Event-Triggered $\mathscr{H}_\infty$ Depth Control of Remotely Operated Underwater Vehicles

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***Abstract*—In this paper, using a novel event-triggered method, a robust $\mathscr{H}_\infty$ depth tracking controller is designed for a remotely operated underwater vehicle (ROV). It is assumed that the desired trajectory of the ROV is determined by an operator outside of the vehicle based on its needed depth and obstacles in its path. It is also assumed that a wireless network is used to connect the user with the ROV. To decrease the communication rate between the controller and the ROV, a novel nonlinear event-triggered $\mathscr{H}_\infty$ controller is designed. The effects of the disturbance on the system performance are also attenuated. Stability of the ROV under the designed event-triggered controller is proved through a theorem. Simulation results demonstrate that the error between the depth of the ROV and its time-varying desired trajectory converges to zero using the proposed event-triggered $\mathscr{H}_\infty$ controller. It is also shown that the communication rate between the designed controller and the ROV is considerably reduced.**

***Index Terms*—Network operating systems, nonlinear systems, time-varying desired trajectory, tracking, underwater equipment.**

I. Introduction

NOWADAYS, ocean sources and industries play important roles in human lives. Among these industries, remotely operated underwater vehicles (ROVs) and autonomous underwater vehicles (AUVs) have become increasingly important tools in a number of applications, such as deep sea inspections, long-distance, long-duration surveys, oceanographic mapping and to detect, locate, and neutralize undersea mines. Improving the performance of these tolls requires the enhancement of engineering studies on all types and components of them.

Due to the rigid body coupling and the hydrodynamic forces, the behavior of an ROV or an AUV is so nonlinear [1]. While some simple linear techniques were used to design controllers for these systems [2]–[4], their performances are degraded for a wide range of the system operation. Therefore, many nonlinear techniques were also used to design proper controllers for the ROVs and the AUVs (see [5]–[15] and some references therein). In [5], an anti-disturbance constrained controller has been recently developed by designing a command governor and a disturbance observer. A neurodynamic optimization method has been also employed to implement the controller in an on-line manner. An output-feedback controller was proposed in [6] to solve the problem of path-following of the AUVs. In this paper, an extended state observer was designed to estimate some unmeasured velocities and uncertainties. In [7], a robust tracking controller for an AUV was designed using the adaptive nonsingular integral terminal sliding mode method in the presence of parametric uncertainties and external disturbances. In [8], using a local recurrent neural network, an adaptive tracking controller was designed for the ROVs. In [9], a nonlinear suboptimal technique was proposed to design a depth control law for the AUVs. In [10], using a model predictive control technique, a nonlinear controller was designed for an AUV to track a desired trajectory in the three-dimensional space. A robust nonlinear controller was designed for an AUV by considering parametric uncertainties and external disturbances [11]. In [12], an actuator failure tolerant control scheme was proposed for an ROV. In [13], paying attention to the difficulty of the accurate velocity measurement, an adaptive output feedback controller was proposed for an AUV based on the dynamic recurrent fuzzy neural network. In [14], an adaptive sliding mode fuzzy technique was used to design a depth control of the ROVs. The backstepping technique was employed to design a depth control law for AUVs [15]. A survey of advanced control techniques in marine systems, including the AUVs and ROVs can be found in [16].

Networked control systems (NCSs) are typical distributed control systems in which sensing devices, control facilities, and actuating agencies are interconnected via communication networks. Traditionally, the NCSs use periodic control scheme where a great number of redundant data might be transmitted. Hence, it is of our interest to reduce the communication for saving energy. Event-triggered communication and control appear to relieve the computational burden and to decrease the network resource utilization in the NCSs [17], [18]. In these techniques, a predefined triggering condition is checked and when it reaches to a specific threshold, the data are transmitted through the communication network [19], [20]. During the current decade, the event-triggered methedologies have attracted increasing attention from the academic community and as a result, many techniques have been proposed to solve different problems of the NCSs based on the idea of aperiodic sampling (see [18], [21]–[24] and some references therein). Two comprehensive surveys of the event-triggered techniques and the related theories can be found in [17] and [25].

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The $\mathscr{H}_\infty$ control theory has received increasing attention in the past years for attenuating the disturbance effect in dynamical systems. For the NCSs, the $\mathscr{H}_\infty$ control schemes have been established via the event-triggered control mechanism [26]–[30]. In [26]–[28], three different approaches have been employed to design the event-triggered $\mathscr{H}_\infty$ controllers for the linear NCSs. In [29], an online event-triggered concurrent learning algorithm has been proposed to design a robust $\mathscr{H}_\infty$ state feedback controller for a class of nonlinear NCSs. For another class of nonlinear NCSs, the problem of event-triggered $\mathscr{H}_\infty$ control has been addressed in [30]. Despite these researches, to the best of our knowledge, there has not been any work on the event-triggered $\mathscr{H}_\infty$ tracking controller design for nonlinear NCSs. In other words, the above techniques are used to stabilize equilibrium points of the system. However, in many practical applications such as ROVs, it is of our interest to design a controller such that the state of the system tracks a desired time-varying trajectory.

In an underwater ROV, a supporting cable is used to control the vehicle depth and to provide power and communication facilities [31]. A wireless channel is used to establish a communication link between the ROV and the user [32], [33]. Therefore, the ROVs can be considered as a class of NCSs and it is desirable to design an event-triggered controller to reduce the usage of the communication network. Moreover, it is important to design a controller such that the ROV tracks a desired time-varying trajectory in complex missions in order to avoid hitting physical obstacles. It is assumed that this desired depth of the vehicle is determined by a user. In this paper, a novel event-triggered $\mathscr{H}_\infty$ tracking controller is proposed for a broad class of nonlinear NCSs. Using the proposed method, a robust event-triggered controller is designed for an ROV in such a way that effects of disturbances on the depth of the ROV are attenuated. The design procedure of the proposed event-triggered $\mathscr{H}_\infty$ tracking controller is straightforward and it is proved that the error between the ROV depth and its desired time-varying trajectory tends to zero. Simulation results show that the proposed event-triggered $\mathscr{H}_\infty$ technique can considerably reduce the communication rate between the controller and the vehicle. In addition, the performance of the obtained closed-loop system is satisfactory even in the presence of uncertainties in the parameters of the ROV. To sum up, the main contributions of this paper are as follows.

1) While the event-triggered $\mathscr{H}_\infty$ regulation problem has been addressed in some recently published papers [26]–[30], to the best of authors' knowledge, it is the first time that the event-triggered $\mathscr{H}_\infty$ tracking controller is established for nonlinear NCSs. The proposed technique leads to an event-triggered controller such that the state of the system asymptotically tracks its desired time-varying trajectory while effects of disturbances on the tracking error are attenuated. Furthermore, the stability of the obtained closed-loop system is guaranteed through a theorem.
2) The proposed event-triggered $\mathscr{H}_\infty$ tracking controller is applied to an ROV which leads to considerable reductions of sending data from the controller to the ROV.

The rest of this paper is organized as follows. In Section II, a robust $\mathscr{H}_\infty$ tracking problem is defined for a broad class of nonlinear NCSs. In Section III, the proposed event-triggered technique is presented in detail. In Section IV, using the proposed method, a robust depth tracking controller is designed for the considered ROV. Simulation results of applying the proposed event-triggered $\mathscr{H}_\infty$ tracking controller to the ROV are also presented in this section. Finally, Section V concludes this paper.

