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ROBUST DESIGN OF POWER SYSTEM STABILIZER: AN LMI APPROACH

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ABSTRACT

This paper addresses an LMI (linear matrix inequalities) based robust control methodology for designing of power system stabilizers (PSS). The PSS design problem is considered as a multi-objective control problem and formulated via a mixed H_2/H_∞ control technique, then for a given power system it is easily carried out to synthesis the desired robust controller by solving standard LMI. A single-machine infinite-bus system example is given to illustrate the developed approach. The results of the proposed control strategy are compared with conventional PSS design. The robust PSS is shown to maintain the robust performance and minimize the effect of disturbance and specified uncertainties.

KEY WORDS

Power system stabilizer, mixed H_2/H_∞ control, robust performance, LMI.

1. Introduction

Power systems continuously experience changes in operating conditions due to variations in generation/load and a wide range of disturbances [1]. The power system stabilizer (PSS) design objectives (i.e. holding the power system stability and performance in the presence of various disturbances and uncertainties) determines the PSS synthesis as a multi-objective control problem. Therefore, it is expected that an appropriate multi-objective control strategy could be able to give a better solution for this problem. However, in the most of reported robust PSS approaches, for example [2], [3] and [4], only one single norm is used to capture design specifications. It is clear that meeting all the PSS design objectives by a single norm-based control approach is difficult. Furthermore each robust method is mainly useful to capture a set of special specifications. For instance, the regulation against random disturbances more naturally can be addressed by LQG or H_2 synthesis. The H_2 tracking design is more adapted to deal with transient performance by minimizing the linear quadratic cost of tracking error and control input, but H_∞ approach (and μ as a generalized H_∞ approach) is more useful to maintain closed-loop stability in the presence of control constraints and uncertainties.

While the H_∞ norm is natural for norm-bounded perturbations, in many applications the natural norm for the input-output performance is the H_2 norm. It is shown that using the combination of H_2 and H_∞ (mixed H_2/H_∞) allows a better performance for a control design problem including both set of above objectives [5].

An application of mixed H_2/H_∞ control technique for tuning of PSS under pole region constraints is given in [6]. In continuation, the present paper provides a more general H_2/H_∞-based control framework for decentralized designing of power system stabilizers.

In this paper, first the robust stability and performance objectives are formulated via a multi-objective control problem and then the desired PSS will be obtained using a mixed H_2/H_∞ control approach. The model uncertainty in power system is covered by an unstructured multiplicative uncertainty block. The proposed strategy is applied to a single-machine infinite-bus system. To show the effectiveness of this methodology, the results of the proposed multi-objective approach are compared with the conventional PSS design.

2. Control Methodology

2.1 Control Background

A general control scheme for the mixed H_2/H_∞ control technique is sketched in Fig. 1. $G_i(s)$ is a linear time invariant system with the following state-space realization.

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_{1i} w_i + B_{2i} u_i \\ z_{\infty i} &= C_{\infty i} x_i + D_{\infty 1i} w_i + D_{\infty 2i} u_i \\ z_{2i} &= C_{2i} x_i + D_{21i} w_i + D_{22i} u_i \\ y_i &= C_{yi} x_i + D_{y1i} w_i \end{aligned} \quad (1)$$

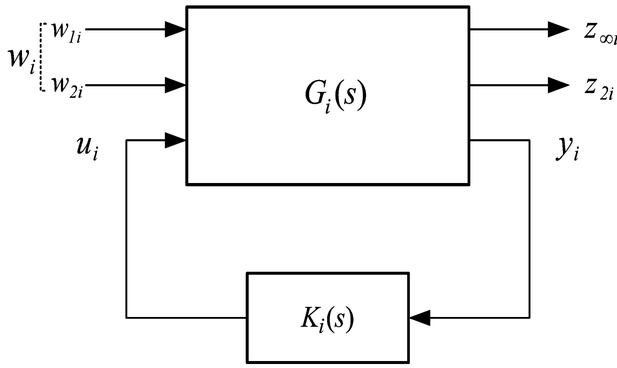


Fig. 1 Closed-loop system via mixed H_2/H_∞ control

Where x_i is the state variable vector, w_i is the disturbance and other external input vector, y_i is the measured output vector and $K_i(s)$ is the controller. The output z_{2i} is associated with the H_2 performance while the $z_{\infty i}$ is associated with the H_∞ performance. Let $T_{z_{\infty i} w_{1i}}$ and $T_{z_{2i} w_{2i}}$ as the transfer functions from $w_i = [w_{1i} \;\; w_{2i}]^T$ to $z_{\infty i}$ and z_{2i} respectively.

