal-to-interference-plus-noise ratio (SINR) of receiver $k \in \mathcal{K}_i$, served by BS_i on subcarrier n with transmit power p_i^n, is

$$\gamma_k^n \triangleq \frac{G_{i,k}^n p_i^n}{I_k^n(p_{-i}^n)} \qquad (2)$$

In (2), the intercell interference on subcarrier n plus the receiver noise variance σ_k^2 is defined as

$$I_k^n(p_{-i}^n) \triangleq \sum_{j=1, j \ne i}^{L} G_{j,k}^n p_j^n + \sigma_k^2 \qquad (3)$$

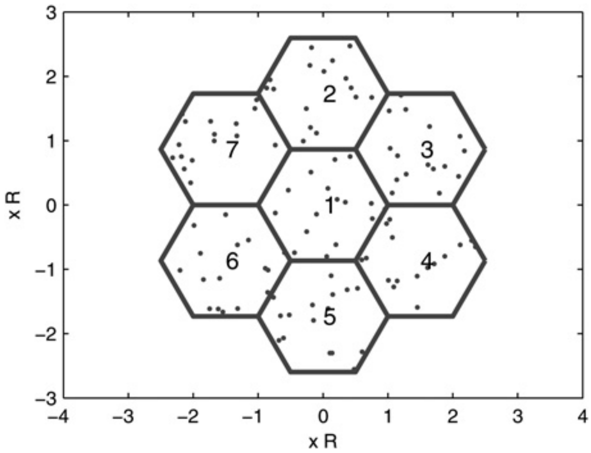

Fig. 1 *Multicell network setup*

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where $p_{-i}^n \triangleq \left[p_1^n, \ldots, p_{i-1}^n, p_{i+1}^n, \ldots, p_L^n\right]$ is the vector of all interfering transmit powers on subcarrier n.

Given a modulation and coding scheme, and a bit-error rate target, let T_q denotes the SINR threshold to load q bits on a subcarrier. Owing to (2), when BS_i decides to serve receiver $k \in \mathcal{K}_i$ with q bits on subcarrier n, that is, $x_k^{n,q} = 1$, the required transmit power is $p_i^n = I_k^n(p_{-i}^n)T_q/G_{i,k}^n$ to ensure $\gamma_k^n = T_q$. The dependency on the receiver/rate allocation becomes explicit. By equivalently rewriting, because of (1), this power becomes

$$p_i^n = \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q} I_k^n(p_{-i}^n)T_q/G_{i,k}^n \tag{4}$$

Moreover, we assume that the total transmit power of BS_i, which is split across the subcarriers, cannot exceed the budget P, that is

$$\sum_{n=1}^{N} p_i^n \le P \tag{5}$$

The objective of the joint resource allocation problem under the aforementioned constraints is to maximise the achievable sum-rate in the network, defined as the sum of bit rates over all subcarriers and cells, that is

$$\max_{\mathrm{X,P}} . \sum_{i=1}^{L}\sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} q x_k^{n,q} \tag{6a}$$

$$\text{s.t.} \quad \text{Equations (1) and (4)} \quad \forall i \in \mathcal{L}, \forall n \in \mathcal{N} \tag{6b}$$

$$\text{Equation (5)} \quad \forall i \in \mathcal{L} \tag{6c}$$

Problem (6) is a MIP [24] with KNQ binary subcarrier/rate allocation variables $\boldsymbol{X} = \left\{x_k^{n,q} \in \{0, 1\}\right\}_{k\in\mathcal{K}}^{n\in\mathcal{N},q\in\mathcal{Q}}$ and LN continuous power variables $\mathbf{P} = \left\{p_i^n \in \mathbb{R}_+\right\}_{i\in\mathcal{L}}^{n\in\mathcal{N}}$. This problem is NP-hard in general [25]. The formulation is non-linear due to the sum in the right-hand side of (4) which consists, due to (3), of bilinear terms of the optimisation variables. Finding the optimal solution requires an exhaustive search with worst-case complexity $O((KQ)^N)$, which is exponential to the number of subcarriers, hence prohibitive for modern broadband networks. This motivates the low-complexity distributed approach that we are proposing in Sections 3 and 4.

