

Dynamic power system equivalence considering distributed energy resources using Prony analysis

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SUMMARY

Power system dynamic equivalence, which has been developed to overcome the computational issues, is faced with new challenges by emerging distributed energy resources (DERs). The switching characteristic between DERs operational modes beside non-dispatchable inherent, that is, generation uncertainties, should be taken into account in power system dynamic equivalencing. This paper proposes a switching-based dynamic equivalence modeling for power system considering DERs. Prony analysis is employed to fit a reduced-order model to a high-order system in frequency domain. The developed modelling strategy consists of high and low frequency equivalences. When the external system reconnects to the main network through the breaker, high-frequency equivalence is replaced with external system. This period contains 4 cycles for communication delay beside 5 cycles governor time constant, which give rise to a six-order equivalence model. Afterwards, a four-order low-frequency equivalence model is fitted to the external system in the connected grid mode. Accuracy of the fitted dynamic equivalences is investigated by calculating the mean square error between the measured signal and the Prony approximated one. A synchronous-based DER, a doubly-fed induction generators and a fuel cell DER are added to the CIGRE benchmark in order to analyse the proposed dynamic equivalence modelling. Simulation results demonstrate that the proposed equivalence modelling could exhibit the same results as the real external system in the frequency domain. Simulation results also reveal that increasing penetration of DERs affect the dynamic equivalence in grid-connected mode by degrading the higher frequencies while it has no impact on transient period equivalence. Copyright © 2014 John Wiley & Sons, Ltd.

KEY WORDS: dynamic equivalence; distributed energy resources (DER); distributed generation (DG); Prony analysis; switching strategy; grid connected mode; islanding

1. INTRODUCTION

Increasing electrical demand gives rise to the emerged bottlenecks in the transmission and distribution infrastructures. These bottlenecks include increased transmission losses and the necessity for continuous upgrading of transmission and distribution systems. In response to the previous concerns, distributed energy resources (DERs) are defined as a new set of electrical power supply sources. Falling investment cost of small-scale power plants, development of data communications and control technologies, emerging potential of DERs, easier DERs allocations in comparison with large central power plants and, finally, short installation times are the main incentives towards DERs application [1–3]. Driven by these issues, the integration of DERs units into distribution networks close to the loads is emerging as a complementary infrastructure to the traditional central power plants [4]. Thus, it is expected that a large number of DERs will be connected to the grid in near future, which gives rise to the high increasing of power system dimension and complexity. In this way, large penetration of DERs may seriously affect distribution systems dynamics, which, in turn, would affect overall power system stability and dynamic behaviour. In these cases, simple representation of the distribution

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network as a constant load (PQ) would not be appropriate [4]. However, to facilitate the dynamic modelling of a distribution system in an actual large-scale power system, a simple equivalent model should be developed so that the overall power system stability studies is possible. Power system dynamic equivalence becomes an interesting solution to deal with this problem since the late 1960s and still continuing today with the newly introduced challenges by emerging of DERs. In dynamic equivalence studies, the power network of interest is commonly divided into the study zone and external system. Transient phenomena occur in the study zone, whereas an encompassed equivalent model is replaced with the external system [5]. In other words, power system modelling requirements fulfil by assigning a reduced order equivalent model, which saves computational time and exhibits the same dynamic characteristics as the external system. It should be noted that distributed generation (DG) and demand response are two common types of DERs, which this paper focuses on the DGs.

