Dynamic equivalencing of an active distribution network for large-scale power system frequency stability studies

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Abstract: This study presents an approach for developing the dynamic equivalent model of an active distribution network (ADN), consisting of several micro-grids, for frequency stability studies. The proposed grey-box equivalent model relies on Prony analysis to establish stop time and load damping as the required modelling parameters. Support vector clustering (SVC) and grouping procedure are employed for aggregation and order-reduction of ADN. This significantly decreases the sensitivity of the estimated parameters to operating point changes which, in turn, guarantees the model robustness. This is done through representing the SVC output, that is, clusters, by cluster substitutes. The final ADN dynamic equivalent model is represented by several groups, in which their mutual interactions are taken into account by a new developed mathematical-based criterion. Simulation results reveal that the proposed model is robust which could successfully take into account the continuous and discontinuous uncertainties.

1 Introduction

Increasing needs for electrical energy besides environmental issues of fossil fuel resources have caused many countries to set an ambitious target for distributed energy resources (DERs) [1]. In spite of DERs benefits, considering individual DERs suffers from several challenges and potential drawbacks [2]. In response to the DERs crudities, micro-grids (MGs) introduce an appropriate concept to realise the emerging potential of DERs [2]. Moreover, conversion of only one-third of fuel energy into electrical power, in bulk power generation units compounded with high voltage transmission network poor efficiency, make MGs in distribution network more pervasive [2].

Increasing penetration of MGs may affect distribution system dynamics, which, in turn, would affect the overall power system stability and dynamic behaviours. It means that a distribution network characterised with MGs, known as active distribution network (ADN), could not be considered as a lumped constant load (PQ) in power system stability studies [3]. Hence, an appropriate dynamical model of distribution system, as well generation and transmission levels, should be considered in large-scale power system dynamic and stability investigations. However, the detailed ADN modelling is impractical due to the size of system and the computational time issues [4]. Therefore, the ADN should be represented by a simple equivalent model, including appropriate details for a given stability problem, with similar dynamic characteristics.

While upward grid equivalencing is a mature topic with significant research works [5], ADN equivalent model proposition becomes an emerged research area. A grey-box model, incorporated with evolutionary particle swarm optimisation, is employed for MG dynamic equivalencing purpose in [1]. Another attempt based on grey-box representation of ADN is made in [3]. It proposes a seventh-order model for an ADN in generic power system stability studies. The attempt to overcome the non-robust feature of parameters estimation fails in [3]. Another grey-box equivalent model and its validation are proposed in [6]. Hankel norm approximation is employed in [7] to fit a dynamic equivalent model to an ADN. Nevertheless, application of linearised-based methods is limited to the small scale power systems and with a limited number of generators. An identification-based dynamic equivalence approach is proposed for ADN in [8]. The external system is considered as a black-box, characterised with voltage and frequency as input and real and reactive powers as output. This model is represented in the form of state space and highly depends on the fault type and location which affect its performance for large-scale power systems. Another black-box equivalent model for a MG is proposed in [9]. It incorporates Prony analysis with non-linear least square optimisation to develop an accurate model. An ANN-based dynamic equivalent model for an ADN in interconnected power systems is proposed in [10]. The proposed ANN-based model has the advantages of robustness in response to operating point changes. Its main drawback is that any change in system topology may result in fail to convergence to accurate model parameters.

Generally, the research reported so far consider the inverter-based DGs in the model proposition. These specific classes of DGs introduce continuous uncertainties, for example, the generation level, in the problem formulation. However, discontinuous uncertainties related to the probable contingencies, such as outage of directly connected DGs to the grid or connection/disconnection of MGs, should be tackled into the problem formulation. Moreover, the non-robust feature of recently proposed approaches gives rise to scenario-based equivalent models. This limits the recent proposed dynamic equivalent models to an ADN with single MG and low dimensions, including a limited number of DERs. However, in a real transmission bus, a significant number of MGs may be coupled to the network. Increasing number of MGs in system significantly increases the scenarios, which are impractical to deal with.

The need for developing of a low-order robust dynamic equivalent model for ADN, driven by the demand for accurate large-scale power system stability studies, is the main motivation of this paper. Owing to the fact that the equivalent model development is an identification process, and taking into account more physically relevant and intuitive of grey-box identification in comparison with black-box [3], the grey-box modelling is employed for equivalent proposition. The proposed grey-box modelling should be formulated based on a specific stability problem, which specifies the overall modelling framework. Generally, in comparison with a bulk power system with dominant synchronous machines, an ADN with installed DERs/DGs has small inertia and damping properties [11]. As, these characteristics play important role in power system stability and control, the ADN dynamic equivalencing approach should properly observe them into model formulation. These characteristics, as will be discussed in the next section, could appropriately adopt to the frequency stability studies requirements. Therefore, in this paper an attempt is made to develop an equivalent model for frequency stability studies, capable of representing the dynamic behaviour of an ADN, in a robust manner. The proposed grey-box modelling relies on Prony analysis to fulfil the overall frequency stability structure (will be discussed in next section). The input data to the Prony analysis toolbox are simple measurable signals, metered at the point of connection of the ADN to the upward grid. The final equivalent model consists of several sub-equivalent models/groups. The modelling methodology tries to aggregate and represent as much as possible MGs in a group to significantly decrease the sensitivity of the estimated parameters to operating point changes. For this purpose, Prony analysis results are applied to support vector clustering (SVC) as input. The proposed modelling strategy employs mathematical and statistical analysis to keep overlap between the sub-equivalents/groups, to establish the model robustness with respect to the changes in operating point. Finally, a new mathematical-based criterion would be developed to systematically assign an appropriate representation, from the developed groups, to a given operating point. Moreover, the proposed criterion quantifies the mutual interactions among the sub-equivalent models, aiming to add some new MGs into an already existing equivalent model with acceptable accuracy. As the model development procedure and parameters adjustment are done offline, the developed criterion eliminates the need to re-arrange all the MGs into the new clusters by SVC algorithm after any structural reconfiguration, that is, MG connection/disconnection. This means that the developed criterion renders the proposed model more applicable for real-time applications. The proposed dynamic equivalent model, which replaced with ADN at the point of connection to transmission network, appropriately tackles the uncertainties into the problem formulation, which are missed in the literature. Developing such model could provide an opportunity for power system operators to systematically deal with the effect of MGs penetration level on power system stability.

The rest of this paper is organised as follows. Section 2 discusses the frequency model requirements in frequency stability studies, Prony analysis and SVC concepts. Section 3 briefly describes the study system. Section 4 describes the proposed equivalencing method. In Section 5, simulation results are explained in details and finally, Section 6 concludes the paper.

2 Theoretical concepts

2.1 Frequency stability studies requirements

Frequency stability assessment, as a long-term dynamic study, requires composite modelling of generation, transmission and load [12]. In view point of ADN dynamic equivalent model proposition, modelling of transmission system requirements would be omitted. Generally, for the purpose of frequency stability synthesis and analysis, a simple low order model is used in literature to represent load and generation. This modelling neglects the fast dynamics related to voltage and angle, and represents generation and load with stop time (M) and load damping (D) in the form of a first degree transfer function, as described by [12, 13]:

$$f \propto \frac{1}{Ms + D} \tag{1}$$

Representation of dynamic equivalence model by (1) could appropriately take both low damping and inertia properties of ADN, as distinguished characteristics of ADNs [11], into stability problem.