2.2 Linear MIP formulation

We revisit the problem in (6) to eliminate the non-linearity in the equality (4). This will transform the formulation into a linear MIP problem that can be solved by off-the-shelf solvers. Inserting (3) to elaborate (4) in terms of interfering transmit powers, we have

$$\begin{aligned} p_i^n &= \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q}\left(\sum_{j=1,j\ne i}^{L} G_{j,k}^n p_j^n + \sigma_k^2\right)T_q/G_{i,k}^n \\ &= \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q}\sum_{j=1,j\ne i}^{L} G_{j,k}^n x_k^{n,q} p_j^n T_q/G_{i,k}^n \\ &\quad + \sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} x_k^{n,q}\sigma_k^2 T_q/G_{i,k}^n \end{aligned} \tag{7}$$

Let $w_{k,j}^{n,q} \triangleq x_k^{n,q} p_j^n$ be an auxiliary variable to be replaced with cross terms $x_k^{n,q} p_j^n$ in (7). This variable is p_j^n if $x_k^{n,q} = 1$, and 0 otherwise. This statement can reasonably be translated into the following constraints

$$w_{k,j}^{n,q} \ge p_j^n - \left(1 - x_k^{n,q}\right)P \tag{8a}$$

$$w_{k,j}^{n,q} \le x_k^{n,q} P \tag{8b}$$

$$w_{k,j}^{n,q} \le p_j^n \tag{8c}$$

$$w_{k,j}^{n,q} \ge 0 \tag{8d}$$

In contrast to the bilinear cross terms in (7), this set of constraints is linear. Including these constraints into (6) and substituting any $x_k^{n,q} p_j^n$ with $w_{k,j}^{n,q}$ in (4), we come up with a linear MIP. Even though the resulting problem is still NP-hard, there exist several techniques, for example, branch-and-cut, and software packages that can find the optimal solution efficiently, by frequently avoiding the exhaustive search.

3 Decomposing the resource allocation

The most significant challenge towards solving (6) is that the resource allocation is not only coupled across the subcarriers, due to the sum in (5), but also across the cells, due to the interference-plus-noise terms $\left\{I_k^n(p_{-i}^n)\right\}$ in (4). In this section, we use dual decomposition in order to decouple the joint problem into subproblems and specify the condition under which these can be solved separately by the BSs. These solutions are obtained synchronously in every iteration of the distributed algorithm that we propose in the following section.

Towards the end, we incorporate the power constraints (6c) in the objective function (6a) to form the partial Lagrangian function

$$L(\boldsymbol{X}, \boldsymbol{P}, \Lambda) \triangleq \sum_{i=1}^{L}\sum_{n=1}^{N}\sum_{k\in\mathcal{K}_i}\sum_{q=1}^{Q} q x_k^{n,q} - \sum_{i=1}^{L}\lambda_i\left(\sum_{n=1}^{N} p_i^n - P\right) \tag{9}$$

where $\Lambda = \{\lambda_i \ge 0\}_{i\in\mathcal{L}}$ is the set of Lagrange multipliers. Optimising with respect to the primal variables $\{\boldsymbol{X}, \boldsymbol{P}\}$ yields the dual function

$$D(\Lambda) \triangleq \max_{\boldsymbol{X},\boldsymbol{P}}\left\{L(\boldsymbol{X}, \boldsymbol{P}, \Lambda)|(1), (4)\forall i \in \mathcal{L}, \forall n \in \mathcal{N}\right\} \tag{10}$$

which provides an upper bound on the optimal value of the primal problem (6) for every feasible value of the dual variables Λ. Hence, the tightest upper bound is obtained by the dual problem

$$\min_{\Lambda \geq 0} D(\Lambda) \tag{11}$$

This problem is always convex and can be solved using iterative methods [26]. Each iteration requires evaluating the dual function. Exploiting the decomposable, per BS, form of (9), we rewrite the dual function as

$$D(\Lambda) = \sum_{i=1}^{L} D_i(\lambda_i) = \sum_{i=1}^{L} d_i(\lambda_i) + \sum_{i=1}^{L} \lambda_i P \tag{12}$$

where $D_i(\lambda_i)$ is the dual function of BS_i.

