Model Predictive Control of a Highly Nonlinear Process Based on Piecewise Linear Wiener Models

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***Abstract* – In this paper a nonlinear model predictive control (NMPC) based on a piecewise linear wiener model is presented. The nonlinear gain of this particular wiener model is approximated using the piecewise linear functions. This approach retains all the interested properties of the classical linear model predictive control (MPC) and keeps computations easy to solve due to the canonical structure of the nonlinear gain. The presented control scheme is applied to a pH neutralization process and simulation results are compared to linear model predictive control. Simulation results show that the nonlinear controller has better performance without any overshoot in comparison with linear MPC and also less steady-state error in tracking the set -points.**

I. INTRODUCTION

There are very few design techniques that can be proven to stabilize processes in the presence of nonlinearities and constraints. Model Predictive Control (MPC) has been one of the successful controllers in manufacturing industries for the past two decades. MPC refers to a class of computer control algorithms that control the future behavior of a plant through the use of an explicit process model. At each control interval the MPC algorithm computes an open-loop sequence of manipulated variable adjustments in order to optimize future plant behavior. The first input in the optimal sequence is injected into the plant, and the entire optimization is repeated at subsequent control intervals [1]. By now, the application of MPC controllers based on linear dynamic models cover a wide range of applications and linear MPC theory can be considered quiet mature. Nevertheless, many manufacturing processes are inherently nonlinear and there are cases where nonlinear effects are significant and can not be ignored. These include at least two broad categories of applications [1]:

1- Regulator control problems where the process is highly nonlinear and subject to large frequent disturbances (pH control, etc.)

2- Servo control problems where the operating points change frequently and span a wide range of nonlinear process dynamics (polymer manufacturing, ammonia synthesis, etc.)

In fact higher product quality specifications and increasing productivity demands, tighter environmental regulations and demanding economical considerations require to operate systems over a wide range of operating conditions and often near the boundary of admissible region [2]. Besides the operating point in some batch processes is not in steady-state and all of the operations are performed in transient conditions [3]. Under these conditions linear models are often not sufficient to describe the process dynamics adequately and nonlinear models must be used.

Many of the current NMPC schemes are based on physical models of the process. However, in many cases such models are difficult to derive, and are often not available at all. In these cases it makes sense to use a nonlinear empirical model, identified from input-output measurements. Some works where this approach has been followed are for instance: [4] where a nonlinear predictive control scheme based on radial basis functions models is proposed, [5] and [6] where the NMPC is based on a Hammerstein model, and [6–9], where the NMPC is based on a Wiener model. In all these works the paradigmatic application has been pH neutralization processes. In other cases (e.g. CSTR and polymerization reactor processes) a nonlinear model predictive control based on a Wiener model with a piecewise linear gain is addressed in [10].

In particular, Wiener models have a special structure that facilitate their application to NMPC. These models consist a linear dynamic element is followed by a static nonlinearity and can represent many of the nonlinearities commonly encountered in industrial processes. Due to the static nature of the nonlinearities, they can be removed from the control problem. This fact generalizes the well-known gain-scheduling concept for nonlinear control. Due to the presence of some potential computational difficulties, an implicit inversion of the nonlinear static gain is necessary.

In this work, the linear dynamic element uses a state space model and the static nonlinear element uses the piecewise linear approximation for the process model. This approach retains all the interesting properties of the classical linear MPC while keeping the computations easy to solve due to the canonical structure of the nonlinear gain.

The paper is organized as follows. In Section 2 a Wiener model with a piecewise linear representation for the nonlinear gain is presented. In Section 3, nonlinear model predictive control based on piecewise linear wiener model is described. The simulation results for identification and control of pH neutralization process are given in section 4. Finally, in Section 5, some concluding remarks are discussed.