2.2 Prony analysis

Prony analysis, as an identification method, models a uniformly sampled signal by a series of damped complex exponentials or sinusoids. Let $y(t)$ be a signal consisting of N evenly spaced samples. Prony analysis fits the following function to an observed record of $y(t)$.

$$\hat{y}(t) = \sum_{i=1}^{N} A_i \,\mathrm{e}^{\sigma_i t} \cos(2\pi f_i t + \phi_i) = \sum_{i=1}^{N} \frac{1}{2} A_i \,\mathrm{e}^{\pm j\varphi_i} \,\mathrm{e}^{\lambda_i t} = \sum_{i=1}^{N} B_i \,\mathrm{e}^{\lambda_i t} \tag{2}$$

(2) in Laplace form is defined by:

$$Y(s) = \sum_{i=1}^{N} \frac{B_i}{s - \lambda_i} \Rightarrow Y(s) = \sum_{i=1}^{N} \frac{1}{(1/B_i)s - (\lambda_i/B_i)} \triangleq \sum_{i=1}^{N} \frac{1}{M_i s + D_i} \tag{3}$$

where:
$\lambda_i = \sigma_i \pm j\omega_i$ are the eigenvalues of the system
σ_i is the damping coefficient
ϕ_i is the phase component
f_i is the frequency component
A_i is the amplitude coefficient
M_i is the stop time
D_i is the load damping factor

It could be clearly seen that, (3) would appropriately map the measured signal components to the essential frequency requirements in (1).

2.3 Support vector clustering

In SVC, a Gaussian kernel function, as a non-linear transformation, maps the data points from the data space to a high dimensional feature space. The Gaussian kernel expression is described as:

$$K(X_i, X_j) = \mathrm{e}^{-q\left\|X_i - X_j\right\|^2} \tag{4}$$

The scale parameter, that is, q, is an important parameter on which the efficiency of the clustering, that is, equivalencing in this paper, is dependent. The SVC explores the smallest sphere that encloses the image of the data in transformed space. It means that an optimisation procedure is incidentally applied to the problem formulation, to achieve the minimum sphere radius [14]. The detailed mechanism on which SVC clusters the data is described in [14]. One thing to be noted here is that the number of the clusters increases by increasing the scale parameter of the Gaussian kernel. Therefore, this parameter is considered as a degree of freedom in the model development procedure. This point will be further discussed in Section 4.

3 Microgrid/ADN system

Fig. 1 shows a single-line diagram of the system used to assess the dynamic equivalencing approach. The line and load data are taken from [15]. The ADN is connected to the 33 kV external grid, represented by the equivalent synchronous generator source. The grid supplies N different 12.47 kV feeders through 33/12.47 kV transformer. Each feeder is typically the network as detailed in the right-hand side of Fig. 1. The 12.47 kV feeders are connected to the point of common coupling via individual step-up 12.47/0.433 kV transformers, with rating varying between 10–20MVA, depending on the load size. Eight DG units, including three synchronous generators with rated power between 2–5MVA and five inverter interfaced units with rated power between 1–3MVA, are considered in each MG topology. The synchronous generators are equipped with excitation and governor control systems as represented in [16, 17]. The inverter based sources utilise frequency and voltage droop characteristics, as represented in [18], to provide control on the output active and reactive powers, that is, PQ control. The aforementioned sources are wind generation units,

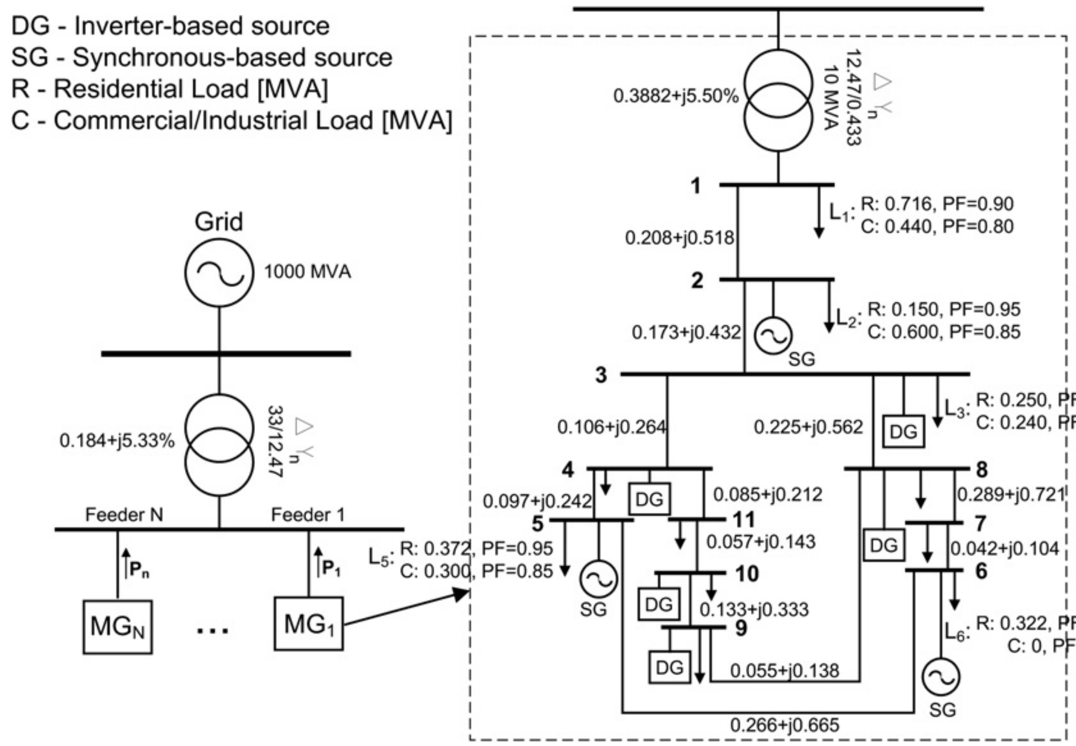

Fig. 1 *ADN network and typical structure of each MG*

that is, DFIG, and micro-turbines/fuel cells. The micro-turbines/fuel cells are implemented through voltage-sourced converters which interface prime sources to the power system [19].

4. Proposed equivalencing method

This section begins with a sub-section devoted to the general overview of the proposed modelling methodology and will be continued by its detailed explanation.

