The effects of ring radius on characteristics of ring modulator

Niloofar Mohamadi
Dept. of Electrical Eng.
University of Kurdistan
Sanandaj,Iran
Email: niloofar.mohamady.71@gmail.com

Mohammad Razaghi
Dept. of Electrical Eng.
University of Kurdistan
Sanandaj,Iran
Email: m.razaghi@uok.ac.ir

***Abstract*—Analysis of ring modulator helps to gain a better perception of modulator operation. There are extracted equations for transmission and modulation depth. These equations are based on loss, index and coupling modulation. Since the size of optoelectronic circuits is concern in nowadays industry, by using these extracted equations the effects of changes in ring radius on bandwidth is studied. In this study, loss, index and coupling modulation considered separately. When the ring radius changes 73 percent, the bandwidth changes $8GHz$ in loss modulation and $55GHz$ in index modulation. For coupling modulation the bandwidth is limited by FSR, where with increase the ring radius FSR decrease. Furthermore, the dependency of loss, index and coupling modulations bandwidth to quality factor parameter is investigated.**

***Keywords*—Bandwidth; Coupling modulation; Index modulation; Loss modulation; Quality factor.**

I. INTRODUCTION

Nowadays, communication community is looking for a way to solve problem of bandwidth for making fast and safe connections. Optical communication solved the limitation of bandwidth by using photon as carrier. Replacing optical communication for data transmission caused a dramatic change in communication industry. High speed data transfer, more bandwidth , high information security were the reasons for importance of optical communication.

Modulators are one of the most important elements that effects the communication links bandwidth. Over the past few years, there have been much efforts in improving modulator characteristics such as bandwidth [1], [2], loss [3], [4] and power consumption[5]. In recent years, with decreasing size of optical and electrical circuits, ring resonator modulators have attracted significant i nterest i n m odulation a rea. Ring resonators have extended use in silicon photonic. A very high index contrast, which caused the ability of making ring with small radius, and availability of CMOS fabrication technology are the reasons for importance of ring resonators in silicon photonic. Recently, novel structures have been reported to increase the ring resonators bandwidth [6], [7], [8], [9] and decrease power consumption [10].

In recent years many efforts have been done in analysis and simulation of ring resonator modulators. In some works by use of Maxwell's equations the analysis of ring modulator is studied [11] and in some cases use numerical methods to solve field's equations of a ring modulator [12] and by use of strong methods like FDTD [13]. For gain a better intuition of limitations in a ring resonator, a fully dynamic analysis is required. In a ring resonator by changing loss of the ring, refractive index and coupling between ring and waveguide an input optical wave can be modulated.

In this paper electric field equations of the ring will be solved. An expression for dynamic transmission will be obtained. This expression is based on mentioned ring resonator parameters. As well as effects of each parameters of modulation on modulation depth will be studied. Bandwidth of the modulator will be calculated by using small signal approximation. At last effects of ring radius on bandwidth and quality factor Q of modulator will be studied.

II. MODELLING

In this section dynamic analysis of the ring modulator using the structure in Fig. 1 is performed. In this structure for study limitations that impose by the ring structure, any specific material is not considered. Also, the Kramers Kronig relation that indicates relation between refractive index and absorption is ignored. By using these assumptions, its possible to study each resonant parameter separately.

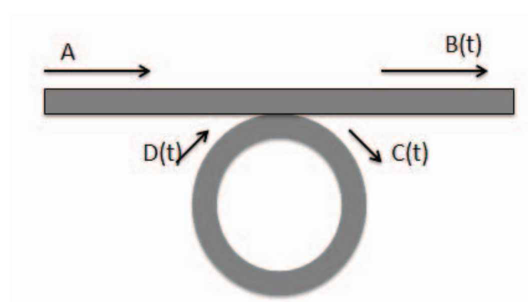

Fig. 1. Schematic of ring resonator modulator.