Power system dynamic equivalence is one of the important power system problems for which there have been considerable research works. Generally, depending on the transient phenomena to be studied the power system equivalent modelling classified into three categories as high frequency, low frequency and wide band modelling [6–8]. The IEEE PES General Systems Subcommittee successfully reviewed the advances and challenges in the state of the art [5]. However, power system dynamic equivalence considering DERs becomes more significant than in the past while it received less attention. Azmy *et al.* [2] introduces artificial neural network based dynamic equivalent for distribution network in interconnected power system. The artificial neural network -based approaches require network learning, which could limit the application of this method. Moreover, the published studies rely on the learning of current network condition. However, as will be discussed later, DGs could be operated in a grid connected or an islanding mode. Therefore, these approaches could not exhibit satisfactory performance by connecting/tripping of DGs in distributed networks. Gray-box modelling is employed in [9] to reduce the order of distribution network in power systems. The gray-box modelling relies on the expert knowledge, which could be crude. Hankel norm approximation method is employed in [10] to fit a dynamic equivalence model to the distribution system considering DERs. The power flow data are employed to calculate a unique linearized model, which takes into account the state space model of generators and the model of the network. Afterwards, the Hankel approximation method reduces the order of the proposed model. It is noteworthy that the linearization is performed around the operating point, and therefore, the proposed model is acceptable only for specific small changes around operating point. Moreover, application of linearized-based methods is limited to the low-dimension power systems and with a small number of generators. An identification-based dynamic equivalence approach is proposed for distributed network cell considering DGs in [11]. The external system model is considered as black box, which characterized with voltage and frequency as input and real and reactive power as output. This model is represented in the form of state space and highly depends on the fault type and location, which affect its performance for large-scale power systems. Zali *et al.* [12] reports the initial results regarding power system equivalencing considering DERs using Prony analysis. A second-order transfer function considers as the dynamic model of external system in steady state. The presented results in Ref. [12] reveal that the Zali [12] model could exhibit satisfactory performance for the system in grid connected mode (will be discussed later) only, and the equivalence model response in transient period significantly differs from the actual system response. Generally, all of the recent published papers in the state of the art only propose an equivalence model for the external system when it is connected to the study zone. However, DERs have two different operation modes on which should be takes into account in equivalencing. Moreover, these researches only model the external system with equivalence for a specific operating point, whereas DERs are non-dispatchable resources. In other words, the uncertainty regarding non-dispatchable inherent of DERs, that is, generation uncertainties, should be investigated in the dynamic equivalencing procedure, which is completely missed in the literature.

Interconnecting/grid connected and islanding are two common operation modes of DERs. Dynamic equivalence modelling of a power system with the installed DERs in the external system should take into account the different operation modes of DERs. Generally, external system dynamic equivalence modelling should consider the following conditions:

- (1) reclosing the breaker and reconnecting the external system to the study zone (main grid) and
- (2) interconnecting operation mode of external system.

Reclosing a breaker and reconnecting the external system to the main grid give rise to transient phenomena similar to switching transient. In simulation of switching transients, synchronous generator in the external system is replaced by a voltage source. However, the special connection of DERs to the network makes the dynamic equivalence model representation in transient studies different from voltage sources. Moreover, the developed model for transient studies could be adequate only for high frequency studies and in situation where DERs with switching operation characteristic are installed in the external system, considering only high-frequency equivalent model would not suffice. The developed model should be adequate for low-frequency investigations in the grid-connected mode, ensuring that at the same time an accurate model for transient studies is achieved. The common coherency based equivalent technique that relies on the investigation of electromechanical behaviour of generator through rotor angles could not satisfy a reasonable response for DERs. Some prominent DERs such as fuel cell, which are not characterized with rotor angle beside special connection of some others through inverter to the grid limit application of coherency based techniques for power system studies considering DERs [2,5]. Therefore, a new general equivalence approach has to be developed in order to take into account DERs characteristics.

In this paper, a generic dynamic equivalent model based on Prony analysis is proposed. The Prony analysis is a widely used identification method in the power system studies [13]. Prony analysis estimates damping coefficients, frequency, phase and amplitude of a measured signal. This information is adequate for power system dynamic equivalencing procedure. Moreover, Prony analysis fits a reduced-order model to a high-order system in time and frequency domains [14]. In this paper, Prony analysis employs to decompose the external system to parallel combination of simple first-order systems. The proposed dynamic equivalence modelling employs switching strategy in order to take into account both operation modes of DERs. In other words, external system responses in both grid-connected mode and transient period are simulated by two reduced-order equivalences. Then a switching strategy with fixed switching time (detail will be discussed later) takes into account the transients between islanding and grid-connected mode operations. In addition to the developing of a reduced-order model, the effect of DERs participation in providing the system load on the developed dynamic equivalence model is investigated.

The rest of this paper is organized as follows. Section 2 discusses dynamic equivalence modelling procedure. Section 3 describes the Prony analysis concept. Section 4 describes the switching time calculation. In Section 5, simulation results are explained in detail, and finally, Section 6 concludes the paper.

2. DYNAMIC EQUIVALENCE MODELING

As stated recently, the developed model should take into account both DER operation modes. The transient phenomena in the time horizon of breaker reclosing should be represented by high-frequency equivalence. The high-frequency equivalence model of external system is represented by the following form [5]:

$$f_{fit}(s) = C_0 + \sum_{k=1}^N \frac{C_k}{s - a_k} \quad (1)$$

where C_k, a_k are the constant parameters to be specified by Prony analysis.

This representation explains the frequency response admittance characteristic of the external system. It is noteworthy that Equation (1) describes the external system characteristics when it reconnects to the study zone (detail about the transient period will be discussed later).