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II. WIENER MODEL IDENTIFICATION

A. Piecewise linear wiener model

Let us assume that the system to be controlled can be described by the following discrete-time, nonlinear, state-space model:

$$x(k+1) = f(x(k), u(k)) \tag{1}$$

$$y(k) = g(x(k) + d(k)) \tag{2}$$

where $f: R^n \times R^{m_i} \to R^n$ and $g: R^n \to \times R^{m_o}$ are twice continuously differentiable functions, $x \in R^n$ is a vector of n state variables, $u \in R^{m_i}$ is a vector of m_i process inputs or manipulated variables, $d \in R^{m_o}$ is a vector of m_o additive disturbance variables $y \in R^{m_o}$ is a vector of m_o process outputs and k is the sample time. Bounds on the manipulated variable as well as on the system outputs are assumed, as follows:

$$y_l \le y(k) \le y_u \quad , \tag{3}$$

$$u_l \le u(k) \le u_u \quad , \tag{4}$$

$$\Delta u_l \le \Delta u(k) \le \Delta u_u \, . \tag{5}$$

In this paper, the possibilities and the advantages of the use of a specific Wiener approximation to represent the model of the process are analyzed. A Wiener model consists of a dynamic linear block (*H1*) in cascade with a static nonlinearity at the output (*H2*), as shown in Fig. 1, where $v(k) \in R^{m_o}$ is an intermediate signal which not necessarily has a physical meaning.

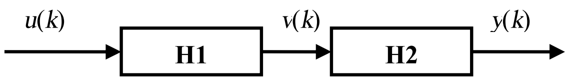

Fig. 1. The wiener model

There are several options to describe the linear dynamic block in wiener models. For examples, some of the used representations include convolution models (step or impulse responses), ARMAX models, ARX models, state-space models, etc [11]. In this application, a state space model is used as follows:

$$\begin{aligned} x(k+1) &= \mathbf{A}x(k) + \mathbf{B}u(k) \\ v(k) &= \mathbf{C}^T x(k) + \mathbf{D}u(k) \end{aligned} \tag{6}$$

For the static nonlinear element (H_2), the use of continuous piecewise linear (PWL) functions is proposed. PWL functions have been proved to be a very powerful tool for modelling and analyzing nonlinear systems [12]. It can be proved that any nonlinear function f, $f: R^{m_o} \to R^{m_o}$ can uniquely be represented as [12]:

$$f(v) = C^T \Lambda(v) \tag{7}$$

where, the vector $\Lambda = [\Lambda^{0T}, \Lambda^{1T}, ..., \Lambda^{m_0 T}]^T$ is the elements of the basis and $C = [C_0^{\,T}, C_1^{\,T}, ..., C_{m_0}^{\,T}]^T$ that every vector C_i is a parameter vector associated to the vector function Λ^i.

In our application, as shown in fig. 2, the function is $f = H_2 \; : D \to D$, being $D \in R^{m_o}$. The domain and the image of the PWL function share the same dimension in this application. Moreover, if we assume that the function f of the system is invertible (this is a reasonable assumption for a large set of process models), it is possible to define the inverse function as f^{-1}, such that $v = f^{-1}(f(v))$. This function is also unique and it is a PWL [11].

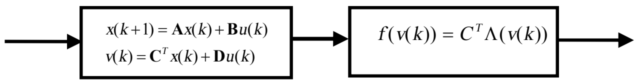

Fig. 2. The piecewise linear wiener model.

B. Test Design

Some important factors which must be considered in designing the identification test for nonlinear systems are: duration of the test signal, amplitude and shape of the test signal, the spectrum of the test signal (the average switching time), correlation of the test signal in each channel, and the number of manipulated variables in each test.

Since in nonlinear systems the test time depends mainly on the number of parameters in the model and the level of noise and unmeasured disturbances, it is recommended longer test time in comparison with linear systems. This is typically considered about 16-25 times of the settling time of the process. The other factors may be included by choosing one of the following test signals [13]:

1- Stair Test: In this type of test the width of the pulses and their numbers must be selected properly.

2- Generalized Multiple-level noise (GMN): This type of test which is also used here is a multi-level extension of generalized binary noise. In this test the amplitude and the number of pulses must be selected suitably. The number of levels on this test is equal or greater than the degree of nonlinear polynomial which must be identified. Moreover, if T_{sw} is the average switching time of the test then $T_{sw}=T_s/3$ where T_s is the 98% of the settling time of the process.

3- Filtered white noise: The flexibility in shaping the spectrum of this type of signal is its main advantage. Each spectrum may be realized with a proper filter. A first order low-pass filter is often suitable for this purpose.