BS_i obtains the term $d_i(\lambda_i)$ as the optimal value of the subproblem

$$\max_{\boldsymbol{X}_i, \boldsymbol{P}_i} \sum_{n=1}^{N} \sum_{k \in \mathcal{K}_i} \sum_{q=1}^{Q} q x_k^{n,q} - \lambda_i \sum_{n=1}^{N} p_i^n \tag{13a}$$

$$\text{s.t.} \quad (1) \text{ and } p_i^n = \sum_{k \in \mathcal{K}_i} \sum_{q=1}^{Q} x_k^{n,q} I_k^n T_q / G_{i,k}^n, \quad \forall n \in \mathcal{N} \tag{13b}$$

by optimising over the cell i variables $\boldsymbol{X}_i = \{x_k^{n,q}\}_{k \in \mathcal{K}_i}^{n \in \mathcal{N}, q \in \mathcal{Q}}$ and $\boldsymbol{P}_i = \{p_i^n\}^{n \in \mathcal{N}}$. Here, we make the assumption that BS_i acquires knowledge of the transmit powers $\{p_{-i}^n\}^{n \in \mathcal{N}}$ of the other BSs, once they have performed the allocation. Hereby, BS_i is able to calculate, using (3), the interference-plus-noise terms $\{I_k^n(p_{-i}^n)\}_{k \in \mathcal{K}_i}^{n \in \mathcal{N}}$ that its users experience. Using these terms as input, the MIP problem (13) is linear as the equality constraints in (13b), differently than (4), are linear.

Inspecting the equalities in (13b), we observe that each of the transmit powers p_i^n depends only on the variables $\{x_k^{n,q}\}_{k \in \mathcal{K}_i}^{q \in \mathcal{Q}}$ of subcarrier n. Hence, substituting the equalities in (13b) into (13a), we can eliminate the power variables and equivalently rewrite problem (13) as

$$\max_{\boldsymbol{X}_i} \sum_{n=1}^{N} \sum_{k \in \mathcal{K}_i} \sum_{q=1}^{Q} x_k^{n,q} \left(q - \lambda_i I_k^n T_q / G_{i,k}^n \right) \tag{14a}$$

$$\text{s.t.} \quad \text{Equation (1)} \quad \forall n \in \mathcal{N} \tag{14b}$$

Owing to (14b), each subcarrier can be used by at most one receiver, with a single rate, in cell i. This fact, along with the decomposable form of (14a) enables decoupling the allocation per subcarrier. For subcarrier n, the optimal receiver/rate allocation (k_n, q_n) is the one corresponding to the maximum of the $K_i Q$ possible terms $H_k^{n,q} \triangleq q - \lambda_i I_k^n T_q / G_{i,k}^n$, that is

$$(k_n, q_n) = \arg\max_{k \in \mathcal{K}_i, q \in \mathcal{Q}} H_k^{n,q} \tag{15}$$

provided that $H_{k_n}^{n,q_n} > 0$. In other words, $x_k^{n,q} = 1$ if $k = k_n$ and $q = q_n$, otherwise $x_k^{n,q} = 0$. Owing to (13b), the transmit power is $p_i^n = I_k^n T_q / G_{i,k}^n$ in the former case and $p_i^n = 0$ in the latter case. However, when $H_{k_n}^{n,q_n} \leq 0$, then $x_k^{n,q} = 0 \; \forall k \in \mathcal{K}_i$, $\forall q \in \mathcal{Q}$ and accordingly $p_i^n = 0$. The complexity of finding (15) is $O(K_i Q)$, so the overall complexity of solving (14) is $O(NK_i Q)$.

Having obtained the terms $d_i(\Lambda_i)$ and accordingly $D(\lambda)$, it is time to solve the dual problem (11) to determine Λ. Thanks to the convexity, this problem can be solved efficiently using the subgradient method [26]. Beginning with an initial $\lambda_i(0)$, BS_i, at iteration t, first obtain $\boldsymbol{P}_i$ from $\boldsymbol{X}_i$ using the equalities in (13b) and then update its Lagrange multiplier using

$$\lambda_i(t+1) = \left[\lambda_i(t) - \alpha \left(P - \sum_{n=1}^{N} p_i^n \right) \right]^+ \tag{16}$$

where $P - \sum_{n=1}^{N} p_i^n$ is the subgradient of $D_i(\lambda_i)$ with respect to λ_i and α is a step size. This approach allows the BSs to contribute separately towards obtaining the overall solution.