The electric field at each location in Fig. 1 can be considered as $E(t) = \xi(t)exp(i\omega_0 t)$. ω_0 is frequency of the optical wave and $\xi = B, C, D$, which these quantities varies slowly respect to the input optical wave. Considering the wave propagation in coupling region, the electric fields relation could be written as [14]:

$$B(t) = \sigma(t)A + i\kappa(t)D(t). \tag{1}$$

$$i\kappa(t)C(t) = \sigma(t)B(t) - A. \tag{2}$$

$\kappa(t)$ and $\sigma(t)$ are the coupling coefficient and transmission coefficient. Without considering loss in coupling region: $\sigma^2(t)+\kappa^2(t) = 1$. Refractive index and attenuation coefficient for loss and index modulation are considered as [14]:

$$n_i(t) = n + \eta(t). \tag{3}$$

$$a_i(t) = a + \gamma(t). \tag{4}$$

In a ring resonator when refractive index and attenuation constant varied as a function of time, a phase shift , ϕ,and attenuation, a, impose to propagation wave by this variation. At frequency ω after a round trip in ring by propagation wave the phase shift and attenuation are [14]:

$$\phi(t, \omega) = \omega\tau + \frac{\omega}{n}\int_{t-\tau}^{t} \eta(t')dt'. \tag{5}$$

$$a(t) = a_0 + \frac{1}{\tau}\int_{t-\tau}^{t} \gamma(t')dt'. \tag{6}$$

Where $\tau = nL/c$ is the resonator round trip time, n is the refractive index, L is perimeter of the ring.

A general expression for dynamical transmission is driven as [14], [15].

$$T_s(t) \equiv \frac{B(t)}{A} = \sigma(t) + \frac{\kappa(t)}{\kappa(t-\tau)}a(t)exp[-i\phi(t)] \\ [\sigma(t-\tau)T(t-\tau) - 1]. \tag{7}$$

By use of Eq.(7) steady state transmission can be expressed as [11]:

$$T_{ss} \equiv \frac{B}{A} = \frac{\sigma - aexp(-i\phi)}{1 - a\sigma exp(-i\phi)}. \tag{8}$$

From Eq.(8) on resonance situation, $exp(-i\phi) \approx 1$, when $a = \sigma$ steady state transmission tends to be zero, this situation is called critical coupling, where the transmission in this case is minimum. Extinction ratio of the modulator becomes maximum in this situation. Another factor that could improve modulation is quality factor. Quality factor is conceptually related to the number of round trips that a wave circulates before its energy decrease to 1/e of its initial value[16]. Quality factor indicates the slope of transmission near the resonance wavelength. By increasing the quality factor of ring resonator, with a small change in modulation parameters, a large change in transmission can be achieved. The quality factor of a ring resonator based on ring parameters can be expressed as [16]:

$$Q_{factor} = \frac{\pi n_g L\sqrt{a\sigma}}{(1-\sigma a)\lambda_{res}}. \tag{9}$$

Where n_g is group index, λ_{res} is wavelength resonance. Based on Eq. (9), when $\sigma, a \approx 1, Q \longrightarrow \infty$. A general solution of Eq. (7) can be expressed as a Fredholm integral equation of the second kind[14]. This integral can be solved by use of a Neumann series solution[17]. This integral for Eq.(7) can be expressed as [14] :

$$Ta(t) = \sigma(t) + \frac{\kappa(t)}{\kappa(t-\tau)}a(t)exp(-i\phi(t)) + \\ \int_{-\infty}^{\infty} \frac{\kappa(t'+\tau)}{\kappa(t')}a(t'+\tau)\sigma(t') \\ exp[-i\phi(t'+\tau)]\delta(t'-(t-\tau)T(t')dt'. \tag{10}$$

Now the effects of modulation parameters is considered, separately.

A. Loss modulation

In this case, loss of the ring is modulated, where $a(t)$ varies as a function of time. In loss modulation the phase shift in one round trip, ϕ, coupling coefficient, κ, and transmission coefficient, σ, are constant. The transmission by Neumann series solution is given by [14]:

$$T_a(t) = \sigma - a(t)e^{-i\phi} + \sum_{n=1}^{\infty}\sigma^n e^{-in\phi}[\sigma - a(t-n\tau)e^{-i\phi}] \\ \prod_{m=0}^{n-1} a(t-m\tau). \tag{11}$$