C. Wiener model identification and inverse model evaluation

Different Wiener model identification approaches can be found in the relevant literature. A general classification of these approaches is the following:

1- The N-L approach. First the output static nonlinearity is determined, using steady-state data. Then the dynamic linear block is identified, being the intermediate signal v generated from the output signal using inverse non-linearity mapping [14].

2- The L-N approach. First the linear block is identified using a correlation technique; after that, the intermediate signal v is generated from the input signal and finally the static non-linearity is estimated [15].

3- The simultaneous approach. Parameters of the linear block and the static non-linearity are estimated at the same time.

The second approach, which is used in this paper, is straightforward and ensures an accurate description of the static nonlinearity.

In this paper a GMN signal is used to generate dynamic data. The nonlinear static gain is identified using dynamic and steady-state input–output data. A standard identification algorithm and the Toolbox [16] based on the least-square method were respectively used to identify the linear dynamic part and the static PWL function. This Toolbox allows not only obtaining a nominal model but also the uncertainty band enclosing all the available data.

In order to implement the NMPC scheme that is described in the next section, a good representation of the inverse of the non-linearity is necessary. To identify it, some approaches are available [10, 17]. Since problems of small dimension are dealt with here and useful data for the identification process are available, the direct identification has been chosen in this paper. This approach identifies the nonlinear element of model by switching inputs and outputs.

III. THE NONLINEAR MODEL PREDICTIVE CONTROL BASED ON PIECEWISE LINEAR WIENER MODEL

The control problem to be solved is to compute a sequence of inputs $\Delta u(k)\ \{k=1,...,M\}$ that will minimize the following dynamic objective:

$$J=\sum_{j=1}^{P}\left\|y(k+j)-r\right\|_{Q_j}+\sum_{j=0}^{M-1}\left\|\Delta u(k+j)\right\|_{R_j} \qquad (8)$$

Subject to model equations and to inequality constraints

$$\begin{aligned} y^l &\le y(k+j)\le y^u \qquad \forall j=1,...,P-1\\ u^l &\le u(k+j)\le u^u \qquad \forall j=1,...,M-1 \end{aligned} \qquad (9)$$

Where P is the prediction horizon, M is the control horizon, r is the desired set point, the relative importance of the objective function contributions is controlled by setting the time dependent weight matrices Q_j and R_j. Beyond the control horizon, the control signal is assumed to be constant ($\Delta(k+j)=0$, $j=M,...,P$). Once $\Delta u(k)$ is computed, following the receding horizon principle, only the first element of the optimal control sequence is used as the current control value. Then the horizon shift one step forward in time and the whole procedure is repeated.

Let us now consider the Wiener model shown in Fig. 2. If at time k, the future state and behavior of the plant is assumed to be known, they can be written in vector form as follows:

$$\mathbf{v}(k)=\left[v^T(k+1)\quad v^T(k+2)\quad\cdots\quad v^T(k+P)\right]^T$$

$$\mathbf{u}(k)=\left[u^T(k+1)\quad u^T(k+2)\quad\cdots\quad u^T(k+M)\right]^T$$

$$\mathbf{y}(k)=\left[y^T(k+1)\quad y^T(k+2)\quad\cdots\quad y^T(k+P)\right]^T$$

$$\mathbf{r}(k)=\left[r^T(k+1)\quad r^T(k+2)\quad\cdots\quad r^T(k+P)\right]^T$$

where $\mathbf{v}(k)$ is the vector of outputs of linear model, $\mathbf{u}(k)$ the vector of manipulating variables, $\mathbf{y}(k)$ the vector of the outputs of the wiener model, and $\mathbf{r}(k)$ the vector consisting set points. Also M , P are the control and prediction horizon respectively. Then the predicted output for the linear model is

$$\hat{\mathbf{v}}(k)=\beta\,\Delta\mathbf{u}(k)+\xi\,x(k)+d(k) \qquad (10)$$

Where

$$\beta=\begin{bmatrix}\mathbf{C}^T\mathbf{B} & \mathbf{D} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{C}^T\mathbf{AB} & \mathbf{C}^T\mathbf{B} & \mathbf{D} & \mathbf{0} & \mathbf{0}\\ \mathbf{C}^T\mathbf{A}^2\mathbf{B} & \mathbf{C}^T\mathbf{AB} & \mathbf{C}^T\mathbf{B} & \mathbf{D} & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{C}^T\mathbf{A}^{P-1}\mathbf{B} & \mathbf{C}^T\mathbf{A}^{P-2}\mathbf{B} & \mathbf{C}^T\mathbf{A}^{P-3}\mathbf{B} & \cdots & \mathbf{C}^T\mathbf{A}^{P-M}\mathbf{B}\end{bmatrix}$$