4 Distributed resource allocation algorithm

Given the solution of the resource allocation subproblems at individual BSs in Section 3, we are now in position to propose a distributed subcarrier, power, and bit level (DSPB) allocation algorithm for the overall multicell network. This algorithm is based on the iterative update of the Lagrange multipliers in (16). As in any iterative implementation, a concern is raised on the computational complexity and signalling overhead exchanged during the algorithm. While the former has been addressed by the efficient problem decomposition per BS and subcarrier in Section 3, the latter is a critical issue in wireless networks and still needs to be investigated. At the beginning of the algorithm, each receiver estimates the gains of the channels from all the BSs, over all the subcarriers, and feeds them back to its serving BS. This is the only information that the receivers need to send over the air interface, assuming that the channel gains remain constant while the allocation is being decided. All other information is exchanged among the BSs, through the high-capacity links that interconnect them. At every iteration, the BSs synchronously solve the cell subproblems to determine their new power allocation over the subcarriers and broadcast their solutions to the other BSs. This information enables each BS to update the intercell interference values experienced by its receivers on all the subcarriers. Finally, at the end of the algorithm, each BS forwards the subcarrier/rate allocation decisions to its receivers.

DSPB is distributed in the sense that each BS takes its allocation decisions autonomously by solving its own version of (14). However, since these subproblems have binary variables, that is, the feasible regions are non-convex, the convergence of this algorithm is not guaranteed. Alternatively, we resort to a finite interval of iterations to terminate the algorithm. Let T denotes the length of this interval in number of iterations that yield updated resource allocations. Analysing the simulation results after every DSPB iteration, we have observed that once a receiver/rate allocation to a subcarrier does not change during the first half of the interval, it is highly probable that it will remain constant during the rest of the interval. Inspired by this observation, we bisect the interval T into $\log_2 T$ disjoint subintervals, each of which has half the length of the previous one, with the last subinterval being one iteration long. Let s denotes the subinterval index and $\mathcal{T}_e$ the set of iterations marking the end of the

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subintervals. At iteration $\mathcal{T}_e(s)$, we introduce a sort of subcarrier filtering; specifically, at each BS_i some subcarriers are allocated a receiver and rate which is kept fixed for the remaining DSPB iterations, so that these subcarriers are excluded from the upcoming resource allocations. Specifically, during each subinterval s, BS_i counts and denotes by $f_s^{i,n}$ the number of iterations in which the allocation per subcarrier n has changed. Let $\bar{f}_s^i$ be the average of the terms $f_s^{i,n}$ with respect to n during subinterval s at BS_i. We take this average value as a threshold for filtering, that is, if $f_s^{i,n} \le \bar{f}_s^i$, then subcarrier n does not change its receiver/rate allocation anymore. Therefore, it is offered the solution achieved at the end of subinterval s and the values of the corresponding binary variables are decided and kept fixed until the algorithm terminates. Since the length of the last subinterval is only one iteration, at DSPB termination all the subcarriers will have a fixed allocation. As a consequence of this procedure, the subcarriers allocated no receiver, that is, achieving zero rate in a particular cell, they will be switched off in that cell, that is, there will be no transmit power on them.

The table below formally presents the proposed DSPB algorithm at each BS_i. All BSs run this algorithm concurrently.

The DSPB algorithm is initialised with uniform distribution of transmit power across the subcarriers, a set of values for the Lagrange multipliers and a set of filtering instants $\mathcal{T}_e$, calculated using the input value for the total iterations T (Fig. 2). At each iteration t of the algorithm, all BSs solve their cell-specific resource allocation subproblems concurrently. Specifically, each BS computes the interference, makes the allocation decisions, updates the Lagrange multiplier and broadcasts the transmit powers to the other BSs. These powers are used by the other BSs at the subsequent iteration to update the interference values. At the iteration instants determined in $\mathcal{T}_e$, the BSs perform subcarrier filtering. Finally, after algorithm termination, each BS forwards its own final allocation decisions to the receivers it serves.