Throughout this paper, the following notation will be used. $\mathbb{R}$ stands for the set of all real numbers. The symbol $\mathbb{R}^+$ denotes the set of all positive real numbers greater than 0. $\mathbb{R}^n$ is the Euclidean space of all n-dimensional real vectors. $\mathbb{R}^{n\times m}$ is the space of all $n \times m$ real matrices. I_n represents the $n\times n$ identity matrix. $0_{n\times m}$ denotes the $n \times m$ zero matrix. The set $\mathbb{Z}^+ = \{0, 1, 2, \ldots, \}$ contains the non-negative integers. $L_2[0, \infty)$ is the space of square-integrable vector functions over the interval $[0, \infty)$. A matrix $P \in \mathbb{R}^{n\times n}$ is said to be positive definite (positive-semidefinite), if for any nonzero vector $x \in \mathbb{R}^n$, it satisfies $x^T Px > 0$ ($x^T Px \geq 0$).

II. Problem Formulation

Consider the following nonlinear continuous-time system:

$$\begin{aligned}\dot{x}(t) &= f(x(t)) + b_1(x(t))u(t) + b_2(x(t))d(t)\\ x(0) &= x_0\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $d(t) \in \mathbb{R}^q$, and x_0 are the state, the control input, the external disturbance, and the initial condition, respectively. $f(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$, $b_1(x(t)) : \mathbb{R}^n \to \mathbb{R}^m$, and $b_2(x(t)) : \mathbb{R}^n \to \mathbb{R}^q$ are assumed to be smooth functions, $b_1(x(t)) \neq 0$ for all $x(t) \in \mathbb{R}^n$, and $f(0) = 0$.

The problem is to design a robust $\mathscr{H}_\infty$ tracking controller such that the system state $x(t)$ tracks a desired time-varying trajectory $x_d(t)$ and the effects of the external disturbance $d(t)$ on the tracking error $e(t) \triangleq x(t) - x_d(t)$ are attenuated. Moreover, it is assumed that the controller and the actuator are connected to each other using a communication network. Paying attention to some technical problems such as limitations in the network bandwidth, it is of our interest to reduce the rate of using this communication channel. In closing, the considered problem is to find the control input $u(t)$ such that the following objectives are simultaneously satisfied.

1) The state of the system asymptotically tracks a desired time-varying trajectory $x_d(t)$.
2) Effects of the external disturbance $d(t)$ on the tracking error are attenuated.
3) The communication rate from the controller to the actuator is decreased.

To formulate the first objective, the following dynamics is assumed for the desired trajectory $x_d(t)$:

$$\dot{z}_d(t) = f_d(z_d(t)), \quad x_d(t) = h_d(z_d(t)), \quad z_d(0) = z_{d0} \tag{2}$$

where $z_d(t) \in \mathbb{R}^{n_d}$, $x_d(t) \in \mathbb{R}^n$, and $z_d(0)$ are respectively the state, the output, and the initial condition of the desired trajectory system. Functions $f_d(z_d(t)) : \mathbb{R}^{n_d} \to \mathbb{R}^{n_d}$ and $h_d(z_d(t)) : \mathbb{R}^{n_d} \to \mathbb{R}^n$ are assumed to be smooth and $f_d(0) = h_d(0) = 0$. Note that the dynamics (2) can be used to describe many commonly used trajectories such as constants and sinusoidals.

The second objective is satisfied by finding a control input $u(t)$ such that the following disturbance attenuation condition is held for every external disturbance $d(t) \in L_2[0, \infty)$:

$$\int_0^\infty e^{-2\alpha t}((x(t) - x_d(t))^T Q(x(t) - x_d(t)) + u^T(t)Ru(t))dt \leq \gamma^2 \int_0^\infty e^{-2\alpha t} d^T(t)d(t)dt. \quad (3)$$

Here, $\alpha > 0$ is called the discount factor, $Q \geq 0$ and $R > 0$ are two weighting matrices with appropriate dimensions, and $\gamma > 0$ is the amount of attenuation from the disturbance $d(t)$ to the tracking error $e(t)$. It is well-known that the solution of an H_∞ control problem can be referred to the solution of a two-player zero-sum game, where the control input $u(t)$ is a minimizing player and the disturbance $d(t)$ is a maximizing one [34], [35]. Therefore, the second objective is formulized as a two-player zero-sum game with the following cost function:

$$J\big(x_0, z_{d_0}, u(t), d(t)\big) = \int_0^\infty e^{-2\alpha t}\Big((x(t) - x_d(t))^T Q(x(t) - x_d(t)) + u^T(t)Ru(t) - \gamma^2 d^T(t)d(t)\Big)dt. \quad (4)$$

Note that by tuning the weighting matrices Q and R, the closed-loop system performance can be directly affected with predictable results. For instance, achieving a faster response is possible through the use of larger values for the elements of Q. In addition, the user-defined parameters α and γ are selected to overcome some imposed limitations on the design procedure of the controller (see Remarks 1 and 2).

Finally, to consider the third objective, i.e., decreasing the communication rate from the controller to the actuator, an event-triggered mechanism is considered as depicted in Fig. 1. The main challenge is to specify a proper triggering condition such that the first two objectives are satisfied. It is worth noting that we consider the so called "local sensor-remote actuator" networked control problem in which all the sensors and the controller being co-located and have access to each other's outputs at all times [36]. Indeed, the sensors are assumed to be coupled to the controller with ideal communication channels or through dedicated (wired) point-to-point connections while the actuators are connected to the controller via a limited bandwidth channel.

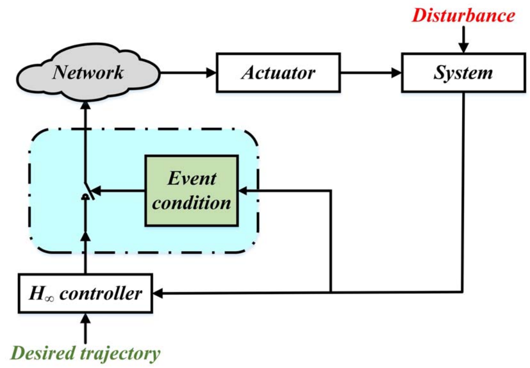

Fig. 1. Structure of the proposed event-triggered $\mathscr{H}_\infty$ controller.

III. EVENT-TRIGGERED $\mathscr{H}_\infty$ TRACKING CONTROLLER

In this section, a novel event-triggered method is proposed to solve the robust $\mathscr{H}_\infty$ tracking control problem defined in Section II. Toward this end, a state-dependent Riccati equation (SDRE)-based method is first reviewed in Section III-A. Then, the proposed event-triggered $\mathscr{H}_\infty$ tracking controller is presented in Section III-B.