4.1 General overview

4.1.1 Procedure and assumptions: The main aim of this paper is to establish a model for ADN in upward frequency studies. In this way the model should be characterised with the inputs, the interactions with the grid as well the outputs. After any disturbance, the injected power of external grid to ADN could be divided into two parts. In the early one, the external grid as a voltage source instantaneously supplies the ADN power requirements. The later describes the decline of the injected power in response to DGs reaction to the fault. ADN dynamic equivalencing purpose justifies that the second part which includes MGs dynamics should be utilised to derive the model. The decline in the injected power is equivalent to the growth of ADN, that is, included MGs, injected power to upward grid. The injected power to the grid should be simulated owing to the fact that, voltage levels in frequency stability studies are considered to be nominal, that is, 1 p.u. [12]. The proposed dynamic equivalent model utilises ideal controllable and balanced voltage source, to keep the voltage at nominal value. The ideal voltage source is fed by a corrective index and angular frequency. While the corrective index is a scalar coefficient that yields the magnitude of the desired voltage when multiplied by the DC voltage, the angular frequency is obtained through applying of the difference between the measured and the desired powers to (1) and comparing the result with the nominal value. Although the corrective index is provided through a simple PI controller, robust representation of the ADN with $(M,\ D)$ is the main contribution of this paper. Here, it should be noted that the instantaneous voltage, current and power are calculated using the rotating $d-q$ reference frame. For this purpose, the instantaneous voltage and current magnitudes are calculated by obtaining the direct (d) and quadrature (q) components with respect to phase-a.

The proposed grey-box model in this paper gets the injected power of each MG to the upward grid as input, and calculates the required frequency parameters, that is, stop time and load damping $(M,\ D)$, as output; through Prony analysis. In this way, it is assumed that the injected power of each MG to the upward grid, that is, (1), could be represented only by dominant frequency component. Dominant frequency refers to the mode with the highest energy in comparison with the involved ones. For theoretical justification of such representation, consider a possible situation where Prony analysis results in multiple dominant oscillatory modes. As the proposed modelling strategy in this paper maps the MG characteristics to the conventional generators, coherent generators concept could be used for the model simplification in such a case. For this purpose, consider two dominant modes with different amplitudes, frequencies and phases, that is, two pairs of $(M,\ D)$, which could be mapped to two conventional generators and associated loads. Based on the coherent identification approaches in [20], speed variation, as coherency criterion, for a generator depends on its inertia and the electrical distance between the studied generator and the other one which should be assessed for coherency. In developing of equivalent model for connected MGs to medium and low voltage buses, the electrical distance would be omitted and hence, the speed variation depends on the inertia as follows:

$$\begin{aligned}\frac{2H}{\omega_0}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} &= P_m - P_e \xrightarrow{\theta=\omega t-\omega_0 t+\theta_0} 2H_i\frac{\mathrm{d}(\Delta\omega_i)}{\mathrm{d}t} \\ &= P_m - P_e \xrightarrow{2H=M} \frac{\mathrm{d}(\Delta\omega_i)}{\mathrm{d}t} = \frac{P_m - P_e}{M_i}\end{aligned} \tag{5}$$

It could be clearly seen that different dominant modes, with different assigned stop times, have different speed variation behaviours, and hence, could be assumed as non-coherent generators. Therefore, the modes are initially reduced based on the non-coherent generators aggregation and after that, are taken into account. Recently, the present authors have numerically dealt with such representation in [17].

4.1.2 General equivalencing framework: A general description of the proposed modelling strategy can be explained by the following steps:

Step 1: Set initial assumption on number of operating points
Step 2: Selecting an operating point from the initial set and going through this point
Step 3: Capturing the injected power of MG to the grid
Step 4: Representing the captured power by dominant frequency through Prony analysis
Step 5: Return to *Step 3*, repeat until all the MGs are represented by dominant frequency
Step 6: Clustering MGs in several sub-clusters by SVC and represent them by equivalent models
Step 7: Evaluating the equivalent model accuracy by numerical methods

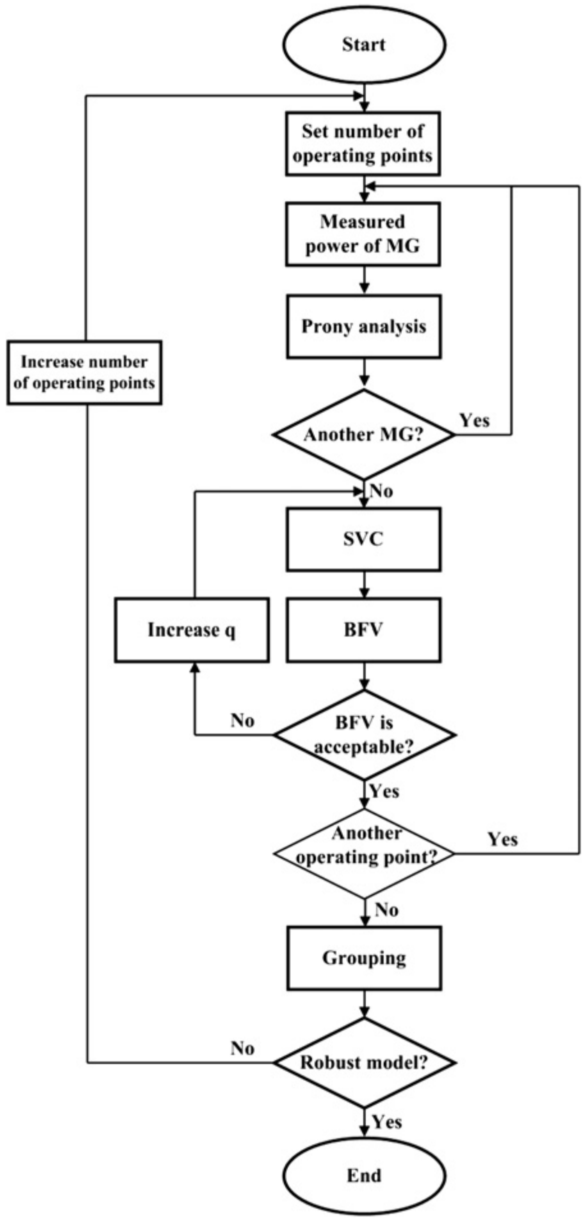

Fig. 2 *Flowchart representation of the equivalencing approach*

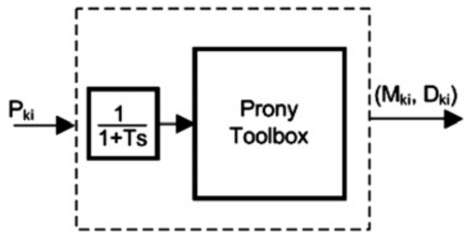

Fig. 3 *(M_{ki}, D_{ki}) calculation block*

Step 8 If the model accuracy is not acceptable, go to *Step 6* and increase the number of sub-clusters
Step 9 Return to *Step 2* and go through another operating point, repeat until the entire initial assumed operating points are represented by equivalent models
Step 10 Summarising the equivalent models, obtained by *Step 6*, into the final equivalent model
Step 11 Assess model robustness, if the robustness does not achieved return to *Step 1* and increase the number of initial operating points

The above steps are summarised in the flowchart of Fig. 2, and detailed discussion will be represented in the next section.

4.2 Detailed modelling procedure

This sub-section explains the proposed dynamic equivalent model in details. Toward a better understanding of the modelling strategy, this sub-section split into three parts: model development, robustness validation and mathematical-based criterion calculation.