In the above equation the first two terms are related to the instantaneous response of the ring and the third term is known as memory of the ring. The existence of memory part originate from the nature of the ring, which after each round trip propagating wave interacts with the wave that coupled to the ring. For study modulation depth of modulator and bandwidth, first the small signal approximation is studied. Attenuation coefficient has the form $a(t) = a_0 + a'cos(\Omega_m t)$, which Ω_m is the modulation frequency, and $a'/a \ll 1$. By using of Fourier transform of $a(t)$ and substituting of $a(\Omega_m)$ into the Fourier transform of the Eq. (7) and some simplification the modulation depth of loss modulation is expressed as [14]:

$$\Delta_a = 2a'(1 - \sigma^2(\sigma cos\phi - a_0 + \sigma a_0 e^{-i\Omega_m\tau}(a_0 cos\phi - \sigma))) \\ |\frac{1}{(1 + a_0^2\sigma^2 e^{-i2\Omega_m} - 2a_0\sigma cos\phi e^{-i\Omega_m\tau})...}| \\ |\frac{1}{...(a_0{}^2 + \sigma^2 - 2a_0\sigma cos\phi)}|. \tag{12}$$

Modulation depth of a signal is expressed as:

$$\Delta = \frac{f_{max}(t) - f_{min}(t)}{(f_{max}(t) + f_{min}(t)}. \tag{13}$$

Where f_{max} and f_{min} are the maximum and minimum amplitude of the signal.

From $Eq.(12)$ as $\sigma, a \simeq 1$ the $3dB$ roll off frequency is decrease, which means the bandwidth of loss modulation is decrease with increasing quality factor.

B. Index modulation

With changing the refractive index of the ring the phase of propagating wave in the ring will be modulated. In this case $\phi(t)$ varies with time and κ and a is constant. The Neumann series solution for index modulation is [14]:

$$T_a(t) = \sigma - ae^{-i\phi(t)} + \sum_{n=1}^{\infty} \sigma^n a^n [\sigma - ae^{-i\phi(t-n\tau}] \prod_{m=0}^{n-1} e^{-i\phi(t-m\tau)}. \quad (14)$$

As in the case of loss modulation, the first two terms in above equation are related to instantaneous terms and the last term known as memory term. Small signal approximation is used to calculate the modulation depth of index modulation.

Modulation depth is expressed as [14]:

$$\Delta_\phi = 2\phi'[\frac{\sigma a(1-\sigma^2)sin\phi_0)}{(\sigma^2+a^2-2\sigma acos(\phi_0))}] \times [\frac{(1+a^4-2a^2cos(\Omega_m\tau)}{((1-\sigma^2a^2)^2+4\sigma^2a^2[cos(\phi_0)-cos(\Omega_m\tau)]^2 ...} \frac{1}{... - 4\sigma a(1-\sigma a)^2 cos(\phi_0)cos(\Omega_m\tau))})]^{0.5}. \quad (15)$$

When the input is on resonance, from Eq. (15) modulation depth is zero. This is because on resonance transmission is minimum. From Eq. (15) as $a, \sigma \longrightarrow 1$ gives modulation depth zero. This means that increasing quality factor makes the bandwidth decrease.

C. Coupling modulation

In this section the coupling strength between the direct waveguide and ring is modulated. The Neumann solution for this case is [14]:

$$T_\sigma(t) = \sigma(t) - \frac{\kappa(t)}{\kappa(t-\tau)} ae^{-i\phi} + \kappa(t) \sum_{n=1}^{\infty} \frac{a^n e^{-in\phi}}{\kappa(t-n\tau)} [\sigma(t-n\tau) - \frac{(\kappa(t-n\tau))}{(\kappa(t-(n+1)\tau))} ae^{-i\phi}] \prod_{m=1}^{n} \sigma(t-m\tau). \quad (16)$$