A. SDRE $\mathscr{H}_\infty$ Tracking Controller

Finding a controller to address the first two objectives of the defined $\mathscr{H}_\infty$ tracking problem needs to solve a Hamilton–Jacobi–Isaacs equation which is too difficult or even impossible [37]. Adding the third objective leads to a much more difficult problem. As a result, finding approximate solutions of this problem is a reasonable alternative. A very efficient method to approximate the solution of these types of problems is the SDRE technique. In this section, the SDRE-based solution of the $\mathscr{H}_\infty$ tracking problem is reviewed. Note that this solution has been recently proposed for nonlinear systems where the control signal is continuously transmitted from the controller to the plant [37]. In Section III-B, this SDRE-based technique will be extended to our $\mathscr{H}_\infty$ tracking problem where a network exists between the controller and the plant and the usage of this network should be reduced.

Let us first find the dynamics of the tracking error $e(t)$ as follows:

$$\dot{e}(t) = f(x(t)) + b_1(x(t))u(t) + b_2(x(t))d(t) - \frac{\partial h_d(z_d(t))}{\partial z_d(t)} f_d(z_d(t)).$$

Since $f(x(t))$, $f_d(z_d(t))$, and $h_d(z_d(t))$ are assumed to be smooth, it is possible to have the following state-dependent coefficient (SDC) representations [37]:

$$\begin{aligned} f(x(t)) &= \mathcal{F}(x(t))x(t) \\ f_d(z_d(t)) &= \mathcal{F}_d(z_d(t))z_d(t) \\ h_d(z_d(t)) &= \mathcal{H}_d(z_d(t))z_d(t) \\ \frac{\partial h_d(z_d(t))}{\partial z_d(t)} f_d(z_d(t)) &= \mathcal{G}_d(z_d(t))z_d(t) \end{aligned}$$

where $\mathcal{F}(x(t)) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $\mathcal{F}_d(z_d(t)) : \mathbb{R}^{n_d} \to \mathbb{R}^{n_d\times n_d}$, $\mathcal{H}_d(z_d(t)) : \mathbb{R}^{n_d} \to \mathbb{R}^{n\times n_d}$, and $\mathcal{G}_d(z_d(t)) : \mathbb{R}^{n_d} \to \mathbb{R}^{n\times n_d}$ are four matrix-valued functions. It should be mentioned that there might be an infinite number of ways to create these SDC forms [38]. This fact can be used as a degree of freedom in our control design procedure.

By defining $X(t) \triangleq e^{-\alpha t}\big[e^T(t) \quad z_d^T(t)\big]^T \in \mathbb{R}^{n+n_d}$, $U(t) \triangleq e^{-\alpha t}u(t) \in \mathbb{R}^m$, $D(t) \triangleq e^{-\alpha t}d(t) \in \mathbb{R}^q$, and $\mathscr{X}(t) \triangleq e^{\alpha t}X(t)$, the following dynamics is obtained for $X(t)$:

$$\dot{X}(t) = A(\mathscr{X}(t))X(t) + B_1(\mathscr{X}(t))U(t) + B_2(\mathscr{X}(t))D(t) \quad (5)$$

where

$$\begin{aligned}A(\mathscr{X}(t)) &= -\alpha I_{n+n_d} \\ &\quad + \begin{bmatrix}\mathcal{F}(x(t)) & \mathcal{F}(x(t))\mathcal{H}_d(z_d(t)) - \mathcal{G}_d(z_d(t)) \\ 0_{n_d\times n} & \mathcal{F}_d(z_d(t))\end{bmatrix} \\ B_1(\mathscr{X}(t)) &= \begin{bmatrix}b_1^T(x(t)) & 0_{m\times n_d}\end{bmatrix}^T \\ B_2(\mathscr{X}(t)) &= \begin{bmatrix}b_2^T(x(t)) & 0_{q\times n_d}\end{bmatrix}^T.\end{aligned} \tag{6}$$

Using these new variables, the cost function (4) is rewritten as follows:

$$J(X_0, U(t), D(t)) = \int_0^\infty \Big(X^T(t)\tilde{Q}X(t) + U^T(t)RU(t) - \gamma^2 D^T(t)D(t)\Big)dt$$

where

$$\tilde{Q} = \begin{bmatrix}Q & 0_{n\times n_d} \\ 0_{n_d\times n} & 0_{n_d\times n_d}\end{bmatrix}.$$

In [37], it has been proved that the first two objectives of our $\mathscr{H}_\infty$ tracking problem are achieved using the control law

$$U(t) = -R^{-1}B_1^T(\mathscr{X}(t))P(\mathscr{X}(t))X(t) \tag{7}$$

where $P(\mathscr{X}(t))$ is the unique symmetric positive-definite solution of the following SDRE:

$$\begin{aligned}&A^T(\mathscr{X}(t))P(\mathscr{X}(t)) + P(\mathscr{X}(t))A(\mathscr{X}(t)) \\ &\quad - P(\mathscr{X}(t))B_1(\mathscr{X}(t))R^{-1}B_1^T(\mathscr{X}(t))P(\mathscr{X}(t)) \\ &\quad + \frac{1}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t)) = -\tilde{Q}.\end{aligned} \tag{8}$$

In addition, the worst-case of disturbance $D(t)$ is

$$D(t) = \frac{1}{\gamma^2}B_2^T(\mathscr{X}(t))P(\mathscr{X}(t))X(t). \tag{9}$$

Let us present some necessary definitions which are needed in the rest of this paper.

Definition 1: The SDC representation (5) is point-wise stabilizable in the bounded open set Ω if the pair $(A(\mathscr{X}(t)), B_1(\mathscr{X}(t)))$ is stabilizable for all $X(t) \in \Omega$ [39].

Definition 2: The SDC representation (5) is point-wise detectable in the bounded open set Ω if the pair $(A(\mathscr{X}(t)), \tilde{Q}^{1/2})$ is detectable for all $X(t) \in \Omega$ [39].

It should be mentioned that SDRE (8) has a unique symmetric positive-definite solution for ufficiently large values of γ if the triple $(A(\mathscr{X}(t)), B(\mathscr{X}(t), \tilde{Q}^{1/2})$ is point-wise stabilizable and point-wise detectable [39].

B. SDRE-Based Event-Triggered $\mathscr{H}_\infty$ Tracking Controller

The main step in the design procedure of the SDRE $\mathscr{H}_\infty$ tracking controller is to find the solution of the SDRE (8). A sample data technique can be used to find $P(\mathscr{X}(t))$ at the sampling instant $t_k (k \in \mathbb{Z}^+)$ and then, the control law $U(t)$ is obtained using (7). This control is applied to the system in the time interval $[t_k, t_{k+1})$. These calculations are done periodically and the control signal is updated at each sampling time. In this way, the control action must be sent to the plant at every sampling instant. Because of the third objective of our control problem, i.e., to reduce the communication rate between the $\mathscr{H}_\infty$ tracking controller and the plant, an event-triggered mechanism is added to the closed-loop system. The main result of the proposed event-triggered $\mathscr{H}_\infty$ tracking controller is explained by the following Theorem 1. Here, $t_0 = 0$ is the initial time, $t_k \in \mathbb{R}^+$ is the times at which the control input has to be transmitted from the controller to the actuator, and $K(\mathscr{X}(t)) = R^{-1}B_1^T(\mathscr{X}(t))\,P(\mathscr{X}(t))$ is the state-dependent gain.

Assumption 1: The triple $(A(\mathscr{X}(t)), B(\mathscr{X}(t)), \tilde{Q}^{1/2})$ is point-wise stabilizable and point-wise detectable in a bounded open set $\Omega \in \mathbb{R}^{n+n_d}$ containing the origin.