The DSPB algorithm takes advantage of the two decomposition levels to overcome the exponential complexity $O((KQ)^N)$ of exhaustive search methods required to solve problem (6). First, decoupling the network problem into cell-specific subproblems (14), the DSPB complexity becomes linear in the number of BSs, due to the autonomous Lagrange multiplier update in (16). Second, the overall complexity, over the T iterations, of the subcarrier and rate allocation in cell i is reduced to $O(TNK_iQ)$, as we

Algorithm 1

Initialisation: $t = 1$, $s = 1$, set filtering instants $\mathcal{T}_e$, $\lambda_i(1) = 10$, $f_s^{i,n} = 0$, and $p_i^n = P/N\ \forall n \in \mathcal{N}$;
Receivers $k \in \mathcal{K}_i$ estimate $\{G_{j,k}^n\}_{j\in\mathcal{L}}^{n\in\mathcal{N}}$ and feed them back to BS_i;
while $t \le T$ **do**
  BS_i computes $\{I_k^n(p_{-i}^n)\}_{k\in\mathcal{K}_i}^{n\in\mathcal{N}}$;
  Using (15), BS_i determines $(k_n(t), q_n(t))$ and accordingly computes $\mathbf{X}_i(t)$;
  Using (13b), BS_i computes $\mathbf{P}_i(t)$ and then updates $\lambda_i(t)$ using (16);
  BS_i broadcasts $\mathbf{P}_i(t)$ to the other BSs;
  for $n \in \mathcal{N}$ **do**
    if $(k_n(t), q_n(t)) \ne (k_n(t-1), q_n(t-1))$ **then**
      $f_s^{i,n} := f_s^{i,n} + 1$;
    end if
  end for
  if $t == \mathcal{T}_e(s)$ **then**
    Define $\bar{f}_s^i = \frac{1}{N}\sum_{n=1}^{N} f_s^{i,n}$;
    for $n \in \mathcal{N}$ **do**
      if $f_s^{i,n} \le \bar{f}_s^i$ **then**
        $(k_n(t), q_n(t))$ does not change anymore;
        Subcarrier n is filtered, i.e., it is removed from the rate/receiver allocation process;
      end if
    end for
    $s := s + 1$;
    $f_s^{i,n} = 0\ \forall n \in \mathcal{N}$
  end if
  $t := t + 1$;
end while
BS_i forwards final allocation decisions $\mathbf{X}_i(T)$ to its receivers;

Fig. 2 *Distributed subcarrier, power and bit-level allocation (DSPB) at BS_i*

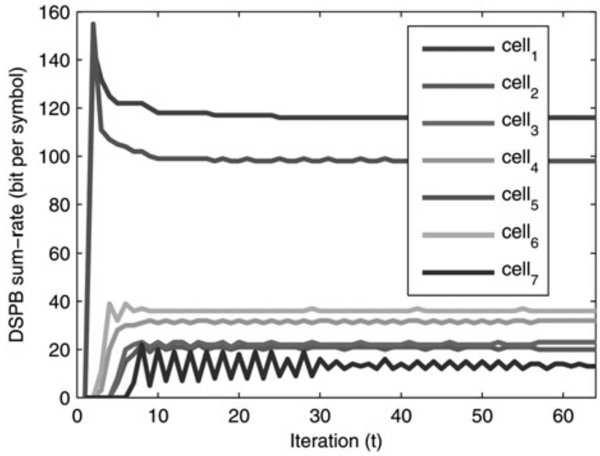

Fig. 3 *Sum-rate variation in DSPB*

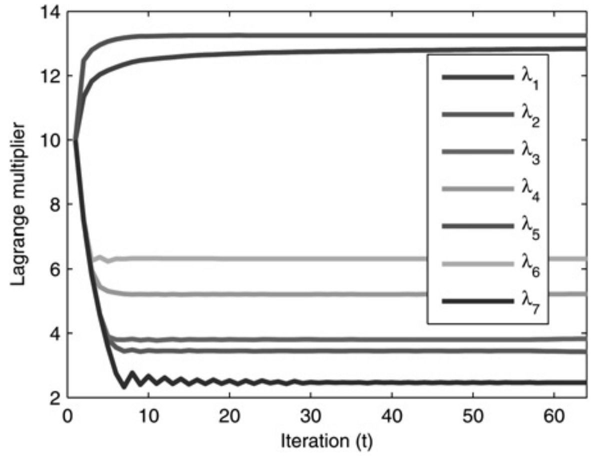

Fig. 5 *Lagrange multiplier variation in DSPB*

decomposed the optimisation per subcarrier in (15). Actually, this is a conservative upper bound, since due to filtering, the algorithm complexity is further reduced, as the number of rate and receiver allocation of subcarriers diminishes at every subinterval. Assuming that $K_i \simeq K/L\ \forall i \in \mathcal{L}$, the overall DSPB complexity is $O(TNQK)$, that is, linear in the number of subcarriers, bit levels and receivers.