$$\xi=\begin{bmatrix}\mathbf{C}^T\mathbf{A}\\ \mathbf{C}^T\mathbf{A}^2\\ \mathbf{C}^T\mathbf{A}^3\\ \vdots\\ \mathbf{C}^T\mathbf{A}^{P-1}\end{bmatrix}$$

and

$$d(k)=[d(k+1|k)\cdots d(k+P|k)$$

Then, the predicted output for the complete model is

$$\mathbf{y}(k)=\begin{bmatrix}h(\hat{\mathbf{v}}(k+1))\\ h(\hat{\mathbf{v}}(k+2))\\ h(\hat{\mathbf{v}}(k+3))\\ \vdots\\ h(\hat{\mathbf{v}}(k+P))\end{bmatrix}=h(\hat{\mathbf{v}}(k)) \qquad (11)$$

Let us now define some points related to the MPC structure [10]:

1. Since the PWL function f was assumed to be invertible, it is possible to write the desired signal referred to the output of the linear model as a transformation of the set point $r(k)$ as,

$$\mathbf{r}^*(k)=f^{-1}(\mathbf{r}(k)) \qquad (12)$$

2. If y_u and y_l are the upper and lower bounds for the outputs variables $y(k)$, then these magnitudes can be translated to the linear model as,

$$\mathbf{v}_u = f^{-1}(\mathbf{y}_u)$$
$$\mathbf{v}_l = f^{-1}(\mathbf{y}_l) \quad (13)$$

3. Disturbances are typically handled by assuming that a step signal has entered at the output and that it will remain constant for all future time ($d(k) = d(k+j),\ j = 1,...,P$). In this case the step disturbance is computed:

$$d(k) = f^{-1}(\mathbf{y}^m(k)) - \hat{\mathbf{v}}(k) \quad (14)$$

where $\hat{\mathbf{v}}(k)$ is the current predicted output for the linear model and $\mathbf{y}^m(k)$ is the current measure output for the process. It is straightforward that introducing this bias in the error, as a perturbation, allows removing any model errors offset in steady-state.

Finally, the WNMPC (Wiener NMPC) can be posed as a quadratic optimization problem (QP),

$$\min_{\mathbf{u}(k)} J = \min_{\mathbf{u}(k)} \left\{ (\hat{\mathbf{v}}(k) - \mathbf{r}^*(k))^T \mathbf{Q}\ (\hat{\mathbf{v}}(k) - \mathbf{r}^*(k)) + \Delta\mathbf{u}(k)^T \mathbf{R}\ \Delta\mathbf{u}(k) \right\}$$

subject to

$$\hat{\mathbf{v}}(k) = \beta\,\Delta\mathbf{u}(k) + \xi\, x(k) + d(k) \quad (15)$$
$$\mathbf{v}_l \le \hat{\mathbf{v}}(k) \le \mathbf{v}_u$$
$$\Delta\mathbf{u}_l \le \Delta\mathbf{u}(k) \le \Delta\mathbf{u}_u$$
$$\mathbf{u}_l \le \mathbf{u}(k) \le \mathbf{u}_u$$

where the relative importance of the objective function contributions is controlled by setting the weight matrices Q and R. Note that minimization of (15) is a classical LMPC;

IV. CASE STUDY: PH NEUTRALIZATION PROCESS

A. Process Description

This process contains HNO3 as the acid stream, NaOH as the base stream and NaHCO3 as the buffer stream. The process is schematically depicted in Fig. 3. The inputs of the system are the base flow rate (u_1), buffer flow rate (u_2), and the acid flow rate (u_3), while the pH level of the solution is considered as output (y). Usually the acid flow rate and the volume of the tank are assumed to be constant and the pH level of the solution is controlled by changing the base flow rate. The governed nonlinear equations which are highly nonlinear and their parameters are described in [9]. Fig. 4 shows the nonlinear behavior of the open-loop response of process for $+/-10\%$ change in the flow rate of the input signal. As it can be seen the gain for $+10\%$ change is about 250% greater than the -10% change.