At the next step, an attempt has been made to develop an equivalence model for external system in grid-connected mode. Grid-connected mode equivalence modelling is performed via the simulation-based method. In simulation-based method, the external system response is simulated sequentially, and reduced-order model fitting techniques such as Prony analysis are used to determine the model parameters [5]. In this time horizon, the electromechanical oscillations in the range of 0–3 Hz are investigated [5], and therefore, the injected electrical power at the border of external system and study zone is modelled by Prony analysis. It is noteworthy that, while both operation modes are represented

by Equation (1), the input data to the Prony analysis tool are different for these operation modes. Generally, for each operation mode, a unique dynamic equivalence model should be replaced instead of the external system. The overall equivalence model could be obtained by simply switching between the developed models for each mode.

The switching time calculation is one of the main issue on which could affect the proposed method efficiency. From the literature, it could be seen that the typical switching time horizon is in the range of $\sim 10^{-2}$ to $\sim 10^{-5}$ [15,16]. Therefore, it seems that the developed dynamic equivalence for electromagnetic transients (EMT) should be replaced with external system for several milliseconds after external system reconnection, and afterwards, the interconnecting mode equivalence model should be employed. In other words, a switching strategy with fixed switching time is employed in order to take into account both operation modes in dynamic equivalence modelling. The exact switching time calculations will be given in Section 4.

3. PRONY METHOD

This section explains a brief description of Prony method. Prony analysis is an extended version of Fourier analysis, which directly estimates the frequency, damping, strength and relative phase of modal components exist in a signal. Ability to extract such important information, which could explain power system characteristics make the Prony analysis attractive for power system studies. Let $f(t)$ be a desired signal. Prony method fits the following function to the observed signal [17,18].

$$\hat{f}(t) = \sum_{i=1}^N A_i e^{\sigma_i t} \cos(2\pi f_i t + \phi_i) = \sum_{i=1}^N \frac{1}{2} A_i e^{\pm\phi_i t} e^{\lambda_i t} \quad (2)$$

where

$\lambda_i = \sigma_i \pm j\omega_i$ are the eigenvalues of the system
 σ_i is the damping coefficient
 ϕ_i is the phase component
 f_i is the frequency component
 A_i is the amplitude coefficient

4. SWITCHING TIME CALCULATION

In order to investigate the proposed modelling, synchronous-based and non-synchronous-based DERs are utilized in the power network of interest. The employed non-synchronous-based DERs in this paper are fuel cell and doubly fed induction generator (DFIG). Fuel cell and DFIG as the most common DERs have special connection to grid through inverters. On the other hand, synchronous-based DER developed by Chang *et al.* [19] is employed in the external system.

Switching time calculation needs a brief argument about frequency and voltage requirements in both operation modes. While the overall discussion is acceptable for both synchronous-based and non-synchronous-based DERs, here, synchronous-based model is explained in detail. Non-synchronous-based DER principles could be found in [20].

The synchronous-based DER model is shown in Figure 1. The employed DER is controlled via the frequency and speed control loop, reactive power compensation loop and voltage regulator. In addition to the mentioned controllers, the DER is equipped with over and under voltage protection, as well as over and under frequency protection, as specified by IEEE 1547 [21].

Synchronous-based DER employs two type of speed control as follows:

- (1) speed droop control or power sharing method, and
- (2) fixed frequency control or isochronous control.

In the speed control loop, DER tries to deliver power to the network at the specific power frequency, whereas fixed frequency control is employed when DER operate in islanding mode. A communication

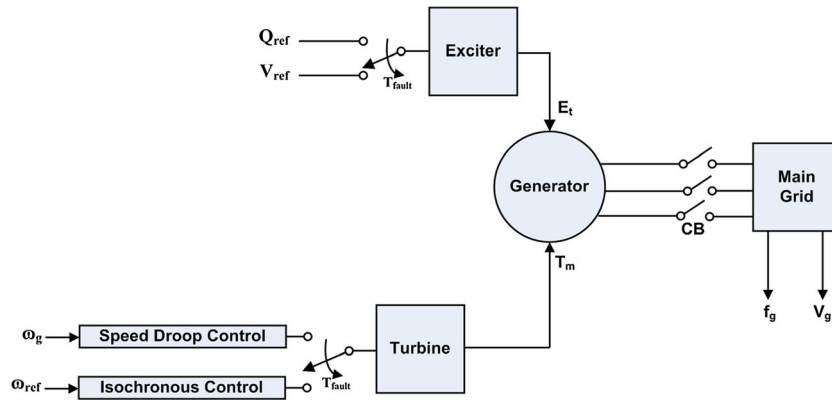


Figure 1. Synchronous-based distributed energy resource model.

delay about 4 cycles should be considered prior to changing the turbine governor control mode from fixed-frequency (isochronous) to speed droop mode. On the other hand, governor reacts to the control signal in about 5 cycles. These 9 cycles (0.15 s) are considered as the switching horizon in reconnection of external system with installed DER to the study zone. In the dynamic equivalence model development, this time is considered as the switching time.