5 Performance evaluation

We consider downlink transmission in a network with $L = 7$ hexagonal cells of radius $R = 2$ km. Every BS, located at the centre of the corresponding cell, serves $K_i = 16$ active receivers, randomly placed within the cell. The path loss (in dB) at a distance d from a BS is given by $L(d) = L(d_0) + 10\alpha\log_{10}(d/d_0)$, where for the reference point it is $d_0 = 50$ m, $L(50) = 0$, and the path loss exponent is $\alpha = 3.5$. The shadowing effect is modelled as an independent log-normal random variable with 8 dB standard deviation. The channel on each link is assumed to be Rayleigh fading, modelled by a six-tap impulse response with exponential power delay profile indicated by $ge^{-(l-1)}$, where $g = 1$ is the first path's average power gain and l is the path index. Moreover, the root-mean-square delay spread is 0.9 µs. The transmission budget of each BS is $P = 5$ W and the noise variance is assumed to be $\sigma_k^2 = -70$ dBm for all receivers. The bit level on each subcarrier is chosen from the set $\mathcal{Q} = \{1, 2, ..., 5\}$, so that the corresponding SINR thresholds, assuming Gaussian signalling and Shannon's capacity equation, are $\mathcal{T}_q = \{1, 3, 7, 15, 31\}$, respectively.

DSPB runs for $T = 64$ iterations with filtering instants $\mathcal{T}_e = \{32, 48, 56, 60, 62, 63, 64\}$, resulting from bisecting the iteration interval. First, to investigate the performance of DSPB for a typical number of subcarriers, for example, $N = 128$, we show in Fig. 3 the sum-rate achievement (in bits per OFDM symbol) of each cell against the iteration number. The sum-rate of the ith cell is evaluated as $u_i = \sum_{n=1}^{N} \sum_{k \in \mathcal{K}_i} \sum_{q=1}^{Q} q x_k^{n,q}$. The total transmit power for each cell and the variation in the Lagrange multipliers within the cells are also shown in Figs. 4 and 5, respectively. After some variations mostly in the first subinterval, these parameters attain fixed values during the determined interval. Fig. 4 shows that all BSs make use of their power budget, as also verified by the convergence of Lagrange multipliers in Fig. 5.

Despite the concurrent run of optimisation subproblems by BSs in the formulation, in our simulation, the order of problem running is from BS1 o BS7. In other words, BS7 is the last BS that performs resource allocation at each iteration and accordingly it is faced with more interference. As a result of this interference, there are more subcarrier back and forth between this BS and the rest of the network, or even more change in the allocated power to assigned subcarriers.