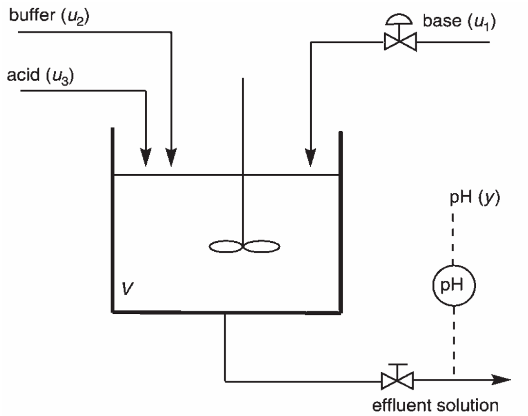

Fig. 3. Schematic representation of the pH neutralization process.

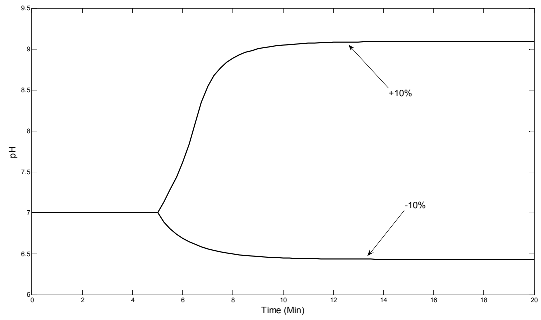

Fig. 4. Open-loop step response of the pH neutralization process for changing the flow rate of the input signal.

B. Identification of the process

To identify this process a GMN signal with six levels 13, 15.55, 17, 18, 20, and 25 is selected to cover the spanned range of the input signal. Switching time between these levels is assumed to be 6 samples. 1280 samples of the input-output data with sampling time of 0.25min are used for identification. Figs. 5 shows the input and output signals of the process (base flow rate and the pH of the solution respectively).

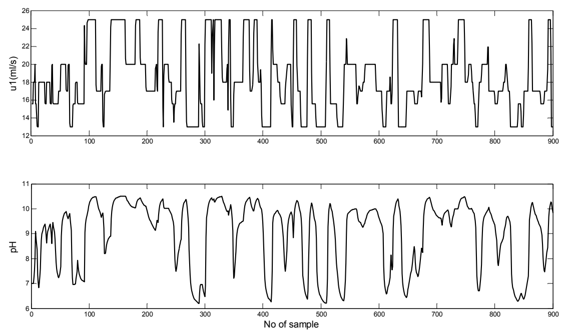

Fig. 5. GMN input (Up) and output (Down) signals for identification of pH process.

By using the input/output data, the model parameters for the linear block were computed. In the identification of the Wiener static gain a dynamic data set and a steady-state data set were used. A state-space description and the toolbox [10] based on the least-square method were respectively used to identify the linear dynamic part and the static PWL function. 900 samples are used for identification and the rest of the signal is used for validation purpose. Fig. 6 shows the validation results. Fig. 7 clearly shows the PWL approximation and the inverse PWL approximation, where the nonlinear nature of the process in the operating region is shown.

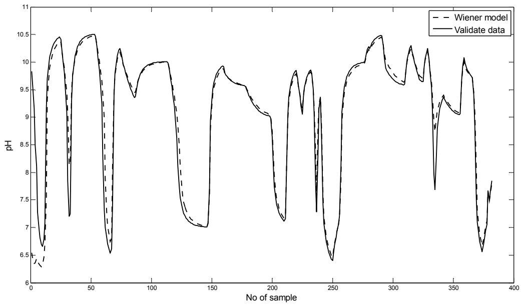

Fig. 6. Validation of the wiener model.

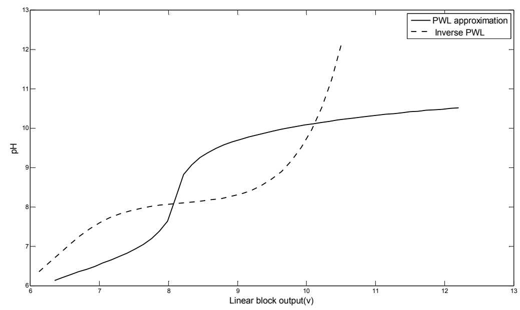

Fig. 7. PWL approximation and inverse PWL approximation.