For the small signal approximation analysis the relation between σ and κ should be considered. So for a sinusoidal coupling coefficient ($\kappa(t) = \kappa_0 + \kappa' cos(\Omega_m t)$), the transmission coefficient is $\sigma(t) = \sigma_0 + \sigma' cos(\Omega_m t)$. With substituting Fourier transform of Eq. (16) and coupling coefficient into Eq. (7) the modulation depth is expressed as [14]:

$$\Delta_a = 2\sigma' |(1 - a^2 e^{-i\Omega_m\tau}) [\sigma_0 - acos\phi + a\sigma_0(a - \sigma_0 cos\phi)e^{-i\Omega_m\tau}] / (\sigma_0^2 + a^2 - 2a\sigma_0 cos\phi)(1 + a^2\sigma_0^2 e^{-2i\Omega_m} - 2a\sigma_0 cos\phi e^{-i\Omega_m\tau})|. \quad (17)$$

On resonance and with the assumption that $\Omega_m\tau \ll 1$, modulation depth can be expressed as [14]:

$$\Delta_{\sigma,res} = 2\sigma'[\frac{((1-a^2)^2 + a^2(\Omega_m\tau)^2)}{((\sigma_0 - a)^2[(1-a\sigma_0)^2 + a\sigma_0(\Omega_m\tau)^2])}]^{0.5}. \quad (18)$$

Eq. (19) shows that magnitude of σ_0 and a are strongly effect modulation depth. Also with changes frequency toward higher frequency, modulation depth is constant. In coupling efficiencythe quality factor doesn't limit the bandwidth.

III. Results and discussion

In this section for gaining a better intuition, a large signal for the refractive index, loss and coupling coefficient is applied to the modulator. Fig. 2(a),Fig 3(a)and Fig. 4(a) show the input signals for Loss, refractive index and coupling modulation, respectively. These inputs are three Gaussian pulses with different free width at half maximum (FWHM) with the same amplitude. By applying these inputs to the ring resonator modulator the effects of frequency and quality factor on transmission can be shown. The results of applying inputs are tabulated in Fig. 2(b),Fig. 3(b) and Fig.4(b). These results are in a good agreement with the results shown in [16]. The difference between amplitude and DC offset of the transmitted signal is due to the different wavelength of the input wave. As shown in Fig. 5(b) and Fig. 6(b), as the FWHM of input decreases, the output signal related to loss and index modulation suffer from more time delay and distortion. This implies that by increasing frequency of input signal the performances of loss and index modulators are limited. The delay in output transmission is due to memory term in Eq. (11) and Eq. (14). In loss and index modulation at any time, transmitted signal is indicates by loss and index quantities at that time and prior times which means that effects of parameters at prior times still exist an cause delay. By increasing quality factor as mentioned before, its take longer time for a circulating wave to disappear in ring. According to this definition, as quality factor increase time for modulator to sense changes in inputs completely is longer and this is the reason for distortion effect in output of modulator. In the other words when the input changes fast the output can not trace the input changes so fast.

In study coupling modulation case we consider an abrupt changes in coupling. When the coupling decreases abruptly to zero. No light could enter or exit in contrast with loss and index modulation. So the changes in coupling modulation impact on output directly. The suppression of Gaussian tail in coupling modulation is due to low frequency, limit of coupling modulation. To suppress this distortion it is necessary to increase quality factor [14]. In coupling modulation case increasing quality factor improves performance of modulators at low frequency without effects on high frequency response of output.

Another structural parameter of the ring resonator is ring radius. This parameter has a great impact on modulator characteristics. These characteristics are bandwidth, quality factor and loss. In this part the impact of rings radius on bandwidth and the limitation impose by the ring radius, is studied. For investigating effects of ring radius on modulator characteristics, ring radius changes from $3\mu m$ to $13\mu m$. Fig. 5(b) and Fig. 6(b) show the bandwidth as a function of ring radius for loss modulation and index modulation, respectively. A comparison between Fig. 5 and Fig. 6 show that in both