4.2.1 Model development: Consider a transmission bus including N MGs. An assumption is made on the number of operating points, that is,N_s. This assumption is made to have a degree of freedom to achieve a robust model structure with acceptable accuracy, and would not affect the model performance. Afterwards, the injected power of each MG to the upward grid, for a specific operating point, is applied to the Prony analysis toolbox (Fig. 3). The Prony toolbox output is a pair of stop time and load damping, represented as:

$$(M_{ki}, D_{ki}) = \mathbb{F}(P_{ki}) \tag{6}$$

where, P_{ki}, M_{ki}, D_{ki} are injected power, stop time and load damping of the kth MG for the ith operating point, respectively. It is noteworthy that the metered P quantity is passed through a low pass filter, as shown in Fig. 3, to attenuate the high frequency components which occur at step load or power changes. Following (6) for all the N MGs gives rise to:

$$C_i = \{(M_{1i}, D_{1i}), \ldots, (M_{Ni}, D_{Ni})\} \tag{7}$$

C_i is then clustered by SVC algorithm. The SVC output is the arrangement of the MGs, characterised with (M_{ki}, D_{ki}), in some sets, called sub-cluster here, S (Fig. 4). The calculated sub-clusters

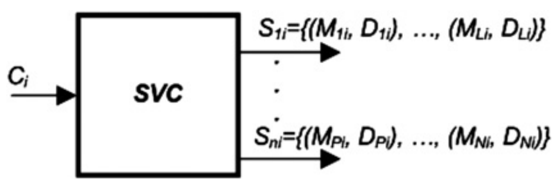

Fig. 4 *Clustering procedure for* i*th operating point*

satisfy the following conditions:

$$S_{1i} \cap S_{2i} \cap \cdots \cap S_{ni} = \emptyset \\ S_{1i} \cup S_{2i} \cup \cdots \cup S_{ni} = C_i \tag{8}$$

where, S_{ni} is the nth sub-cluster for the ith operating point.

To develop a simple model, equivalencing is followed by representing each sub-cluster by an substitute. In this way, the injected power of the sub-cluster is applied to the Prony analysis to calculate a unique (M, D), called sub-cluster substitute here. The injected power of each sub-cluster is defined by:

$$P_j^{\text{set}} = \frac{\sum_{k \in j} P_k^{\text{MG}}}{Q} \tag{9}$$

where, Q is the number of MGs in sub-cluster j. The aim is to replace the sub-cluster substitute with the corresponding sub-cluster. In this way, the accuracy of such representation should be validated before the algorithm proceeds. To validate the fitting and clustering procedures, the performance of the mentioned steps is evaluated by the Best Fit Value (BFV) criterion as described by (10) [3].

$$\text{BFV} = 1 - \frac{y - \hat{y}}{y - \bar{y}} \tag{10}$$

where y is the measured signal, $\hat{y}$ is the simulated model signal, and $\bar{y}$ is the mean of y. To proceed with model proposition, the BFV between the measured power of sub-cluster (ADN) and the simulated one, by the sub-cluster substitute(s), is calculated for the studied operating point. If the BFV is acceptable, then, the model is followed by another operating point. Otherwise, the scale parameter (q) in the clustering procedure should be increased. This would increase the number of sub-clusters, which, in turn, improves the BFV. Unacceptable BFV for a sub-cluster reveals that the allocated MGs to the sub-cluster are not identical and could not be represented by an substitute. Hence, the MGs should be re-arranged into more sub-clusters. The above steps follow for all the preliminary assumed operating points. Sometimes, the clustering procedure corresponding to several operating points gives rise to the same number of sub-clusters with the same arrangement of MGs. Therefore, once the clustering for all the operating points is validated, they are further arranged into groups (Fig. 5). Each group characterises with several operating points which are same in the number of sub-clusters and the arrangements of MGs. The developed groups satisfy the following conditions:

$$(M_{ki}, D_{ki}) \in S_{ji} \subset C_i \subset G_p \rightarrow S_{ji} \subset G_p \\ G_1 \cap G_2 \cap \cdots \cap G_n = \emptyset \\ G_1 \cup G_2 \cup \cdots \cup G_n = \{C_1, \ldots, C_{N_S}\} \tag{11}$$

Based on (11), while a set of sub-clusters with the same arrangements of MGs forms a cluster, a group encapsulates several clusters. The number of clusters in each group specifies the number of the required equivalent model, that is, (M, D), for the ADN. The cluster substitute, that is, a unique (M, D), for each cluster in a given group is calculated by applying the injected

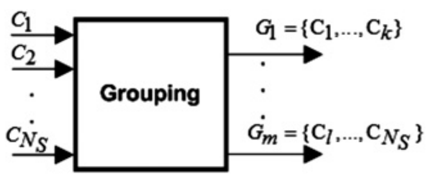

Fig. 5 *Grouping procedure for all the operating points*

power of the cluster to the Prony analysis toolbox. The injected power of a cluster is calculated by:

$$P_j^{\text{cluster},m} = \frac{\sum_{i=1}^{N_O} \sum_{k=1}^{Q} P_{ki}^{\text{MG}}}{Q \times N_O} \tag{12}$$

where, $P_j^{\text{cluster},m}$, Q, N_O are the injected power of cluster m in group j to the upward grid, number of MGs in each sub-cluster and the number of operating point represented by group j, respectively. This representation requires validation based on which, one could accurately replace each cluster in the developed groups, including several sub-clusters with various (M_i, D_i), with an equivalent cluster substitute. While the eligibility of sub-cluster substitute for representing included MGs is discussed, eligibility of the cluster substitute in representing the included sub-clusters and model robustness will be dealt in the next sub-section in a mathematical way.

4.2.2 Robustness validation: Representing a cluster by cluster substitute means that, the cluster substitute should exhibit acceptable BFV for all the included sub-clusters. For such eligibility assessment, consider a cluster in a given group represented by its corresponding cluster substitute, that is, (B, λ). Each sub-cluster in the assumed cluster may be represented by a (B', λ'), and explained based on the cluster substitute as:

$$B_i{}' = B_i + \Delta B_i \\ \lambda_i{}' = \lambda_i + \Delta\lambda_i \tag{13}$$

ΔB, $\Delta\lambda$ may be positive or negative. Ignoring (ΔB_i, $\Delta\lambda_i$) terms in (13) and representing all the sub-clusters in a cluster with cluster substitute, that is, (B, λ), would degrade the BFV and model efficiency. To evaluate model accuracy, that is, BFV, in such a case, $\hat{y}(t)$ would be expanded around the cluster substitute, that is, (B, λ). Applying Taylor expansion to (2), gives rise to:

$$\hat{y}'(t) = \hat{y}(t) + \Delta\hat{y}(t) = \hat{y}(t) + \frac{\partial(\hat{y})}{\partial B}\Delta B + \frac{\partial(\hat{y})}{\partial \lambda}\Delta\lambda \Rightarrow \\ \hat{y}(t) + \mathrm{e}^{\lambda t}\Delta B + tB\,\mathrm{e}^{\lambda t}\Delta\lambda = \hat{y}(t) + \frac{\hat{y}(t)}{B}\Delta B + t\Delta\lambda\hat{y}(t) \tag{14}$$