The H_2/H_∞ based output feedback controller $K_i(s)$ minimizes a trade-off criterion of the following form [5, 7], with $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$.

$$\alpha_1 \left\| T_{z_{\infty i} w_{\infty i}} \right\|_\infty^2 + \alpha_2 \left\| T_{z_{2i} w_{2i}} \right\|_2^2 \tag{2}$$

2.2 Proposed Control Framework

The main control framework to formulate the PSS design problem via a mixed H_2/H_∞ control design for a given power system "i" is shown in Fig. 2. $G_i(s)$ and $K_i(s)$ correspond to the nominal dynamical model of the given power system and PSS, respectively. Also y_i is the measured output, u_i is the control input and w_i includes the perturbed and disturbance signals in the control area.

The model uncertainties in power system can be considered as multiplicative and/or additive uncertainties [4]. In Fig. 2, Δ_i models the structured uncertainty set in the form of multiplicative type and W_i includes the associated weighting function. The output channel $z_{\infty i}$ is associated with the H_∞ performance while the fictitious output z_{2i} is associated with LQG aspects or H_2 performance.

The η_{1i}, η_{2i} and η_{3i} in Fig. 2 are constant performance weighting coefficients. Experience suggests that one can fix the weights η_{1i}, η_{2i} and η_{3i} to unity and use the method with regional pole placement technique for performance tuning [8]. The PSS design problem as a multi-objective control problem can be expressed by the following optimization problem: design a controller that minimizes the 2-norm of the fictitious output signal z_{2i} under the constraints that the ∞-norm of the transfer function from w_{1i} to $z_{\infty i}$ is less than one. On the other hand, the PSS design is reduced to find an internally stabilizing controller $K_i(s)$ such that,

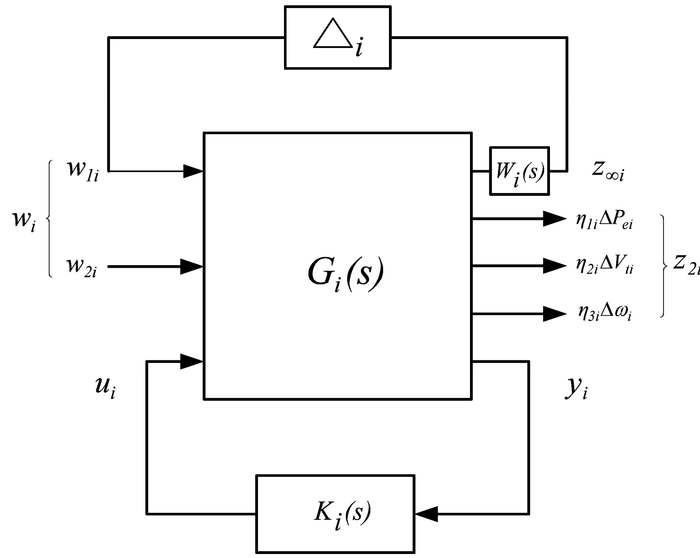


Fig. 2. Mixed H_2/H_∞-based PSS synthesis framework

$$\textit{minimize}\ \gamma_2 = \left\| T_{z_{2i} w_{2i}} \right\|_2 \ \ \textit{subject to}\ \gamma_\infty = \left\| T_{z_{\infty i} w_{1i}} \right\|_\infty < 1 \tag{3}$$

This problem can be solved by convex optimization techniques using linear matrix inequalities (LMI) [9]. Using conventional linear models for the given power system "i", it will be easy to find the state-space realization in form of (1). Here, disturbance input and output vector z_{2i} are considered as follows:

$$w_i^T = [w_{1i} \quad w_{2i}],\ w_{2i} = \Delta V_{tdi}$$

$$z_{2i}^T = [\eta_{1i}\Delta P_{ei} \quad \eta_{2i}\Delta V_{ti} \quad \eta_{3i}\Delta\omega_i]$$

where w_{1i} is the perturbed input and,

ΔV_{td} voltage disturbance,

ΔP_e electrical power,

ΔV_t terminal voltage,

$\Delta\omega$ machine speed.

The H_2 performance is used to minimize the effects of disturbances on electrical power (ΔP_e), speed ($\Delta\omega$) and terminal voltage (ΔV_t). Also in PSS design, it is important to keep up the stability and desired performance in the face of uncertainties affecting the power system. The H_∞ performance is used to meat the robustness against specified uncertainties and reduction of its impact on closed-loop system performance.