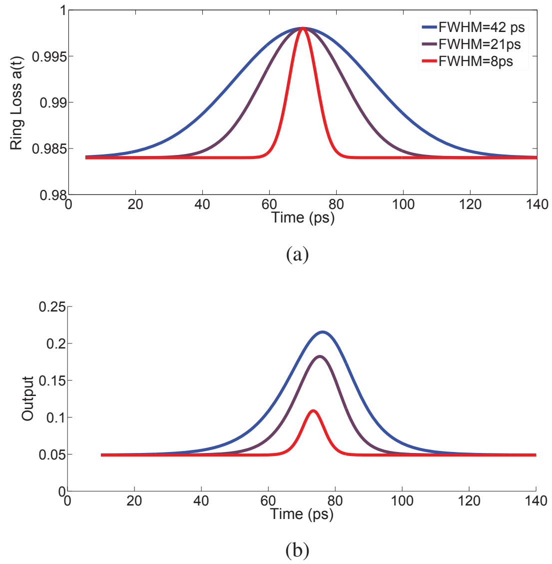

Fig. 2. (a) Input signal of loss modulation, $\sigma = 0.9928$. (b)The outpot transmission of optical wave

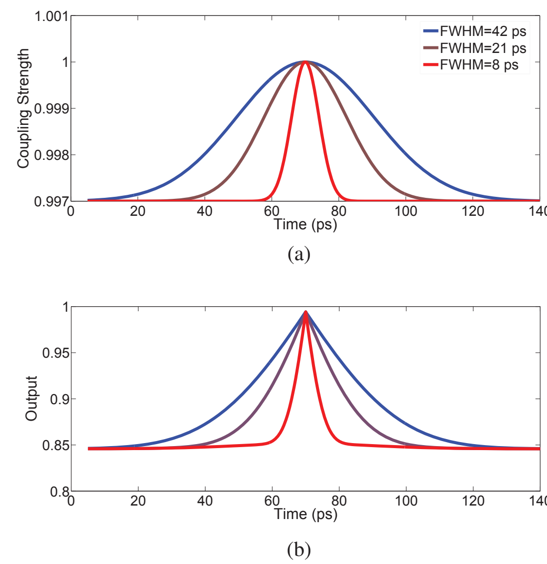

Fig. 4. (a) Input signal of coupling modulation. (b)The outpot transmission of optical wave.

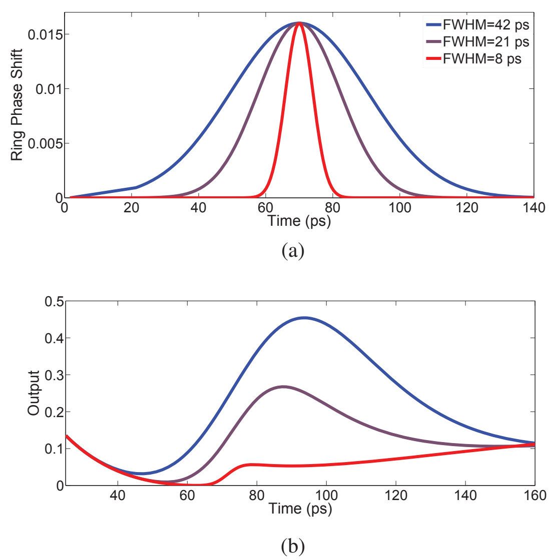

Fig. 3. (a) Input signal of index modulation, $\phi_0 = 0.039477$, $\sigma = 0.9928$. (b)The outpot transmission of optical wave.

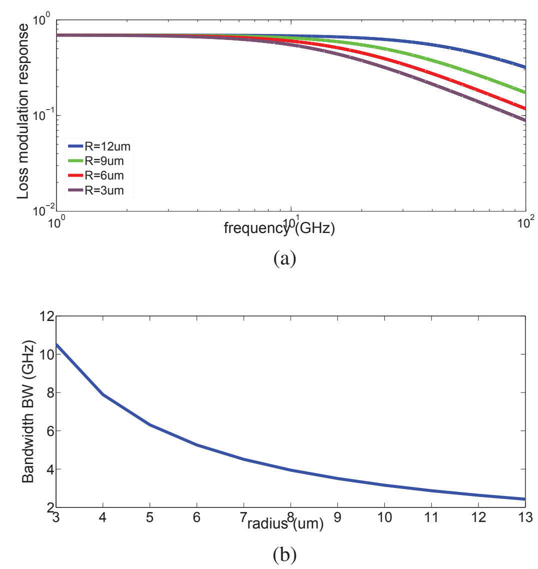

Fig. 5. (a)Modulation depth versus frequency in loss modulation for four different ring radius. (b) Bandwidth as a function of ring radius.

cases with decreasing of ring radius, bandwidth is increased and as mentioned before reduction of ring radius is one of the desired requirement in designing of a modulator. Furthermore index modulation at resonance has larger bandwidth than loss modulation. The bandwidth in this study is the bandwidth that limited by the structure. In reality the bandwidth of a ring resonator can be limited by the photon and carrier lifetimes that in this analysis is ignored. On the other hand as shown in Fig. 5(a) and Fig. 6(a) index modulation has smaller modulation depth rather than loss modulation.