Assume BFV for the cluster substitute is X_1, and X_2 is the BFV for a sub-cluster in the stated cluster. Therefore:

$$\text{If } 1 - \frac{y - \hat{y}}{y - \bar{y}} = X_1 \Rightarrow 1 - \frac{y - \hat{y}'}{y - \bar{y}} \\ = X_2 \xrightarrow{\text{Equation (14)}} 1 - \frac{y - \hat{y}(1 + (\Delta B/B) + t\Delta\lambda)}{y - \bar{y}} = X_2 \tag{15}$$

where, $\hat{y}, \hat{y}'$ are the estimated signals characterised by cluster substitute and sub-cluster substitute, respectively. There is no known mathematical method for solving (15), that is, calculation X_2 based on X_1. As a result, statistical approaches may be used to deal with such equation. For this purpose, it is required to extract a relationship between X_1 and X_2 based on several measured and modelled signals. The extracted relationship should be validated by the statistical criteria, such as correlation coefficient (R) and Root Mean Squared Error (RMSE), before being used in the model development. These criteria are employed to measure the strength and directions of the extracted relationship between two variables. The extracted relation between X_1 and X_2 is utilised to analyse the sensitivity of the cluster substitute BFV to operating point changes, that is, cluster substitute justification. In other words, the extracted relation is employed to verify the model robustness in a limited range of operating points, corresponding to a given group. However, the model could be justified as robust, if all the possible operating points could be represented by the developed groups. For this purpose, an overlap should be provided between the groups in representing the operating points. If such overlap exists, the developed groups are adequate for ADN modelling, otherwise, the number of groups should be increased.

Investigation of the stated overlap is done by following the same procedure as (15). For this purpose, X_1 and X_2 are considered as the BFV for two cluster substitutes corresponding to group i and j, respectively. The overlap is verified if, unacceptable BFV in representation of a given operating point by a cluster substitute in group i gives rise to acceptable value in group j. Otherwise, the preliminary assumption regarding to the number of operating points should be reset, and all the stated procedures in Fig. 2 should be re-done. After robustness validation, the ADN could be replaced by the developed model with acceptable accuracy. Finally, a mechanism, on which, one could select a group, from the developed ones, for a given operating point is discussed in the next sub-section.

4.2.3 Mathematical-based criterion development: To systematically deal with the transients between the groups, initially a mathematical-based criterion is developed on the basis of connecting a new MG to a modelled ADN, and afterwards, the transients will be adapted to the developed criterion.

Consider a situation where a new MG is added to an already modelled system. It is assumed that in the present operating point, the equivalent model of the system is known. The aim is to analytically add the stated MG to the developed groups/clusters in such a way that the model accuracy is kept at a desired value. Adding a new MG to a cluster would change the cluster configuration and may not be necessarily represented by the previous group and cluster substitute. To investigate the impact of adding a new MG on the developed model accuracy and cluster substitute, the new configuration would be represented by a new cluster substitute. The new cluster substitute gives rise to a different BFV from the latest one which could be represented as follows:

$$1-\frac{y-\hat{y}'}{y-\bar{y}}=X_2 \xrightarrow{\text{Equation (14)}} 1-\frac{y-\hat{y}(1+(\Delta B/B)t\Delta\lambda)}{y-\bar{y}}=X_2$$
$$\Rightarrow 1-\frac{y-\hat{y}}{y-\bar{y}}-\frac{\hat{y}([\Delta B/B]+t\Delta\lambda)}{y-\bar{y}} \Rightarrow X_1^L-\frac{\hat{y}([\Delta B/B]+t\Delta\lambda)}{y-\bar{y}}=X_2 \tag{16}$$

where, X_1^L is the BFV for the cluster substitute before connection of the new MG. To allocate the new MG to the stated cluster with an acceptable accuracy and to keep the existing overlap between the developed groups, the following condition should be satisfied:

$$\frac{\hat{y}([\Delta B/B]+t\Delta\lambda)}{y-\bar{y}}\le 0 \tag{17}$$

However, (17) is the Euclidean norm which is characterised by a non-negative value. Therefore, (17) is re-written as:

$$\frac{\hat{y}([\Delta B/B]+t\Delta\lambda)}{y-\bar{y}}=0 \tag{18}$$

While B, $\Delta\lambda$, ΔB are specific constant values in (18), some considerations regarding t are required. To satisfy (17) for every situation, (18) is solved for the worst case condition. Owing to the fact that (14) is expanded according to the cluster substitute for period of interest, t in (18) would be the study time, as the worst case condition. It should be noted that, the mean of the measured signal appears at the denominator of (18) which reduces the dependency of the results to parameter t. Therefore, one could re-write (18) as:

$$\frac{\hat{y}([\Delta B/B]+T\Delta\lambda)}{y-\bar{y}}=0\Rightarrow\frac{\Delta B}{B}+T\Delta\lambda=0\Rightarrow 1-\frac{B'}{B}=-T\Delta\lambda \tag{19}$$

where T is the study time. It should be noted that, setting t equal to the study time is absolutely a pessimistic assumption which guarantees an acceptable accuracy for the new cluster allocation. The calculated value in (19) is the border on which the new MG could be added to a cluster with acceptable accuracy. Generally, the allocation of a new MG to the clusters is done based on the following rules:

$$\text{if}\begin{cases}0<1-\dfrac{B'}{B_1}\le T\Delta\lambda_1 & \text{then MG}\in\text{cluster1}\\ T\Delta\lambda_1<1-\dfrac{B'}{B_2}\le T\Delta\lambda_2 & \text{then MG}\in\text{cluster2}\\ \text{etc.}\end{cases} \tag{20}$$

Moreover, after MG allocation, (9) in combination with the Prony analysis could be employed to re-calculate the new cluster substitute, to achieve a better BFV.

After all, the transients between the groups should be investigated to settle the model. As the proposed modelling strategy feeds by the injected power of each MG to the upward grid, any change in operating point could be assumed as the connection of a new MG, with injected power equal to the change in operating point. Therefore, the change in operating point could be assigned to the clusters corresponding to the groups based on the proposed criterion in (20). This gives rise to selection of an appropriate equivalent model which could explain the ADN dynamic behaviour. A new group including the selected cluster is specified as the present ADN model.

5 Simulation and results

5.1 Dynamical simulation

The capability of the proposed modelling strategy is investigated on several systems and is illustrated here on two ADNs including 4 and 20 MGs. The total installed local generation in each MG could be equal, larger or smaller than the local loads. This paper considers all three situations in each case study. A definition, called MG capacity is used in what follows and is described by:

$$MG_i^{\text{Capacity}}=P_{\text{MG}_i}^{\text{Max}}-L_{\text{MG}_i} \tag{21}$$

where $P_{\text{MG}_i}^{\text{Max}}$, L_{MG_i} are the maximum local generation and local load corresponding to the ith MG. Based on this definition, 0%, 100% of MG capacity correspond to pure load and maximum local power generation, respectively.

The ouput signal, that is, the injected power to the grid, following the reconnection of the ADN to the upward network, is captured with the sampling of 0.05 s over a period of 20 s after the fault. The captured response is applied to the Prony toolbox to identify the equivalent model parameters. The Prony toolbox developed by Singh [21] is employed for the Prony analysis. Several operating conditions, in the range of 0 to 100% of each MG capacity, are considered to generate ADN initial operating points. It means that the ADN initial operating points include various combinations of MGs operating conditions. A BFV equal to 80% is considered as the margin of model accuracy. For dynamical simulation of the developed equivalent model, the difference between the measured and the desired powers is applied to each (M, D), corresponding to a specific cluster, through a participation index. Participation index for each cluster in a given group is defined as the ratio between the capacities of the cluster and the group. Summation of the participation indices for all of the clusters in a group is 1.