Theorem 1: Consider the nonlinear continuous-time system (5). Under Assumption 1 and for $D(t) = 0$ and any sufficiently large values of γ, the following control law with $U(t_0) = -K(\mathscr{X}(t_0))X(t_0)$ leads to a closed-loop system with a locally asymptotically stable equilibrium at the origin

$$U(t) = \begin{cases}U(t_k), & \eta(\mathscr{X}(t)) < 0 \\ -K(\mathscr{X}(t))X(t), & \eta(\mathscr{X}(t)) \geq 0\end{cases} \tag{10}$$

where

$$\begin{aligned}\eta(\mathscr{X}(t)) &= \begin{bmatrix}X^T(t) & E^T(t)\end{bmatrix}\Psi(\mathscr{X}(t))\begin{bmatrix}X(t) \\ E(t)\end{bmatrix} \\ E(t) &= B_1(\mathscr{X}(t))K(\mathscr{X}(t))X(t) \\ &\quad - B_1(\mathscr{X}(t))K(\mathscr{X}(t_k))X(t_k).\end{aligned} \tag{11}$$

In the above, the matrix-valued function $\Psi(\mathscr{X}(t))$ is

$$\Psi(\mathscr{X}(t)) = \begin{bmatrix}\Psi_{1,1}(\mathscr{X}(t)) & P(\mathscr{X}(t)) \\ P^T(\mathscr{X}(t)) & 0_{(n+n_d)\times(n+n_d)}\end{bmatrix} \tag{12}$$

where

$$\begin{aligned}&\Psi_{1,1}(\mathscr{X}(t)) \\ &= (\sigma - 1)\Big(\tilde{Q} + K^T(\mathscr{X}(t))RK(\mathscr{X}(t)) \\ &\quad - \frac{1}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t))\Big)\end{aligned}$$

and $0 < \sigma \leq 1$ is a constant parameter.

Proof: Consider the Lyapunov candidate function $V(X(t)) = X^T(t)P(\mathscr{X}(t))X(t)$. For sufficiently small values of $X(t)$, the derivative of $V(X(t))$ along the trajectory (5) is given as [37]

$$\begin{aligned}&\dot{V}(X(t)) \\ &= X^T(t)\Big(A_{\text{cl}}^T(\mathscr{X}(t))P(\mathscr{X}(t)) + P(\mathscr{X}(t))A_{\text{cl}}(\mathscr{X}(t)) \\ &\quad + \frac{2}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t))\Big)X(t)\end{aligned} \tag{13}$$

and the following Lyapunov equation is held [37]:

$$\begin{aligned}&A_{\text{cl}}^T(\mathscr{X}(t))P(\mathscr{X}(t)) + P(\mathscr{X}(t))A_{\text{cl}}(\mathscr{X}(t)) \\ &\quad + \frac{2}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t)) \\ &= -\Big(\tilde{Q} + K^T(\mathscr{X}(t))RK(\mathscr{X}(t)) \\ &\quad - \frac{1}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t))\Big)\end{aligned} \tag{14}$$

where

$$A_{\text{cl}}(\mathscr{X}(t)) \triangleq A(\mathscr{X}(t)) - B_1(\mathscr{X}(t))K(\mathscr{X}(t)).$$

It should be mentioned that the Lyapunov equation (14) is simply obtained using (8) and $K(\mathscr{X}(t)) = R^{-1}B_1^T(\mathscr{X}(t))P(\mathscr{X}(t))$. Using (14) in (13) yields

$$\dot{V}(X(t)) = -X^T(t)\Big(\tilde{Q} + K^T(\mathscr{X}(t))RK(\mathscr{X}(t)) - \frac{1}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t))\Big)X(t).$$

On the other hand, if the following weaker inequality is satisfied for sufficiently large values of γ, the stability of the origin of the system (5) is guaranteed:

$$\dot{V}(X(t)) \le -\sigma X^T(t)\Big(\tilde{Q} + K^T(\mathscr{X}(t))RK(\mathscr{X}(t)) - \frac{1}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t))\Big)X(t). \quad (15)$$

Let t_k shows the instants at which the control input is computed and transmitted through the communication channel to the actuator ($\eta(\mathscr{X}(t_k)) \ge 0$). In the time interval $[t_k, t_{k+1})$, the closed-loop system dynamics is as follows:

$$\dot{X}(t) = A(\mathscr{X}(t))X(t) - B_1(\mathscr{X}(t))K(\mathscr{X}(t_k))X(t_k). \quad (16)$$

By defining $E(t) = B_1(\mathscr{X}(t))K(\mathscr{X}(t))X(t) - B_1(\mathscr{X}(t))K(\mathscr{X}(t_k))X(t_k)$, (16) can be rewritten as follows:

$$\dot{X}(t) = A_{\mathrm{cl}}(\mathscr{X}(t))X(t) + E(t).$$

Additionally, for sufficiently small values of $X(t)$, it is possible to approximate $P(\mathscr{X}(t))$ with its value at the origin and therefore, the derivative of $V(x(t))$ is as follows:

$$\dot{V}(X(t)) = X^T(t)\Big(A_{\mathrm{cl}}^T(\mathscr{X}(t))P(\mathscr{X}(t)) + P(\mathscr{X}(t))A_{\mathrm{cl}}(\mathscr{X}(t)) + \frac{2}{\gamma^2}P(\mathscr{X}(t))B_2(\mathscr{X}(t))B_2^T(\mathscr{X}(t))P(\mathscr{X}(t))\Big)X(t) + 2X^T(t)P(\mathscr{X}(t))E(t).$$

Now, using the Lyapunov equation (14), inequality (15), and the above equality, the triggering times are obtained when the following inequality is violated:

$$\begin{bmatrix} X^T(t) & E^T(t) \end{bmatrix} \Psi(\mathscr{X}(t)) \begin{bmatrix} X(t) \\ E(t) \end{bmatrix} < 0$$

where $\Psi(\mathscr{X}(t))$ is defined by (12). This completes the proof. ∎

According to the definition of $X(t)$ and from the above theorem, it can be concluded that $\bar{e}(t) = e^{-\alpha t}(x(t) - x_d(t))$ asymptotically tends to zero.

Remark 1: As mentioned in Theorem 1, the pair $(A(\mathscr{X}(t)), B(\mathscr{X}(t)))$ must be point-wise stabilizable to use the proposed event-triggered $\mathscr{H}_\infty$ tracking controller. To this end, the parameter α can be tuned since $A(\mathscr{X}(t))$ is dependent on this parameter according to (6).

Remark 2: Although the minimum value of γ is desirable, there is no way to find the smallest amount of the disturbance attenuation for general nonlinear systems [35], [40]. Nevertheless, the value of γ should be selected in such a way that the SDRE (8) has a unique symmetric positive-definite solution. Therefore, a large enough value is usually predetermined for γ.

Remark 3: While it is not generally possible to find the value of the triggering factor σ corresponding to a specific percentage of the communication rate reduction, the effects of the triggering factor σ on the obtained results are predictable. Indeed, according to (15), a smaller value of σ leads to more reduction of sending messages from the controller to the actuator. However, the price of this reduction is a decrease in the performance of the closed-loop system. Hence, σ can be used to make a tradeoff between the communication rate reduction and the value of the corresponding cost function. This issue is investigated in the following simulations (see Table III).