2.3 Modeling of Uncertainties

Power systems are constantly subjected by variations in generation/load as well as changes in transmission networks. These variations can be expressed as a parametric or unstructured uncertainty in the small signal linearized model of the system. It is clear that modeling of uncertainties due to parameters variations increases the complexity of computations and control structure, so that finding a tighter control solution by a simple structure will be difficult.

Here, the power system uncertainties are modeled as an unstructured input multiplicative uncertainty, as shown in Fig. 3. The uncertainty block W_i contains all possible variations for the uncertain parameters in the assumed ranges.

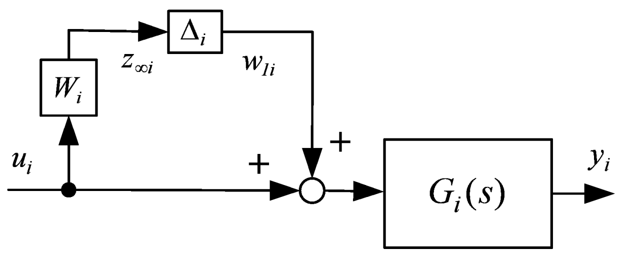


Fig. 3. Modeling the parameters variation as an input multiplicative uncertainty

Let $\hat{G}_i(s)$ denotes the transfer function from the control input u_i to control output y_i at operating points other than nominal point. Following a practice common in robust control, we can represent this transfer function as

$$\left|\Delta_i(s)W_i(s)\right| = \left|[\hat{G}_i(s) - G_{0i}(s)]G_{0i}(s)^{-1}\right| \tag{4}$$

where,

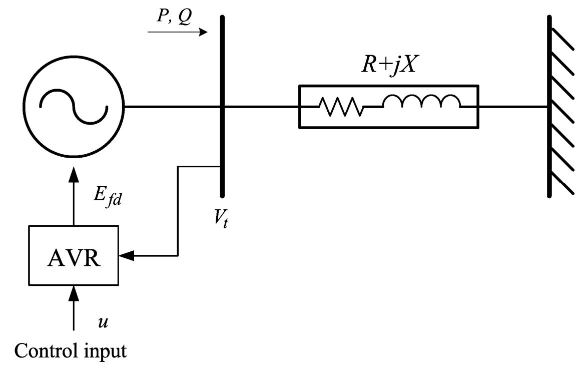
$$\left\|\Delta_i(s)\right\|_\infty = sup_\omega\left|\Delta_i(s)\right| \le 1;\ \ G_{0i}(s) \ne 0 \tag{5}$$

$\Delta_i(s)$ shows the uncertainty block corresponding to the perturbed terms and $G_{0i}(s)$ is the nominal transfer function model. Thus, $W_i(s)$ is such that its respective magnitude bode plot covers the bode plots of all possible open-loop structures.

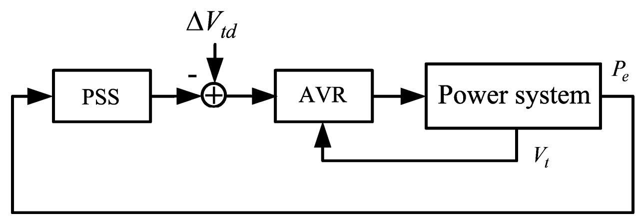
3. Application to a Single-Machine Infinite-Bus System

To illustrate the effectiveness of the proposed control strategy, one-machine infinite-bus system is considered as a test system. A single line representation of the power system is shown in Fig. 4(a) and the block diagram of the closed-loop system is shown in Fig. 4(b). The electrical power signal is considered as the input of PSS. The power system parameters are given in Appendix. The state variables and the measured output signal are chosen as (6), where v_R, E_{fd} and e'_q are AVR voltage, field excitation voltage and the quadratic-axis transient voltage, respectively.

$$\begin{aligned} x^T &= \begin{bmatrix}\Delta\omega & \Delta\delta & \Delta E_{fd} & \Delta e'_q & \Delta v_R\end{bmatrix} \\ y &= \Delta P_e \end{aligned} \tag{6}$$



(a)



(b)

Fig. 4. Single-machine infinite-bus power system; (a) Single line representation, (b) Closed-loop block diagram.

3.1 Weights Selection

In this example with regards to uncertainty, it is assumed that the parameters of connected line to infinite bus (R_l and X_l) have uncertain parameters. Considering a more complete model by including additional uncertainties is possible and causes less conservative in synthesis. However, the complexity of computations and the order of resulted controller will increase. These uncertainties are modeled as an unstructured multiplicative uncertainty block that contains all the information available about R_l and X_l variations.