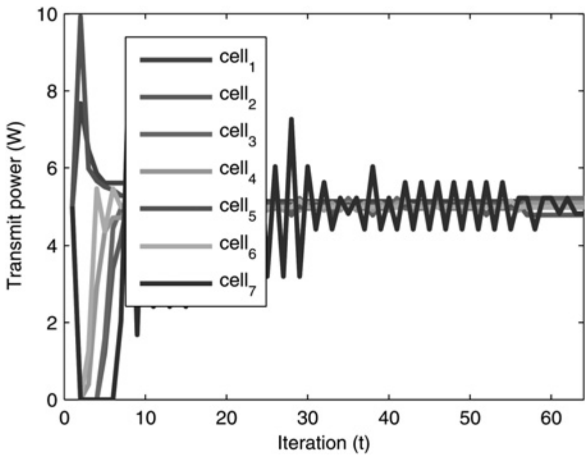

Fig. 4 *Power variation in DSPB*

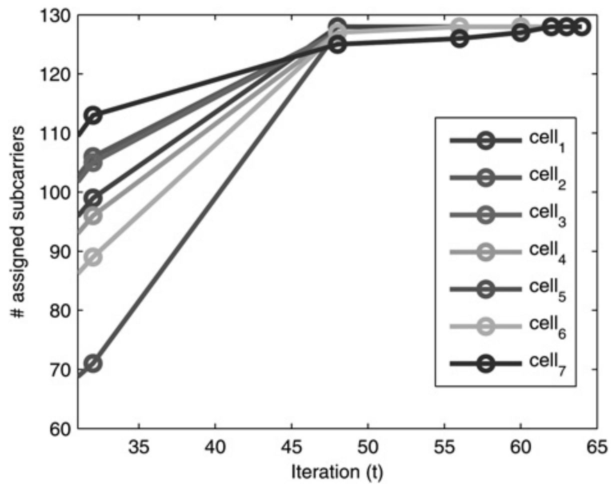

Fig. 6 *Number of assigned subcarriers after subintervals*

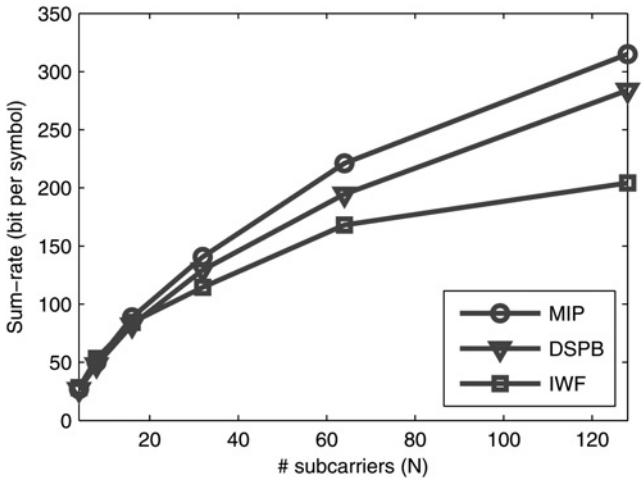

Fig. 7 *Average sum-rate in MIP, DSPB and IWF*

The fluctuations in the curves of cell 7 in Figs. 3–6 could be justified in this viewpoint.

As stated in the DSPB algorithm, at the end of each subinterval, some subcarriers are filtered, that is, they are allocated a fixed rate and receiver. The number of filtered subcarriers after each subinterval is indicated in Fig. 6 for all cells. As shown, at least 60% of subcarriers have been assigned a fixed rate and receiver in the network. This observation verifies the intuition of proposed filtering method in DSPB algorithm.

In the following, we compare, in the aforementioned setup, the performance of DSPB with the results obtained by solving the linear MIP version of problem (6) presented in Section 2.2, as a benchmark. The optimal solution of linear MIP is achieved calling the GNU linear programming kit (GLPK) [27]. As a lower bound, we also include the sum-rate values achieved from the IWF algorithm [21, 28] customised to OFDMA systems using joint subcarrier and power allocation as in [3, 4]. Since the subcarrier rates in IWF are assumed to be continuous, we round off each achievable rate to the largest integer value not greater than that rate. We compare the aggregate rate of the network in the aforementioned schemes with different number N of subcarriers. For each value of N, we obtain the sum-rates for 50 realisations of the fading channel gains and show the average rates in Fig. 7. It is observed that DSPB algorithm follows the optimal results obtained from the solution of linear MIP. DSPB performance also outperforms that of IWF. The performance gap between DSPB and IWF becomes larger as the number of subcarriers increases. This is due to the degradation effect of the rounding operation in IWF which increases with the number of subcarriers.

6 Conclusion

We formulated the spectrum, power and rate allocation problem that maximises the sum-rate of multicell OFDMA networks as a non-linear MIP, which is computationally intractable to solve for practical problem sizes. The capability of the receivers to estimate channel gains per subcarrier enabled us to decouple the global problem into local subproblems that can be solved at each cell. Given these local solutions, we proposed a distributed algorithm, in which the BSs concurrently participate in the solution of the global problem, by performing the optimisation per subcarrier and updating a Lagrange multiplier at each iteration. The complexity of this solution in every iteration is linear to the number of allocation variables of each BS. We demonstrated with numerical results that the proposed algorithm follows the performance of the optimal solution obtained by branch-and-cut solvers.

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