Remark 4: To implement the proposed event-triggered $\mathscr{H}_\infty$ tracking controller, the time-triggered SDRE implementation technique, represented in [39], is extended. To this end, the positive-definite solution of the sampled-data algebraic Riccati equation

$$A^T(\mathscr{X}(iT))P(\mathscr{X}(iT)) + P(\mathscr{X}(iT))A(\mathscr{X}(iT)) - P(\mathscr{X}(iT))B_1(\mathscr{X}(iT))R^{-1}B_1^T(\mathscr{X}(iT))P(\mathscr{X}(iT)) + \frac{1}{\gamma^2}P(\mathscr{X}(iT))B_2(\mathscr{X}(iT))B_2^T(\mathscr{X}(iT))P(\mathscr{X}(iT)) = -\tilde{Q}$$

is computed periodically with the sample time T. Then, the control action $U(iT) = -R^{-1}B_1^T(\mathscr{X}(iT))P(\mathscr{X}(iT))X(iT)$ ($i \in \mathbb{Z}^+$) is obtained at the current state $X(iT)$. To determine whether this computed control must be sent to the actuator or not, the event-triggering condition (11) is checked at this current sampling instant $t = iT$. If $\eta(\mathscr{X}(iT)) \ge 0$, the computed control input $U(iT)$ is sent to the actuator and t_{k+1} is set to iT. Therefore, just like the event-triggered control techniques for discrete-time systems [41], [42], the interevent times of the proposed event-triggered $\mathscr{H}_\infty$ tracking controller are always greater than or equal to the sampling time T. Hence, the proposed method overcomes the problem of the minimum interevent time and, the Zeno free execution of the control updating instants is always guaranteed.

Remark 5: According to Remark 4, to use the proposed event-triggered $\mathscr{H}_\infty$ tracking controller, the SDRE (8) must be solved at each sampling instant iT ($i \in \mathbb{Z}^+$). From a practical point of view, the sample time T should be much longer than the time needed to find the solution of (8). Although this may seem to limit the practical applicabilities of the SDRE-based techniques such as the proposed event-triggered strategy, there are many successful implementations of the SDRE techniques. For example, in a missile autopilot example containing five state variables and three control inputs, an SDRE-based technique was implemented at speeds greater than 600 Hz and up to 2 kHz sample rates [39]. In our problem, i.e., the depth control problem of the ROV, the sampling time $T = 0.2$ s is used which is extremely larger than the needed time to solve the SDRE (8).

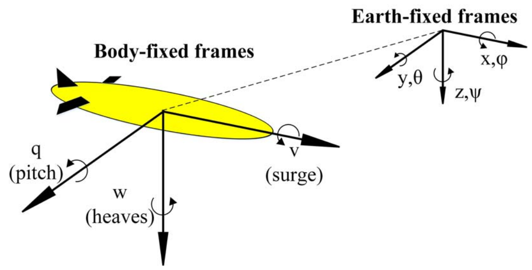

Fig. 2. General scheme of the considered ROV.

IV. ROV Depth Tracking Controller Design

A. Mathematical Model of the ROV

The schematic of the considered ROV with its body-fixed co-ordinate system is depicted in Fig. 2. The earth-fixed frame is treated as an inertial frame and the motion of the ROV lies in a vertical plane. Assume the triple (x_B, y_B, z_B) is the co-ordinate of the center of buoyancy and the origin of the body-fixed co-ordinate system is fixed at the center of buoyancy [i.e., $(x_B, y_B, z_B) = 0$]. In the following, the co-ordinates of the center of gravity of the vehicle with respect to the center of buoyancy are denoted by (x_G, y_G, z_G).

The heave and pitch equations of motion of the vehicle with respect to the body-fixed moving frame are modeled by a set of four nonlinear differential equations as follows [3]:

$$\begin{aligned}
& m\Big(\dot{w}(t) - vq(t) - x_G\dot{q}(t) - z_G q^2(t)\Big) \\
&\quad = Z_{q|q|}q(t)|q(t)| + (W - B_0)\cos(\theta(t)) + Z_{vq}vq(t) \\
&\qquad + Z_{w|w|}w(t)|w(t)| + Z_{vw}vw(t) + v^2 Z_{vm}u(t) \\
&\qquad + Z_{\dot{q}}\dot{q}(t) + Z_{\dot{w}}\dot{w}(t) \\
& I_{yy}\dot{q}(t) + m(x_G(vq(t) - \dot{w}(t)) + z_G w(t)q(t)) \\
&\quad = M_{\dot{q}}\dot{q}(t) + M_{\dot{w}}\dot{w}(t) + M_{vq}vq(t) + M_{vw}vw(t) \\
&\qquad + M_{q|q|}q(t)|q(t)| + M_{w|w|}w(t)|w(t)| + M_{vv}v^2 u(t) \\
&\qquad - (x_G W - x_B B_0)\cos(\theta(t)) - (z_G W - z_B B_0)\sin(\theta(t)) \\
& \dot{z}(t) = w(t)\cos(\theta(t)) - v\sin(\theta(t)) \\
& \dot{\theta}(t) = q(t)
\end{aligned} \tag{17}$$

where $w(t)$, $q(t)$, $z(t)$, and $\theta(t)$ are the heave velocity, the pitch velocity, the depth, and the pitch angle, respectively; $u(t)$ is the fin angle which is the control input of the ROV in its dive plane mode; I_{yy} is the moment of inertia about the pitch axis; v is the forward velocity; W denotes the weight of the vehicle; m and B_0 are the mass and buoyancy of the ROV, respectively. In this paper, it is assumed that the forward velocity v is held constant by a control mechanism as $v = 2$ m/s and the lateral velocity is zero. The aim is to design a controller such that by applying the control signal $u(t)$ to the ROV fins, the depth of the ROV tracks a desired time-varying trajectory. The hydrodynamic and physical parameters values of the ROV are represented in Tables I and II, respectively [43].

B. Controller Design and Simulation Results

In this paper, it is assumed that the states of the ROV are measured using some sensors and the collected data are sent to the actuator. In addition, the calculated control signal is

TABLE I
HYDRODYNAMIC PARAMETERS OF THE ROV

Parameter	Value	Parameter	Value
$M_{\dot{q}}$	-4.88 kg m^2/rad	$M_{\dot{w}}$	-1.93 kg m
$M_{q\|q\|}$	-188 kg m^2/rad	$M_{w\|w\|}$	3.18 kg
M_{vq}	-2 kg m/rad	M_{vw}	24 kg
$Z_{\dot{q}}$	-1.93 kg m/rad	$Z_{\dot{w}}$	-35.5 kg
$Z_{q\|q\|}$	-0.632 kg m/rad^2	$Z_{w\|w\|}$	-131 kg/m
Z_{vq}	-5.22 kg/rad	Z_{vw}	-28.6 kg/m
M_{vv}	-6.15 kg/rad	Z_{vv}	-6.15 kg/(m rad)