Initially, a system consisting of four MGs is investigated. The model proposition begins with assuming 81 operating conditions as initial ADN operating points. This consists of all possible combinations from 0%, 70%, 100% of MGs capacities. The injected power of each MG, for a specific operating point, is applied to Prony toolbox to calculate (M_i, D_i). The four calculated

Table 1 Developed groups for system with 4 MGs with BFV = 80%

Group	Cluster 1 (substitute)	Cluster 2 (substitute)	Cluster 3 (substitute)	$P_{MG} - L_{MG}$				RMSE	Computing time [Sec]	
				1	2	3	4		Full model	Equivalent model
1	1–3 ($M = 0.54$; $D = 0.86$)	4 ($M = 0.12$; $D = 0.52$)	-	4.15	−7.80	2.30	1.35	0.5413	565	9
2	2, 4 ($M = 0.27$; $D = 0.63$)	1 ($M = 0.09$; $D = 0.92$)	3 ($M = 0$; $D = 0$)	−6.5	4.75	0	1.75	0.6123	630	13
3	1–4 ($M = 0.49$; $D = 0.21$)	—		5.35	1.25	0.5	−7.1	0.4523	385	5
4	1, 2 ($M = 0.19$; $D = 0.88$)	3 ($M = 0.13$; $D = 0.72$)	4 ($M = 0$; $D = 0$)	3.15	1.50	−4.65	0	0.5981	540	11

(M_i, D_i), corresponding to the four MGs, is applied to SVC for clustering. This procedure is done for all 81 assumed operating points. Afterward, the SVC results, corresponding to all the 81 operating points, are checked for the same arrangements of MGs in the same number of sub-clusters. Calculations reveal that, the grouping procedure, followed by the SVC, arranges the MGs in four groups. Table 1 illustrates the developed groups, the corresponding estimated parameters of clusters, specific operating condition that each group corresponds to and the numerical verification of the developed equivalent model. The reported *RMSE* values in Table 1 are calculated for a set of specific operating points. Comparison between the reported computing times for the full and the equivalent models in Table 1 reveals the significant simplicity of the equivalent model in terms of computational complexity. Owing to the fact that the computational complexity refers to the time requirements, as a function of the size of inputs, to solve a problem [22], such simplification could be justified through two simple inputs of the developed equivalent model, that is, corrective index and angular frequency.

To verify the model robustness, eligibility of cluster substitute in representing sub-clusters dynamics is investigated by sensitivity analysis. For this purpose, in each cluster corresponding to the groups, the MG with greater participation factor is selected to be employed in the sensitivity analysis. MG participation factor is calculated by:

$$\mathrm{PF}_i = \frac{M - M_i}{M_i} \tag{22}$$

where, M, M_i are the cluster substitute stop time and cluster stop time after changing the capacity of ith MG from 0% to 100%, respectively. The sensitivity analysis corresponding to group 1, cluster 1 and sub-cluster1 gives rise to the following equation based on (15):

$$\begin{aligned} &\text{If } 1 - \frac{y - \hat{y}}{y - \bar{y}} = X_1 \Rightarrow 1 - \frac{y - \hat{y}'}{y - \bar{y}} = X_2 \\ &\Rightarrow 1 - \frac{y - \hat{y}(1 + [0.003/0.029] + t0.01)}{y - \bar{y}} = X_2 \end{aligned} \tag{23}$$

In (23), 0.003, 0.029, 0.01 are the difference between B for the cluster substitute and sub-cluster substitute 1, B for cluster 1 in group 1, the difference between λ for the cluster substitute and sub-cluster substitute 1, respectively. All the values are reported in percentage. As stated, (23) could be solved through statistical analysis. In this way, X_1 is measured in several sample times, that is, t, and X_2 are calculated based on (23). Curve fitting technique, applied to the obtained data, gives rise to:

$$X_2 = 1.05X_1 - 0.005 \tag{24}$$

Fig. 6 demonstrates the sampled data and the fitted curve to X_1 and X_2, as described by (24) with R equal to 0.89 and *RMSE* to 0.4823. To investigate the effect of operating point changes on cluster substitute efficiency, MG number 2, the MG with the highest PF in comparison with the included MGs, is ignored and the BFV is re-calculated, that is, $X_2 = \mathrm{BFV} = 0.856$ and $RMSE = 0.6731$. From (24), it could be clearly seen that the cluster substitute would represent the ADN dynamic behaviour with an acceptable accuracy, that is, BFV = 0.82 and $RMSE = 0.8412$. It means that the proposed method even could be employed in $N-1$ criterion stability assessment with acceptable accuracy. On the other hand, it is obvious that the operating point corresponding to $X_2 = 0.835$ causes the ADN to be marginally represented by this cluster substitute, that is, $X_1 = 0.80$. It means that the ADN operated in operating points corresponding to $X_2 \leq 0.835$ could not be replaced with this group. In such a case, another cluster substitute corresponding to another group should represent the stated operating points. As stated, the overlapping between the groups should be investigated in this stage. (25) is calculated by following the same procedure as (24) with R equal to 0.87 and *RMSE* to 0.6374.

$$X_2 = 1.16X_1 - 0.05 \tag{25}$$

where X_1 and X_2 are the BFV for two of the cluster substitutes in groups 1 and 2, respectively. (25) reveals that there is an effective overlap between groups 1 and 2, on which, an unacceptable BFV for one group causes an acceptable one for the other. Following the same procedures as described above for all the clusters in the groups reveal that, the proposed model, with four reported groups, could represent the ADN dynamic behaviour with an acceptable accuracy in a robust manner. Simulation results reveal that the proposed model characterised with some simple first order transfer functions not only exhibits acceptable response, but also significantly reduces computational time to obtain results. The duration of time required to obtain results is 11 s with step size of 0.001 for the equivalent model, while it takes about 9 min for the full model.

Another test is done on the four MG system to validate the accuracy of the proposed equivalent modelling when the operating condition is not included in the assumed initial operating points. For this purpose, consider a situation where the operating points of MGs set to 12%, 38%, 57%, 83% of their respected capacities. Applying Prony analysis and clustering to this system gives rise to an equivalent model, different from those in Table 1, with BFV equal to 85.34% and $RMSE = 0.7112$. However, the group number 2 in Table 1 could appropriately, BFV equal to 83.06% and $RMSE = 0.8320$, represent the system dynamic behaviours. Simulation results reveal that the estimated parameters by