5. SIMULATION AND RESULTS

5.1. Benchmark system

Figure 2 shows a modified 12.47 kV distribution network CIGRE benchmark. The DFIG as a renewable DER in downward grid and the synchronous-based DER at bus 3 are added to the standard benchmark. This system is used to investigate the dynamic equivalence modelling considering DER. The nominal parameters of the systems are taken from [22] and presented in the Appendix A.

In this study, the benchmark system and DER are modelled in PSCAD/EMTDC version 4.20 (Manitoba HVDC Research Centre Inc., Winnipeg, Manitoba, Canada). The synchronous-based DER is a 4.7 MVA, 4.16 kV unit connected to bus 3, with parameters extracted from [13]. The

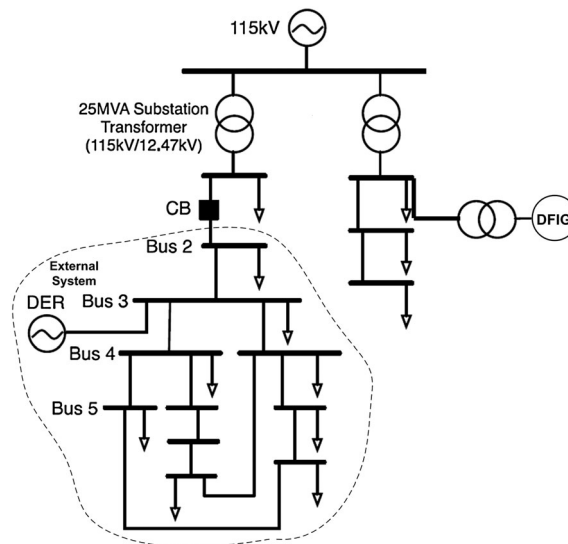


Figure 2. Benchmark system.

synchronous-based DER could operate in both grid connected and islanded mode and considered in the external system. In grid-connected mode, the governor functions are in speed-droop mode and will be shifted to fixed-frequency (isochronous) mode when operating as an island.

5.2. Dynamic equivalence

In order to investigate the dynamic equivalence modelling, a fault is applied to bus 1 at 4 s. Therefore, the breaker reacts and the DER switches to the islanding mode. At $t=20$ s, the fault is cleared and thereafter, the breaker recloses and the DER reconnect to the study zone. The breaker status is shown in Figure 3.

Figure 4 demonstrates the DER behaviour (active and reactive output power, rotor speed and terminal voltage) in response to the applied fault. It could be clearly seen that in the time of fault occurrence, the breaker opened and the DER switches to the islanding mode. In islanding mode, the DER delivers the power to the local loads at the predetermined frequency and with the specified ramp rate. Figure 4 reveals that the DER reduces its power to 0 in 9 cycles (as described in Section 3) when the fault is cleared, that is, $t=20$ s. In other words, the local loads are supplied through the main grid. Afterwards, the DER output starts to increase with low oscillations. Figure 5 demonstrates the enlarged DER output power signal for $t=30-35$.

The voltage–current characteristic in the switching period (9 cycles) is applied to the Prony toolbox in order to calculate dynamic equivalence in EMT period. Figure 6 demonstrates the voltage–current characteristic of bus 1 where the external system connects to the study zone via transmission line and breaker. The Prony analysis give rise to a six-order equivalence model with the calculated parameters in Table I. Therefore, the dynamic equivalence model in the EMT period could be explained by the following equation.

$$f_{fit}(s) = 2.32 + \frac{0.81}{s - 500} + \frac{0.0061}{s - 120} \tag{3}$$

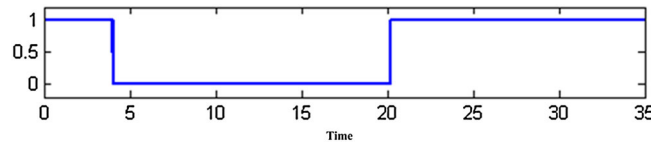


Figure 3. Breaker response to the fault.