Reduction in ring radius, increases bandwidth, but with decreasing radius, quality factor is decreased [16]. This reduction in quality factor with micrometer radius is relatively linear . In addition to reduction in quality factor with decreasing ring radius the bending loss of ring resonator increase [16] . As discussed, there is a trade-off between bandwidth, modulation depth, quality factor and loss in a ring resonator. Depends on the requirements for a special application a ring radius for an optimum performance is chosen.

For the case of coupling modulation, bandwidth of the ring resonator not limited by quality factor. As shown in Fig. 7 changes in ring radius, only shifts the modulation depth to higher frequencies.

In coupling modulation the bandwidth of the modulator is limited by free spectral range (FSR). With decreasing radius the FSR of the ring resonators is increased[16]. So in coupling

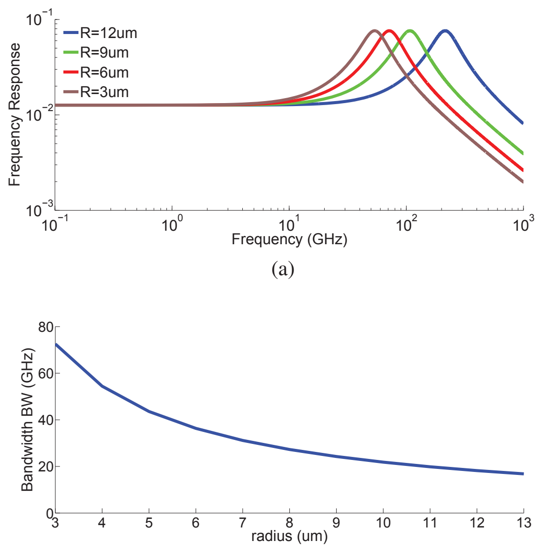

Fig. 6. (a) Input signal of coupling modulation. (b)The outpot transmission of optical wave.

modulation decreasing ring radius increase bandwidth and the modulators can have compact size. Unlike the loss and ring modulation, coupling modulation is not limited by quality factor.

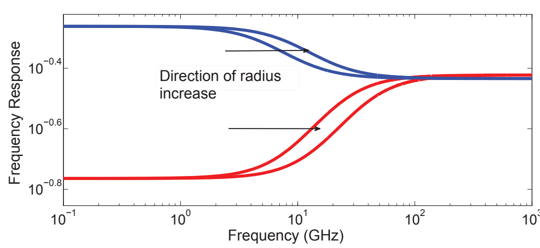

Fig. 7. Modulation depth versus frequency for over-coupled (red graphs)and under-coupled (blue graphs) for two different radius.

IV. Conclusion

In summery, the effects of loss, index and coupling modulation on transmitted signal were investigated. Limitation of bandwidth by increasing quality factor in loss and index modulation were explained. Furthermore, the relation between bandwidth and quality factor were investigated. The reasons for improving the characteristics of modulator with increasing the quality factor were explained. At last, the effects of ring radius on modulator parameters were studied. In loss modulation when radius varied from $3\mu m$ to $13\mu m$ the bandwidth of modulator changed from $2.428GHz$ to $10.52GHz$. For same range in radius change in index modulation, the bandwidth of modulator changed from $16.82GHz$ to $72.56GHz$. The index modulation have larger bandwidth than loss modulation. This is the reason of extended use of refractive index in modulation based on ring resonators. Also in coupling mode as were shown, the bandwidth of coupling modulation is limited by FSR. As were shown here coupling modulation has better performance rather than index and loss modulation. A challenge that exist in coupling modulation is, finding an optimum way for modulation coupling coefficient in ring modulator.

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