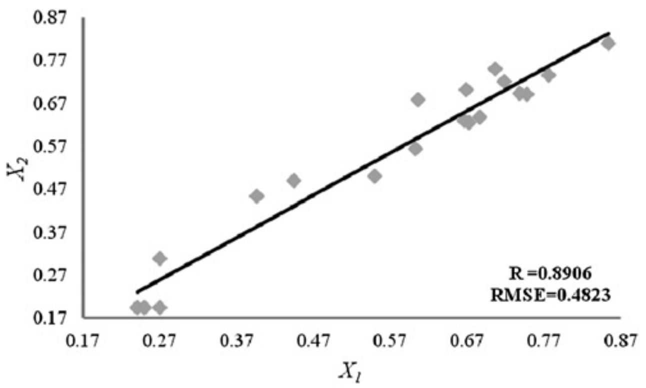

Fig. 6 *Extracting relationship between* X_1 *and* X_2 *based on sampled data*

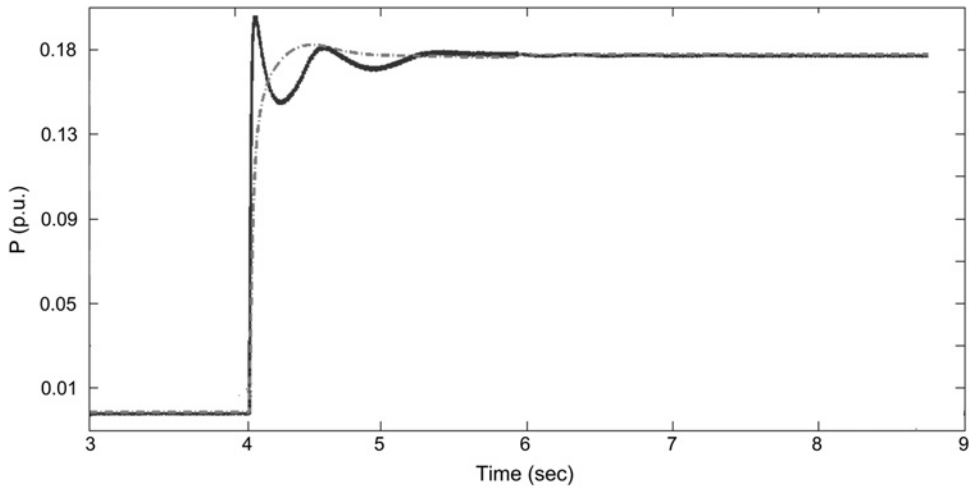

Fig. 7 *Difference between P of the cluster for the new and initial operating points: new operating point with complete model and initial with cluster substitute (solid line), both operating points by cluster substitute (dashed line)*

Table 2 Developed groups for system with 4 MGs, with BFV = 87%

Group	Cluster 1	Cluster 2	Cluster 3	Cluster 4	$P_{MG} - L_{MG}$				RMSE	Computing time [Sec]	
					1	2	3	4		Full model	Equivalent model
1	1, 2, 4	3	—	—	4.17	−3.23	1.75	−2.69	0.3412	615	12
2	1, 4	2	3	—	−5.12	1.47	3.13	−0.52	0.4208	720	14
3	1–4	—	—	—	3.17	−5.17	−4.12	6.12	0.1397	545	8
4	1, 3,4	2	—	—	0.5	3.75	−4.25	0	0.5419	610	12
5	1	2	3	4	−3.13	5.13	−2	0	0.2198	800	17
6	1, 3	2	4		7.12	0	−5.37	−1.75	0.3961	750	15
7	1, 4	2, 3	—	—	4.32	2.76	−4.32	−2.76	0.3123	745	14

re-applying Prony analysis and SVC leads to maximum difference of 0.05 in compare with group 2 parameters. Considering robustness of power system in respect to small variation of parameters, such small difference in ADN parameters could be neglected without any concern. This means that, while the developed groups could not represent all the possible operating points with the optimum model, but could represent the ADN with acceptable accuracy for upward studies. Furthermore, based on the assumed initial operating points, the groups and clusters configuration and sequentially cluster substitute may differ, but the existing overlap between groups guarantees representation of all operating points with acceptable accuracy. It means that, two different sets of initial operating conditions may cause different but acceptable BFV for a specific operating point. Therefore, the proposed modelling strategy tries to compromise between the accuracy and robustness. The test is followed by investigation on transient between the groups. Assume that, initially the system is operated at group 4 in Table 1. The difference between the injected power of the ADN for the new and initial (obtained by equivalence model, here group 4) operating points is considered as injected power of a new connected MG (Fig 7, solid line). The variations in the cluster substitutes after connection of the new assumed MG are calculated by the Prony analysis. (20) gives rise to allocation of the MG, that is, transients from group 4, to group 2 (cluster 1) which verifies the modelling results. Fig. 7 also illustrates the difference between the simulated, by cluster substitutes, injected power of the group 4 and group 2 (dashed line). Comparison between the solid and dashed lines in Fig. 7 reveals that (20) could appropriately specify the transient between the groups.

In another attempt, effect of BFV on the developed model is investigated. For this purpose, in the modelling procedure of four MG system, the BFV increases to 87% and all the stated procedures in Fig. 2 are re-done. Simulation results reveal that the number of groups increases to 7 groups as depicted in Table 2. It is noteworthy that BFV is a degree of freedom and is determined based on the required accuracy. Increasing BFV, gives rise to a more accurate model and causes the MGs to be clustered in more sub-clusters. Numerical comparison between the reported *RMSE* values in Tables 1 and 2 also confirms the better accuracy of the model for greater BFV. While there is no standard value for BFV, the value of BFV would not affect the proposed approach efficiency.

The reported specific operating condition that each group corresponds to in Tables 1 and 2 reveal that the total local generation is equal to the total load in ADN. As another scenario, the efficiency of the proposed modelling strategy is investigated for a case when there is power exchange between the ADN and upward grid. For this purpose 20 [MW] load is switched on at the 1st second of simulation. Fig. 8 illustrates the injected power of ADN to grid after fault. As stated, at the fault time, the switched on 20 [MW] load will be instantaneously provided by upward grid. Afterwards, the ADN starts to provide the load based on the calculated parameters. However, it could be seen from Fig. 8 that the injected power of ADN to upward grid reaches to 13.5 [MW] at steady state. This means that, the ADN has not enough generation to track the load and the mismatch should be compensated by upward grid. Fig. 8 reveals that, the proposed

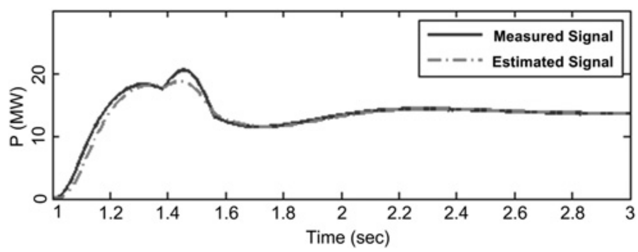

Fig. 8 *P response of the full model and the proposed equivalent model for load increase*

IET Gener. Transm. Distrib., pp. 1–10

Table 3 Developed groups for the system with 20 MGs associated with 200 operating points

Group	Cluster 1 (substitute)	Cluster 2 (substitute)	Cluster 3 (substitute)	$P_{cluster} - L_{cluster}$			RMSE	Computing time [Sec]	
				1	2	3		Full model	Equivalent model
1	1–4, 7–9 (M = 0.59; D = 0.16)	5,6 10–13 (M = 0.91; D = 0.65)	14–20 (M = 0.22, D = 0.81)	12.35	−5.13	−7.22	0.5370	2000	20
2	1,2,4–9 (M = 0.99; D = 0.11)	3, 13–17 (M = 0.82; D = 0.49)	10–12, 18–20	−6.5	6.5	0	0.4961	1850	17
3	1–15 (M = 1.28; D = 0.21)	15–20 (M = 0.64; D = 0.71)	—	9.86	−9.86	—	0.5412	1680	19