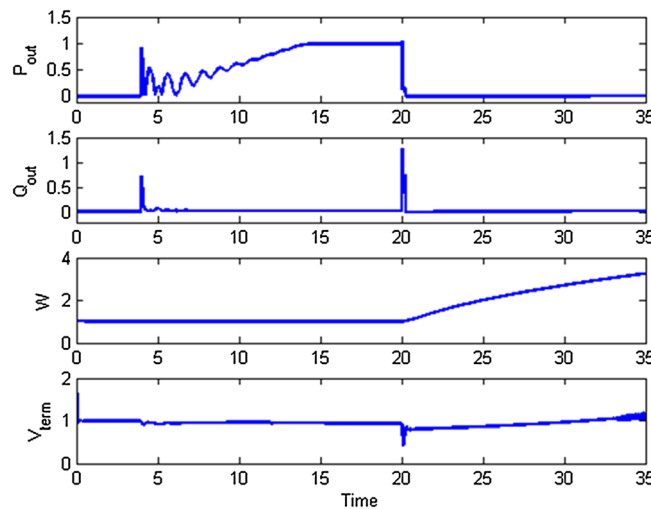


Figure 4. Distributed energy resource response to the fault.

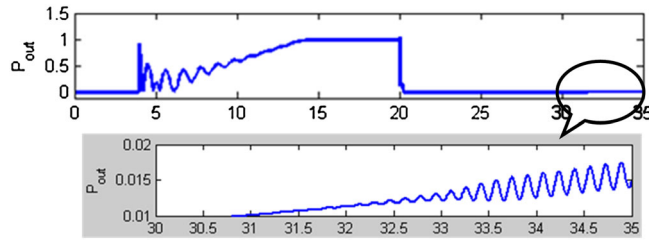


Figure 5. Distributed energy resource output power in connected grid mode.

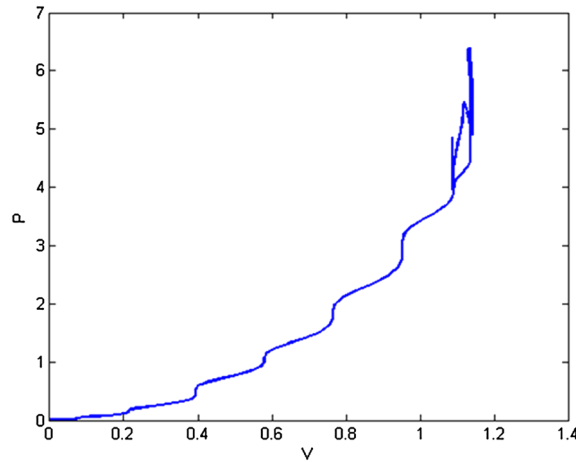


Figure 6. Voltage–current characteristic.

Table I. Electromagnetic transients dynamic equivalence parameters.

Mode	Amplitude	Frequency (Hz)
1, 2	2.2+0.12	0
3, 4	0.0405 × 2	500
5, 6	0.0061 × 2	120

Figure 7 demonstrate the accuracy of the calculated six-order dynamic equivalence model by mean square error (MSE) criterion. It is clear that measured signal has no significant difference with the modelled Prony approximation. It could be clearly seen that the developed model could simulate the behaviour of external system with high accuracy (MSE < 0.0002).

In order to calculate the dynamic equivalence in the connected grid mode, the delivered power to the bus 1 is investigated. Figure 8 shows the injected power to bus 1. This power supplies the local load at bus 1 beside the installed loads in the external system. By doing the same procedure as the EMT modelling, the Equation (4) is obtained based on Table II parameters.

$$f_{fit2}(s) = 0.82 + \frac{0.13}{s - 0.67} + \frac{0.065}{s - 1.86} + \frac{0.2}{s - 3.4} \quad (4)$$

Equation (4) exhibits the same result as the external system in the interconnecting mode. Figure 9 demonstrate the accuracy of the four-order dynamic equivalence model. It could be clearly seen that the developed model could simulate the behaviour of external system with high accuracy (MSE < 0.0025).

The final calculated MSE between the proposed switching-based model and the actual system reveals that the addressed dynamic equivalence modelling strategy could successfully simulate the external system behaviour in frequency domain.