C. Nonlinear model predictive control

The WNMPC described in (15) was connected to the simulation model of the process. The control and prediction horizons are tuned 10 and 5 respectively and weighting matrices are selected as $\mathbf{Q} = 100\mathbf{I}$ and $\mathbf{R} = 80\mathbf{I}$. Also, saturation constraints in the manipulated variable are imposed to take into account the minimum and the maximum aperture of the valve regulating the base flow rate. For both cases (NMPC and linear MPC) a lower limit of 13 ml/s and an upper limit of 25 ml/s were chosen for this variable. Parameters of both linear and nonlinear model predictive controllers are tuned and the best obtained results are compared.

The behavior of the following regulating points with NMPC controller has been studied. The comparison of these results with linear MPC controller in Fig. 8 is shown. As it is clear from this figure the NMPC controller has better performance without any overshoot, while in linear MPC especially when the operating point goes far from the point where linear model is identified the performance is poor. The control effort which is the base flow rate, for NMPC and LMPC is shown in Fig. 9. This figure shows that the control signal for NMPC controller is relatively smooth and has not large step changes. Fig. 10 shows the behavior of the NMPC when a measurement noise with the SNR of 20dB is added to the output signal; demonstrating that NMPC shows good performance.

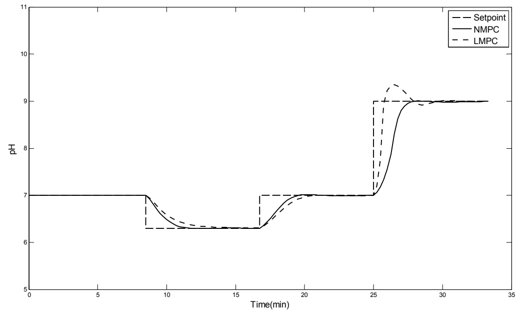

Fig. 8. Comparison of NMPC (solid line) with linear MPC (dashed line) for set point changes.

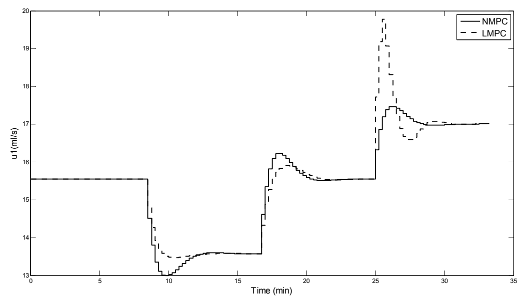

Fig. 9. Comparison control signal of NMPC (solid line) with linear MPC (dashed line) for set point changes.

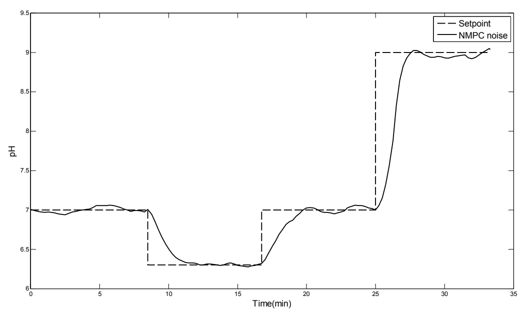

Fig. 10. Process output with noise for set point changes

V. CONCLUSIONS

In this paper a nonlinear model predictive control (NMPC) based on a piecewise linear wiener model for control of pH neutralization process is applied and simulated. This approach has all the interesting features of classical MPC and since it considers a standard structure for computing nonlinear gain, will result in quadratic programming problem which has easy computations. On the other hand, the identification of nonlinear gain of process using piecewise linear approximation needs both dynamic and steady state input-output data. Simulation of the NMPC controller for a wide range of operating point shows superior performance of the NMPC compare to linear MPC. This is especially true, when the operating condition of the controller is far from the point where the model for linear MPC is identified. Results show that in such conditions the linear MPC follows the set point with overshoot, while the nonlinear MPC exhibits a desirable fast response with smoother changes in the control effort. Simulations also confirm that when a measurement noise is added to the output signal, the NMPC shows good performance.

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