TABLE II
PHYSICAL PARAMETERS OF THE ROV

Parameter	Value	Parameter	Value
x_G	0	z_G	0.0196 m
x_B	0	z_B	0
W	299 N	B_0	306 N
m	30.48 kg	I_{yy}	3.45 kg m^2

also sent to the ROV using the channel. It is worth noting that employing the proposed event-triggered $\mathscr{H}_\infty$ tracking control can lead to a reduction in transmitting messages from the controller to the vehicle. To apply the proposed event-triggered $\mathscr{H}_\infty$ tracking controller to the ROV, let us first find the SDC representation of the system dynamics (17) as follows:

$$\begin{aligned}
\begin{bmatrix}\dot{w}(t)\\ \dot{q}(t)\end{bmatrix} &= \underbrace{\begin{bmatrix} f_{1,11} & f_{1,12}\\ f_{1,21} & f_{1,22}\end{bmatrix}}_{\mathcal{F}_1(x(t))}\begin{bmatrix}w(t)\\ q(t)\end{bmatrix} + \underbrace{\begin{bmatrix} 0 & f_{2,12}\\ 0 & f_{2,22}\end{bmatrix}}_{\mathcal{F}_2(x(t))}\begin{bmatrix}z(t)\\ \theta(t)\end{bmatrix} \\
&\quad + \underbrace{M^{-1}v^2\begin{bmatrix}Z_{vv}\\ M_{vv}\end{bmatrix}}_{b_{1,1}}u(t) + \underbrace{M^{-1}\begin{bmatrix}(W - B_0)\\ (x_B B_0 - x_G W)\end{bmatrix}}_{d_1} \\
\begin{bmatrix}\dot{z}(t)\\ \dot{\theta}(t)\end{bmatrix} &= \underbrace{\begin{bmatrix} f_{3,11} & 0\\ 0 & 1\end{bmatrix}}_{\mathcal{F}_3(x(t))}\begin{bmatrix}w(t)\\ q(t)\end{bmatrix} + \underbrace{\begin{bmatrix} 0 & f_{4,12}\\ 0 & 0\end{bmatrix}}_{\mathcal{F}_4(x(t))}\begin{bmatrix}z(t)\\ \theta(t)\end{bmatrix}
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
f_{1,11} &= Z_{w|w|}|w(t)| + Z_{vw}, f_{1,21} = M_{w|w|}|w(t)| + M_{vw} \\
f_{1,12} &= Z_{q|q|}|q(t)| + Z_{vq} + m(z_G q + v) \\
f_{1,22} &= M_{q|q|}|q(t)| + M_{vq} + m(x_G v + z_G w(t)) \\
f_{2,12} &= M^{-1}(W - B_0)(\cos(\theta(t)) - 1)\theta^{-1} \\
f_{2,22} &= M^{-1}(x_B B_0 - x_G W)(\cos(\theta(t)) - 1)\theta^{-1} \\
&\quad\ (z_G W - z_B B_0)\sin(\theta(t)) \\
f_{3,11} &= \cos(\theta(t)), \quad f_{4,12} = -\theta^{-1}\sin(\theta(t)) \\
M &= \begin{bmatrix} m - Z_{\dot{w}(t)} & -mx_G - Z_{\dot{q}(t)} \\ -mx_G - M_{\dot{w}(t)} & I_{yy} - M_{\dot{q}(t)}\end{bmatrix}.
\end{aligned}$$

Based on (18), the state space representation of the ROV can be rewritten as the following standard form:

$$\dot{x}(t) = \underbrace{\begin{bmatrix}\mathcal{F}_1(x) & \mathcal{F}_2(x)\\ \mathcal{F}_3(x) & \mathcal{F}_4(x)\end{bmatrix}}_{\mathcal{F}(x(t))}x(t) + \underbrace{\begin{bmatrix}b_{1,1}\\ 0_{2\times 1}\end{bmatrix}}_{b_1}u(t) + \underbrace{\begin{bmatrix}I_2\\ 0_{2\times 2}\end{bmatrix}}_{b_2}d_1 \tag{19}$$

where $x(t) = \begin{bmatrix}w(t) & q(t) & z(t) & \theta(t)\end{bmatrix}^T$. In the following simulations, the initial condition $x(0) = \begin{bmatrix}0.5 & 0 & -1 & 0\end{bmatrix}^T$ is considered. Assume that the following dynamics of the desired trajectory is considered by the user because of the needed level of the ROV and some physical obstacles:

$$\dot{z}_d(t) = 0_{4\times4} z_d(t) = \mathcal{F}_d z_d(t)$$
$$x_d(t) = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} z_d(t) = \mathcal{H}_d z_d(t) \quad (20)$$

with initial condition $z_d(0) = \begin{bmatrix}0 & 0 & \beta & 0\end{bmatrix}$. Note that β is determined by the location of the obstacles. The robust $\mathscr{H}_\infty$ control problem is to find $u(t)$ such that the following cost function is minimized:

$$J(x_0, u(t), d(t)) = \int_0^\infty e^{-2\alpha t}\Big(\tilde{Q}(z(t) - x_d(t))^2 + Ru^2(t) - \gamma^2 d^T(t)d(t)\Big)dt.$$

Due to physical limitations, the amplitude of the fin angle $u(t)$ should be in a specific bound, which is $[-45°, 45°]$ in this paper. To address this important problem in the design procedure of the controller, a technique discussed in [39] is employed. At first, (19) is rewritten as follows:

$$\dot{x}(t) = \mathcal{F}(x(t))x(t) + b_1 \mathrm{sat}\big(u(t), 45°\big) + b_2 d_1 \quad (21)$$

where

$$\mathrm{sat}\big(u(t), 45°\big) = \begin{cases} -45°, & u(t) \le -45° \\ u(t), & |u(t)| < 45° \\ 45°, & u(t) \ge 45°. \end{cases}$$

Now, an auxiliary input $\tilde{u}(t)$ is defined and the following dynamics is considered for $u(t)$:

$$\dot{u}(t) = \tilde{u}(t). \quad (22)$$

Augmenting (21) and (22) yields

$$\begin{bmatrix}\dot{x}(t) \\ \dot{u}(t)\end{bmatrix} = \begin{bmatrix}\mathcal{F}(x(t)) & b_1 S(u(t)) \\ 0_{4\times1} & 0\end{bmatrix}\begin{bmatrix}x(t) \\ u(t)\end{bmatrix} + \begin{bmatrix}0_{4\times1} \\ 1\end{bmatrix}\tilde{u}(t) + b_2 d_1 \quad (23)$$

where $S(u(t))$ is as follows:

$$S(u(t)) = \begin{cases} \frac{\mathrm{sat}(u(t),45°)}{u(t)}, & u(t) \ne 0 \\ 1, & u(t) = 0. \end{cases} \quad (24)$$

By comparing (23) with (1), it can be said that the ROV depth control problem is in our standard form without any input limitation on the auxiliary input $\tilde{u}(t)$ and therefore, the proposed event-triggered $\mathscr{H}_\infty$ tracking controller can be used to solve this problem.

Remark 6: The ROVs may suffer from the input and the state constraints [5]. While the input saturation is considered in the proposed control design procedure, we do not directly address the problem of state constraints. However, the user-defined desired trajectory $x_d(t)$ can be used to handle this problem. Indeed, it is possible to consider the state constraints by selecting a suitable desired trajectory $x_d(t)$.