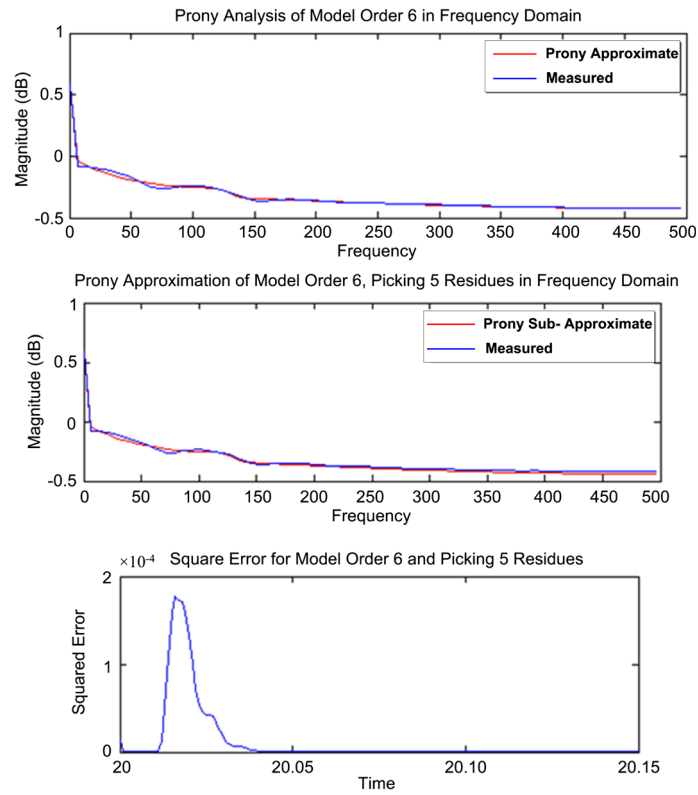


Figure 7. Electromagnetic transients dynamic equivalence accuracy.

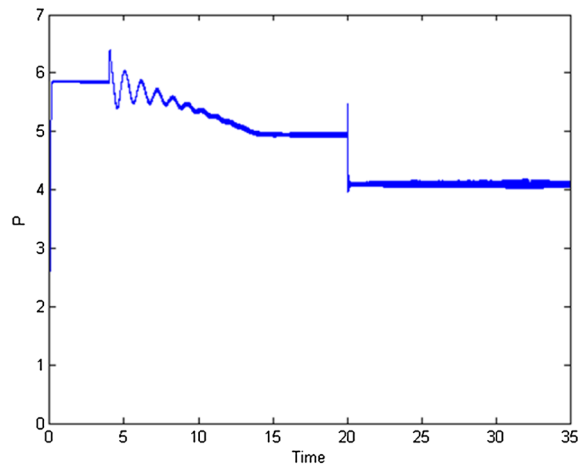


Figure 8. Injected power to bus 1.

In order to investigate the effect of DERs penetration on the developed dynamic equivalence, a fuel cell with 2 MVA rated and DFIG with 4.7 MVA rated capacity are added to the external system as shown in Figure 10. Following the same procedure as Equations (3) and (4), the following equivalences are obtained for the external system:

$$f_{fit3}(s) = 2.32 + \frac{0.76}{s - 500} + \frac{0.0067}{s - 120} \quad (5)$$

Table II. Interconnecting mode dynamic equivalence parameters.

Mode	Amplitude	Frequency (Hz)
1	8.2	0
2	0.13	0.67
3	0.065	1.86
4	0.2	3.4

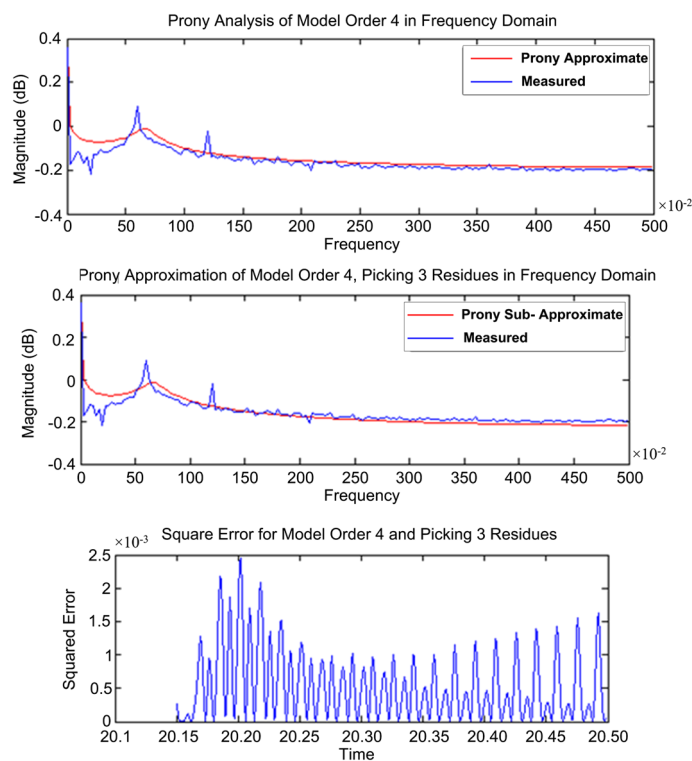


Figure 9. Connected grid mode dynamic equivalence accuracy.