Using (4), some sample uncertainties for $\pm 50\%$ changes are shown in Fig. 5. To keep the complexity of obtained controller low, we can model uncertainties from both parameters variation by using a norm bonded multiplicative uncertainty to cover all possible plants as follows

$$W_i(s) = \frac{0.0001s^3 + 0.0059s^2 + 0.3505s + 3.3128}{-0.001s^3 + 0.0215s^2 + 0.2078s + 3.781}.$$

Fig. 5 clearly shows that attempts to cover the uncertainties at all frequencies and finding a tighter fit using higher order transfer function will result in high-order controller. The weight W_i used in our design provides a conservative design at high frequencies but it gives a good trade-off between robustness and controller complexity.

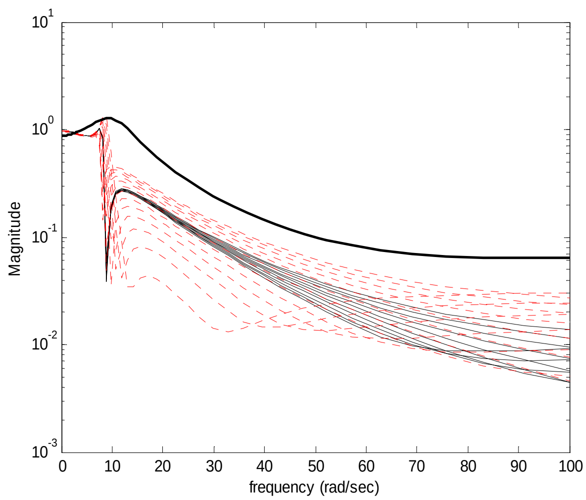


Fig. 5. Uncertainty plots; R_i (solid), X_i (dotted) and the upper bound W_i (bold line).

The selection of performance constant weights η_{1i}, η_{2i} and η_{3i} is dependent on specified performance objectives. In fact an important issue with regard to selection of these weights is the degree to which they can guarantee the satisfaction of design performance objectives. The selection of these weights entails a trade off among several performance requirements [1]. Here, the values of constant weights are fixed in 0.01.

3.2 H_2/H_∞-based PSS

According to the proposed synthesis methodology described in section 2 and using the LMI control toolbox in MATLAB [7], a robust PSS satisfying optimization problem (3) is obtained with the following state space form:

$$\begin{aligned} \dot{x}_{ki} &= A_{ki}x_{ki} + B_{ki}y_i \\ u_i &= C_{ki}x_{ki} + D_{ki}y_i \end{aligned} \qquad (6)$$

The order of resulted controller is 8 (almost it is equal to the size of linearized power system model plus $W_i(s)$). Finally, Hankel norm model reduction yielded a 2nd order controller with virtually no performance degradation.

4. Simulation Results

In order to demonstrate the effectiveness of the proposed strategy, some simulations were carried out. The performance of the closed-loop system in comparison of a conventional PSS is tested in the presence of voltage disturbances, short circuit fault on transmission line and parameter variations. For this purpose, a quite popular structure for the conventional PSS with the following transfer function is considered [10]. Many existing generators are commissioned with a PSS of this form.

$$K(s) = \frac{K_C(1+T_1s)^2}{(1+T_2s)^2}$$

For the problem at hand, the gain and the time constants of conventional PSS are properly selected (given in Appendix). In first test case, the performance of two controllers was evaluated in the presence of a 0.1 pu step disturbance injected at the voltage reference input of the AVR at 1 second. Fig. 6 shows the closed-loop response of the power systems fitted with the conventional and proposed PSS.

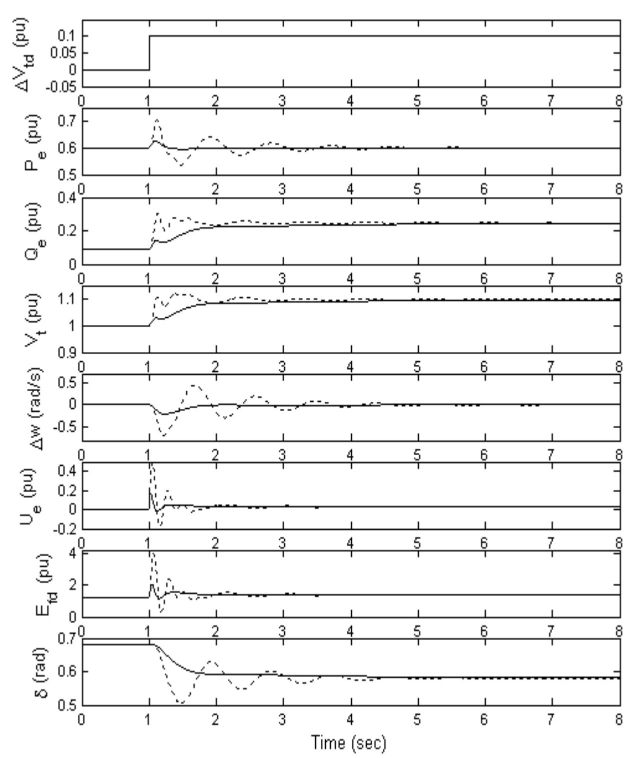


Fig. 6. System response to a step disturbance at the voltage reference input; Solid (Robust PSS), dotted (Conventional PSS).