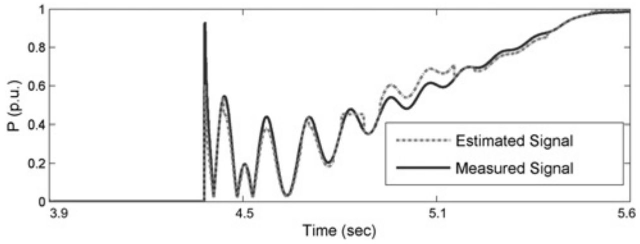

Fig. 9 *P response of the full model (solid line) and the proposed equivalent model (dashed line) in time domain*

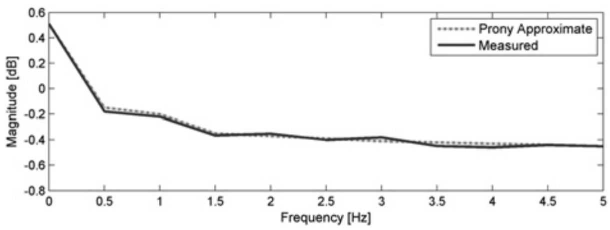

Fig. 10 *P response of the full model and the proposed equivalent model in frequency domain*

model could appropriately represent the ADN dynamic behaviour for a case when there is power exchange between the ADN and the upward grid.

The same procedure is applied to a second system with 20 MGs. Initially, 200 operating points are assumed for model development. Applying Prony analysis, clustering and grouping procedures reveal that 3 groups, as shown in Table 3, should be adapted to the 200 operating points. The developed groups required to be investigated in the case of existing overlap, as done in (25), to cover all the possible operating points. The overlap assessment between two clusters in groups 2 and 3 gives rise to:

$$X_2 = 0.98X_1 - 0.005 \quad (26)$$

From (26) it could be clearly seen that there is no efficient overlap between the developed groups. Representation of the system with the developed groups/clusters could not satisfy the pre-specified accuracy, that is, BFV, for all the possible operating points. In other words, there are several operating points which could not be represented appropriately by any of the developed groups. In such a case, the initial assumption regarding the operating points should be modified. The initial number of the operating points set to 400, which gives rise to 7 groups. Simulation results reveal that the developed groups could appropriately represent the ADN dynamics. Figs. 9 and 10 compare the actual ADN response, obtained using simulations with the full model and the response, obtained with the proposed equivalent model, in time and frequency domains, respectively. Fig. 10 is obtained by calculating the frequency response for some specific frequencies, for which, a linear relationship between the data points is assumed. It could be clearly seen that the proposed equivalent method could appropriately represent the ADN dynamic behaviour in both time and frequency domains. Furthermore, the equivalent model could simulate the ADN dynamics in 17 s which is negligible in comparison with 28 min for full model.

Finally, assume a new MG is to be added to the transmission bus. Initially, the variations in the cluster substitutes after the connection of the new MG are calculated by the Prony analysis. Afterwards, (20) with a study time of 20 s, that is, $(\Delta B_i/B_i) = 20\Delta\lambda_i$, is employed to, approximately, select the associated group and cluster. For this case, $(\Delta B/\Delta\lambda)$ becomes 0.0084 which satisfies (20), associated with cluster 1. To validate the above assignment, the proposed approach in Fig. 2 is applied to the new system with 21 MGs and the results are compared with those based on (20). Simulation results reveal that while both approaches accuracies are acceptable, but the arrangements of the MGs in groups/clusters are different. Fig. 11 compares the actual ADN response obtained using simulations with the full model, the response obtained with the proposed equivalent model, and the response obtained based on (20), respectively. Following the same procedure as presented in Fig. 2, the BFV becomes 86.33% and $RMSE = 0.4318$, while adding a new MG to the old modelled system degrades it to 83.68% and $RMSE = 0.5960$, using (20). These different assignments could be justified by the existing overlap between the groups/clusters. This overlap causes various arrangements of MGs, with different but acceptable accuracy. Owing to the fact that the

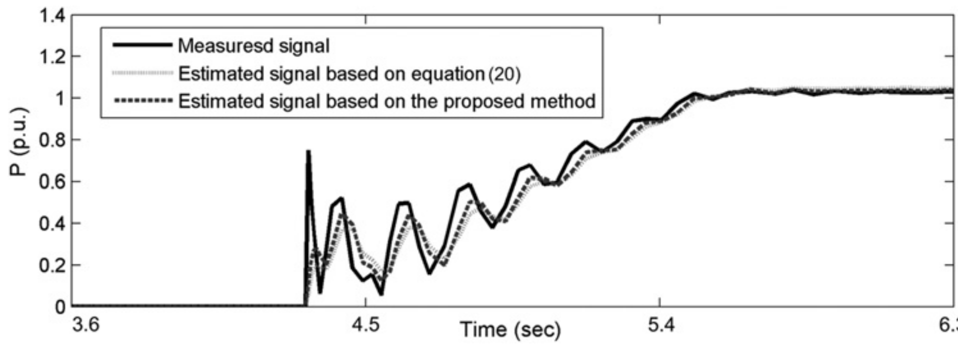

Fig. 11 *P responses of the full model, the proposed equivalent model and the criterion-based model in time domain*

proposed method in Fig. 2 relies on the performed optimisation for the SVC, it exhibits optimal accuracy for the obtained equivalent model. However, (20), which originates from a pessimistic assumption, gives rise to an equivalent model with acceptable accuracy, but not necessarily an optimum one.

5.2 Discussion

The proposed model accommodates variation in the structure of ADN by using statistically derived parameters based on large number of different operating points which include various combinations of DGs and loads. This means that MGs control and management are equivalent to different case studies considered in developing model parameters. More importantly the proposed algorithm gives a robust model which appropriately tackles the operating point changes as well as connection/disconnection of MGs into equivalencing formulation. On the other hand, power system is robust enough so that small variation of ADN parameters will not affect its performance significantly.

Another important point is the model domain of validity. As stated, in spite of the fault type, the ADN responses to the disturbance by providing the power based on the calculated (M, D) in this paper. Nevertheless, the proposed model could not represent the ADN dynamics in the transient horizon between the islanding and grid connected modes.

6 Conclusion

A robust grey-box dynamic ADN model, for frequency stability studies, based on Prony analysis and SVC is presented in this paper. The model uses merely measurable signals to map the ADN characteristics to the conventional generators, and hence can be accurately employed in the upward power system frequency stability studies. The model is a group-based dynamic equivalent model, which systematically takes into account the transients between the groups. It tackles the ADN reconfiguration and transient between the groups into account by a developed mathematical-based criterion, which guarantees the model robustness in respect to operating point changes.

Simulation data from two networks with 4 and 20 MGs are used to evaluate the performance of the proposed method. The obtained results revealed that, the proposed method could simulate the full model response by several simple first order transfer functions in a considerably short period of time. The simulation results indicated that the proposed methodology attempts to make a bridge between the accuracy of modelling and robustness to satisfy the power system stability studies requirements.

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