$$f_{fit4}(s) = 0.82 + \frac{0.16}{s - 0.67} + \frac{0.07}{s - 1.86} + \frac{0.08}{s - 3.4} \tag{6}$$

Equations (5) and (6) explain the external system dynamic equivalences in transient and grid-connected mode following increasing of DERs penetration, respectively. Tables III and IV illustrate the Prony analysis results in response to the increasing penetration of DERs. It is noteworthy that the results in Tables III and IV are validated by MSE less than 0.0025. In Table III, it could be clearly seen that the developed dynamic equivalence for transient period do not change significantly with increasing penetration of DERs. However, increasing DERs penetration may significantly affect the developed model in grid-connected mode as described later.

Generally, the dominant frequency, that is, the frequency with the highest amplitude as compare with the others, as obtained from the Prony analysis represents the distribution network model. This gives rise to a simple reduced model, which simplifies the studies, ensuring, at the same time, the results accuracy. The dominant frequency-based equivalence model parameters, in highlighted forms, are given in Table V. Simulation results reveal that representing the external system, only with the dominant frequency obtained by the Prony analysis, could exhibit the same characteristics as the detailed system with high precision (MSE < 0.0087). In Table V,

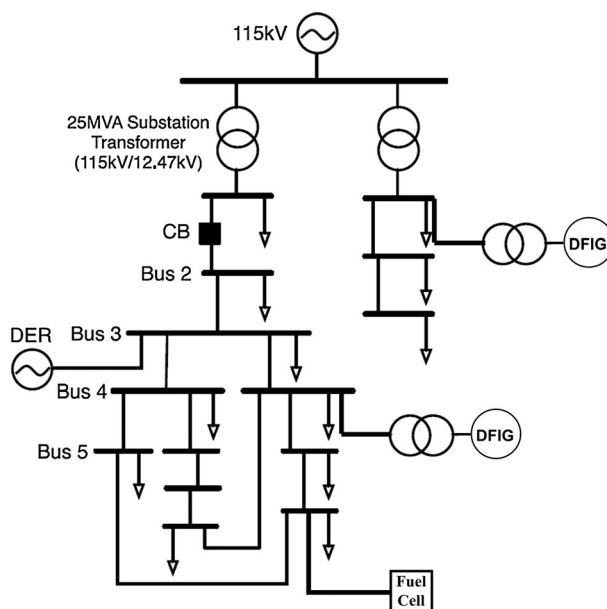


Figure 10. Benchmark system with fuel cell.

Table III. Prony analysis results (transient period) in response to increasing distributed energy resources penetration.

Case	DFIG (MW)	Fuel cell (MW)	C_0	C_1	C_2	a_1	a_2
1	4.7	2	2.32	0.0067	0.76	120	500
2	4.7+5%	2	3.03	0.0065	0.76	120	500
3	4.7+10%	2+5%	2.04	0.0070	0.075	120	500
4	4.7+20%	2+10%	2.53	0.0068	0.075	120	500
5	4.7+50%	2+10%	1.78	0.0064	0.073	120	500

DFIG, doubly fed induction generator.

Table IV. Pony analysis results (grid-connected mode) in response to increasing distributed energy resources penetration.

Case	DFIG (MW)	Fuel cell (MW)	C_0	C_1	C_2	C_3	a_1	a_2	a_3
1	4.7	2	0.82	0.16	0.07	0.08	0.67	1.86	3.4
2	4.7+5%	2	0.91	0.173	0.072	0.065	0.67	1.86	3.4
3	4.7+10%	2+5%	0.95	0.178	0.072	0.053	0.67	1.86	3.4
4	4.7+20%	2+10%	1.02	0.174	0.074	0.021	0.66	1.86	3.4
5	4.7+50%	2+10%	1.17	0.180	0.075	0.001	0.67	1.85	3.4

DFIG, doubly fed induction generator.

it could be clearly seen that by increasing the penetration of inverter-based sources into the system, the dominant frequency is changed. In other words, the equivalent model representing the external system changes significantly by increasing the DERs penetration. From technical view point, overall power system studies should take into account such model variation. Moreover, it could be seen from Table V that by increasing DERs penetration, connected to the main grid through inverters, the overall system inertia, that is, $\frac{a_i}{C_i}$ for dominant frequency, reduces. This concern could be of interest for frequency stability studies and beyond the scope of dynamic equivalencing.

Table V. Dominant frequency-based reduced model in grid-connected mode in response to increasing distributed energy resources penetration.