Augmenting (20) and (24) leads to the following SDC representation for the new state $X(t) = e^{-\alpha t}[x^T(t)\ z_d^T(t)\ u(t)]^T$:

$$\dot{X}(t) = \underbrace{\left[-\alpha I + \left[\begin{array}{c|c|c} \mathcal{F}(x(t)) & 0_{4\times4} & b_1 S(u(t)) \\ \hline 0_{4\times4} & \mathcal{F}_d & 0_{4\times1} \\ \hline 0_{1\times4} & 0_{1\times4} & 0 \end{array}\right]\right]}_{A(\mathscr{X}(t))} X(t) + \underbrace{\begin{bmatrix}0_{4\times1} \\ \hline 0_{4\times1} \\ \hline 1\end{bmatrix}}_{B_1}\tilde{U}(t) + \underbrace{\begin{bmatrix}b_2 \\ \hline 0_{4\times2} \\ \hline 0_{1\times2}\end{bmatrix}}_{B_2} D(t) \quad (25)$$

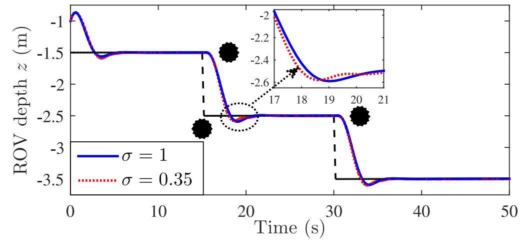

Fig. 3. Diagrams of the ROV depths and its desired trajectory.

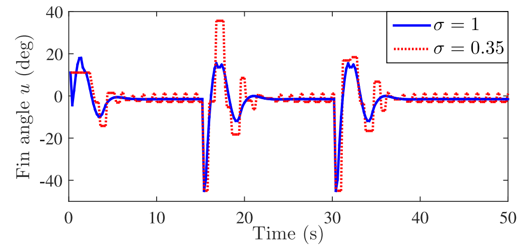

Fig. 4. Diagrams of the control signal $u(t)$.

where $\tilde{U}(t) = e^{-\alpha t}\tilde{u}(t)$ and $D(t) = e^{-\alpha t}d_1(t)$.

Paying attention to Theorem 1, the pair $(A(\mathscr{X}(t)), B_1)$ must be point-wise stabilizable. In [9], it is shown that the states $e^{-\alpha t}x(t)$ and $e^{-\alpha t}u(t)$ are point-wise controllable. Moreover, the states related to the desired trajectory, i.e., $e^{-\alpha t}z_d(t)$, are stable for any $\alpha > 0$ (note that the eigen-values of $-\alpha I + \mathcal{F}_d$ are in the left half-plane). Therefore, the pair $(A(\mathscr{X}(t)), B_1)$ is point-wise stabilizable for any $\alpha > 0$ and sufficiently large values of γ. It should be noted that the point-wise detectability of the pair $(A(\mathscr{X}(t)), \tilde{Q}^{1/2})$ is guaranteed by selecting a positive value for Q. According to [38], the point-wise stabilizability and detectability of the SDC representation (25) is sufficient for the existence of the set Ω in Theorem 1.

In the following simulations, the parameters of the $\mathscr{H}_\infty$ tracking controller are $\alpha = 10^{-4}$, $\gamma = 1$, $Q = 10$, $R = 10$, and $\beta = -1.5u_1(t) - u_1(t-15) - u_1(t-30)$ where $u_1(t)$ is the unit step function. For two values of the triggering factor $\sigma = 1$ and $\sigma = 0.35$, the diagrams of the ROV depths are depicted in Fig. 3. From this figure, one can see that the desired trajectory is successfully tracked by the ROV. Note that the sampling time $T = 0.2$ s is used in both cases. The obtained control inputs are also shown in Fig. 4, where the fin angle variations of the ROV are in the range $[-45°, 45°]$.

In the first case, i.e., $\sigma = 1$, the control law is updated and transmitted to the ROV at every sampling instants. Yet, for $\sigma = 0.35$, the communication rate of using the utilized network between the controller and the ROV is decreased 76%. In addition, Fig. 3 shows that this reduction does not affect the performance of the closed-loop system. Fig. 5 shows the triggering condition (11) for $\sigma = 0.35$. One can see that whenever the condition $\eta(X(t)) < 0$ is violated, the computed control

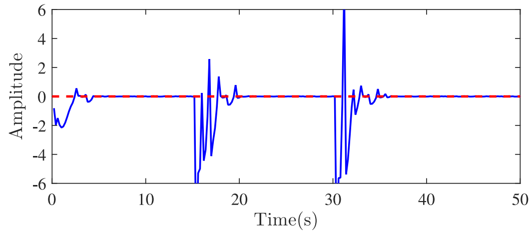

Fig. 5. Diagram of the triggering condition for $\sigma = 0.35$.

TABLE III
VALUES OF THE COST FUNCTION AND THE COMMUNICATION RATE REDUCTION PERCENTAGE BASED ON σ

σ	Reduction percentage	Cost function
1	0	32.16
0.9	70	35.49
0.5	74.4	37.16
0.35	76	38.54

input is transmitted from the controller to the ROV and held until the triggering condition is not again satisfied.

To investigate the effects of the triggering factor on the results, the above simulation is done for four different values of σ. Table III reports the corresponding values of the cost function J and the communication rate reduction. As expected, the smaller value of σ leads to more reduction of sending messages from the controller to the ROV. In addition, this reduction leads to decrease the performance of the closed-loop system and the value of the cost function is increased. Therefore, σ is an important factor in making tradeoff between the usage of the network and the closed-loop performance.

To apply the proposed event-triggered $\mathscr{H}_\infty$ tracking controller to the ROV, the parameters of the ROV should be known. However, the actual values of these parameters are different from their nominal values represented in Tables I and II. Therefore, the designed tracking controller should be robust enough against the parametric uncertainties. Fortunately, the SDRE technique has intrinsic robustness and hence, our event-triggered $\mathscr{H}_\infty$ tracking controller is expected to inherit this interesting property. To investigate this issue, the designed controller is applied to 100 sets of ROV parameters with $\pm 50\%$ uncertainties. Indeed, the controller is designed based on the nominal values of these parameters (p) and is applied to 100 different ROVs with random parameters in the bound $p \pm 0.5p$. For $\sigma = 0.35$, Fig. 6 shows the graphs of the ROVs under the designed controller where the depths of the ROVs successfully track the desired trajectory. The average value of the mean square value of the tracking error is 0.0094 which is close to the nominal value of this index (0.0091).

Remark 7: It is possible to improve robustness of the ROV against the uncertainties using the proposed technique in [6]. In this method, an observer is designed to estimate the disturbance and the unmeasured states and then, the obtained estimations are used in the design procedure of the controller.

Note that the proposed event-triggered $\mathscr{H}_\infty$ tracking controller can be used when the ROV must track more complex time-varying trajectories. For example, Fig. 7 shows the graph of the ROV depth when its desired trajectory is a combination

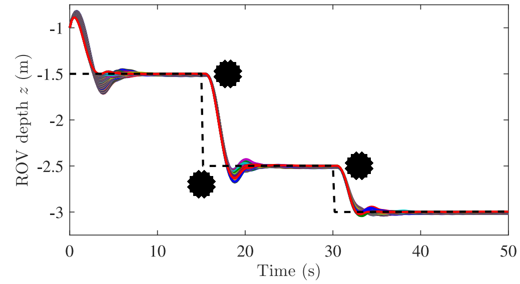

Fig. 6. Diagrams of the ROV depths for 100 set of parameters values with $\sigma = 0.35$.