Fig. 7 shows the electrical power, terminal voltage and machine speed for a 10% and 50% decrease in uncertain parameters in addition to the applied voltage step disturbance in the previous test case. It can be seen that the power system with conventional PSS is more sensitive to the parameter variation and it fails to even stabilize the system for a large change in parameters.

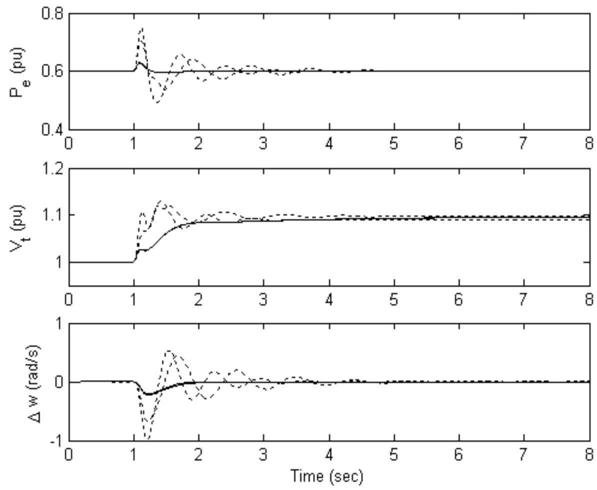


Fig. 7. System response to a simultaneous step voltage disturbance and 10% (and 50%) changes in R and X parameters; Solid (Robust PSS), dotted (Conventional PSS).

Fig. 8 shows the electrical power, terminal voltage and machine speed following a fault on the transmission line during 1 to 10 seconds (the fault is removed at 10s).

Comparing the simulation results with both types of controllers shows that the robust design achieves robustness against the uncertainties/disturbance and a quite better performance with less control effort.

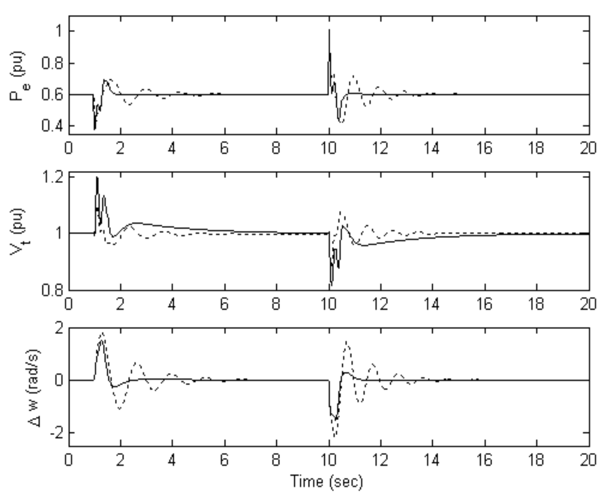


Fig. 8. System response following a fault on transmission line during 1 to 10 seconds; Solid (Robust PSS), dotted (Conventional PSS).

5. Conclusion

The power system stabilizer design problem is formulated as a decentralized multi-objective optimization control problem using the mixed H_2/H_∞ control technique. The proposed method was applied to a single-machine infinite-bus power system, and the results are compared with the conventional PSS design. The performance of the resulting fixed structure robust PSS is shown to be satisfactory over a wide range of operating conditions.

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Appendix

Power system parameters (Dimensions, notation and labels are considered the same as given in [11]):

Generator:

$x_d = 0.905,\ x'_d = 0.144,\ x_q = 0.542,\ x'_q = 0.542,$
$T_{d0} = 1.49,\ T_{q0} = 0.13$

Conventional PSS: $K_C = 5.0,\ T_1 = 0.11,\ T_2 = 0.17$

AVR: $K_R/(1 + sT_R);\quad K_R = 10.0,\ T_R = 0.05$

Line: $R = 0.0269,\ X = 0.6231$

Excitation: $K_A = 6.48,\ T_A = 0.02$

Initial state: $f_0 = 60\ Hz,\ V_{t0} = 1\ pu,\ P_0 = 1\ pu$