Case	DFIG (MW)	Fuel cell (MW)	C_1	C_2	C_3	a_1	a_2	a_3
1	4.7	2	0.160	0.070	0.080	0.67	1.86	3.4
2	4.7 + 5%	2	0.173	0.072	0.065	0.67	1.86	3.4
3	4.7 + 10%	2 + 5%	0.178	0.072	0.053	0.67	1.86	3.4
4	4.7 + 20%	2 + 10%	0.174	0.074	0.021	0.66	1.86	3.4
5	4.7 + 50%	2 + 10%	0.180	0.075	0.001	0.67	1.85	3.4
6	4.7 + 50%	2 + 15%	0.183	0.077	~0.001	0.67	1.86	3.4
7	4.7 + 50%	2 + 20%	0.125	0.520	~0.001	0.67	1.86	3.4
8	4.7 + 50%	2 + 40%	0.119	0.590	~0.001	0.67	1.85	3.4

DFIG, doubly fed induction generator.

The italic/bold boxed values specify variation pattern of dominant frequency by increasing DGs Penetration.

5.3. Discussion

As stated previously, power system dynamic modelling, simulation and analysis in the presence of DGs become very difficult, as there are much more separate generating units in different levels (distribution and transmission) with different dynamic inherent. Therefore, the whole distributed system could not be represented in details, due to the size of the system and the computational time constraints associated with dynamic simulation of large power systems. Obviously, a distribution network with the significant installed DERs could not be considered as passive network anymore. In other words, considering a simple constant power (PQ) model for distribution network in power system dynamic modelling is not acceptable, as it was performed before. The proposed method in this paper takes into account the switching characteristic of DERs, which is completely missed in the literature. A new switching-based strategy with fixed switching time is proposed in order to make the upward grid studies, possible in the presence of DERs. Moreover, there are several parameters uncertainties such as weather condition and shine situation, which affect the DERs operating point and the resulting developed dynamic equivalence. However, most of the proposed methods in the state of the art depend on the operating point, whereas, the effect of operating point changes is neglected. Analysing the impact of operating point changes on the developed dynamic equivalence is crucial in proposing a comprehensive model, which takes into account continues and discontinues changes of distributed network operating point. This paper appropriately addresses this concept. Generally, the proposed method in this paper overcome some crudities of the recent published papers while it could not consider the probabilistic inherent of DGs in equivalencing, yet. In viewpoint of subject importance, the proposed method could be employed for the overall system studies in the presence of DERs in transient and dynamic horizons. In these horizons, the downward grid could be described by a simple and appropriate transient/grid-connected model in the viewpoint of the upward network.

6. CONCLUSION

The need for equivalent models driven by the demand for fast and accurate simulation tools are the main motivations of this paper. Owing to the increased interest to DER for supplying of electrical demand, dynamic equivalence modelling in the presence of DER has recently become an emerged research area.

A frequency-domain reduction method for external system considering DER is presented. Prony analysis is employed to model the external system in both grid-connected and islanding mode. A high-frequency equivalence model is calculated for the EMT, that is, study zone connection to main network, whereas low-frequency equivalence is proposed for the interconnecting mode. A switching strategy with constant switching time is employed in order to take into account the different modes of DER. Simulation results reveal a direct relation between the developed equivalence in grid-connected mode with DERs penetration. This result may propose a new aspect of studies, which takes into account the probability problems in power system dynamic equivalencing.

7. LIST OF SYMBOLS AND ABBREVIATIONS

DER	Distributed Energy Resources
DG	Distributed Generation
IEEE PES	IEEE Power Engineering System
EMT	Electromagnetic transients
DFIG	doubly fed induction generator
MSE	Mean square error
CB	Circuit breaker
T_m	Mechanical torque
f_g	Grid frequency
V_g	Grid voltage
E_t	Terminal voltage
T_{fault}	Time of fault occurrence

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APPENDIX A

Table AI. MV equivalent network parameters.

Nominal system voltage	Short circuit	R/X ratio
12.47 LL	100	10

Table AII. Transformer data.

No.	X (m Ω)	Rating (MVA)
1	24.96	25
2	35.94	300

Table AIII. Grounding impedances.

Pole grounding impedance (Ω)	Transformer grounding impedance (Ω)
3	3

Table AIV. Load parameters.

Node	Apparent power (MW)	Power factor
1	10	0.90
2	7	0.90
3	10	0.90
4	6	0.95
5	4	0.95
6	7	0.95
7	5	0.95
8	7	0.90
9	10	0.90
10	15	0.90
11	20	0.90
12	25	0.85