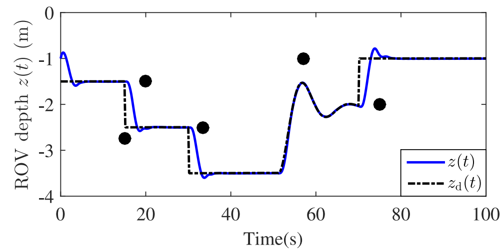

Fig. 7. Diagram of the ROV depth with steps and damped sinusoids as its desired trajectory.

of steps and damped sinusoids. As it can be seen from this figure, the designed controller leads to a closed-loop system with acceptable performance. In this case, $\sigma = 0.35$ is selected and the communication rate of using the network between the controller and the ROV is decreased 67.86%.

V. CONCLUSION

In this paper, a novel event-triggered $\mathscr{H}_\infty$ tracking controller has been proposed for a broad class of nonlinear NCSs. Through a mathematical theorem, it has been proved that the tracking error between the system state and its time-varying desired trajectory asymptotically tends to zero. The proposed method has been systematically applied to solve the depth control problem of an ROV. Simulation results have shown that the designed controller is so effective in the reduction of sending messages from the controller to the ROV while the performance of the closed-loop system is acceptable even in the presence of parametric uncertainties. In future work, the communication rate between sensors and controllers will also decreased by extending the proposed method to a dual-side event-triggered $\mathscr{H}_\infty$ tracking controller.

REFERENCES

[1] J. Yuh, “Modeling and control of underwater robotic vehicles,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 20, no. 6, pp. 1475–1483, Nov./Dec. 1990.

[2] T. I. Fossen, *Guidance and Control of Ocean Vehicles*. New York, NY, USA: Wiley, 1994.

[3] T. T. J. Prestero, “Verification of a six-degree of freedom simulation model for the REMUS autonomous underwater vehicle,” Ph.D. dissertation, Dept. Mech. Eng., Massachusetts Inst. Technol., Cambridge, MA, USA, 2001.

[4] S. P. Hou and C. C. Cheah, “Can a simple control scheme work for a formation control of multiple autonomous underwater vehicles?” *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 5, pp. 1090–1101, Sep. 2011.

[5] Z. Peng, J. Wang, and J. Wang, "Constrained control of autonomous underwater vehicles based on command optimization and disturbance estimation," *IEEE Trans. Ind. Electron.*, vol. 66, no. 5, pp. 3627–3635, May 2019.

[6] Z. Peng and J. Wang, "Output-feedback path-following control of autonomous underwater vehicles based on an extended state observer and projection neural networks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 4, pp. 535–544, Apr. 2018.

[7] L. Qiao and W. Zhang, "Adaptive non-singular integral terminal sliding mode tracking control for autonomous underwater vehicles," *IET Control Theory Appl.*, vol. 11, no. 8, pp. 1293–1306, Dec. 2017.

[8] Z. Chu, D. Zhu, and S. X. Yang, "Observer-based adaptive neural network trajectory tracking control for remotely operated vehicle," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 7, pp. 1633–1645, Jul. 2017.

[9] Y. Batmani, M. Davoodi, and N. Meskin, "Nonlinear suboptimal tracking controller design using state-dependent Riccati equation technique," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 5, pp. 1833–1839, Sep. 2017.

[10] C. Shen, Y. Shi, and B. Buckham, "Nonlinear model predictive control for trajectory tracking of an AUV: A distributed implementation," in *Proc. 55th Conf. Decis. Control (CDC)*, 2016, pp. 5998–6003.

[11] N. Fischer, D. Hughes, P. Walters, E. M. Schwartz, and W. E. Dixon, "Nonlinear RISE-based control of an autonomous underwater vehicle," *IEEE Trans. Robot.*, vol. 30, no. 4, pp. 845–852, Aug. 2014.

[12] M. L. Corradini, A. Monteriu, and G. Orlando, "An actuator failure tolerant control scheme for an underwater remotely operated vehicle," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 5, pp. 1036–1046, Sep. 2011.

[13] L.-J. Zhang, X. Qi, and Y.-J. Pang, "Adaptive output feedback control based on DRFNN for AUV," *Ocean Eng.*, vol. 36, nos. 9–10, pp. 716–722, 2009.

[14] W. M. Bessa, M. S. Dutra, and E. Kreuzer, "Depth control of remotely operated underwater vehicles using an adaptive fuzzy sliding mode controller," *Robot. Auton. Syst.*, vol. 56, no. 8, pp. 670–677, 2008.

[15] L. Lapierre, "Robust diving control of an AUV," *Ocean Eng.*, vol. 36, no. 1, pp. 92–104, 2009.

[16] Y. Shi, C. Shen, H. Fang, and H. Li, "Advanced control in marine mechatronic systems: A survey," *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 3, pp. 1121–1131, Jun. 2017.

[17] C. Peng and F. Li, "A survey on recent advances in event-triggered communication and control," *Inf. Sci.*, vols. 457–458, pp. 113–125, Aug. 2018.

[18] X.-M. Zhang, Q.-L. Han, and B.-L. Zhang, "An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems," *IEEE Trans. Ind. Informat.*, vol. 13, no. 1, pp. 4–16, Feb. 2017.

[19] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *Proc. IEEE Conf. Decis. Control (CDC)*, 2012, pp. 3270–3285.

[20] H. Li *et al.*, "Event-triggered distributed average consensus over directed digital networks with limited communication bandwidth," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 3098–3110, Dec. 2016.

[21] X. Xie, Q. Zhou, D. Yue, and H. Li, "Relaxed control design of discrete-time Takagi–Sugeno fuzzy systems: An event-triggered real-time scheduling approach," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 12, pp. 2251–2262, Dec. 2018.

[22] W. M. H. Heemels, A. R. Teel, N. Van de Wouw, and D. Nesic, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1781–1796, Aug. 2010.

[23] D. Wang, C. Mu, H. He, and D. Liu, "Event-driven adaptive robust control of nonlinear systems with uncertainties through NDP strategy," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 7, pp. 1358–1370, Jul. 2017.

[24] D. Wang, C. Mu, X. Yang, and D. Liu, "Event-based constrained robust control of affine systems incorporating an adaptive critic mechanism," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 7, pp. 1602–1612, Jul. 2017.

[25] Q. Liu, Z. Wang, X. He, and D. Zhou, "A survey of event-based strategies on control and estimation," *Syst. Sci. Control Eng. Open Access J.*, vol. 2, no. 1, pp. 90–97, 2014.

[26] A. Selivanov and E. Fridman, "Event-triggered H_∞ control: A switching approach," *IEEE Trans. Autom. Control*, vol. 61, no. 10, pp. 3221–3226, Oct. 2016.

[27] Y. Qi, P. Zeng, W. Bao, and Z. Feng, "Event-triggered robust H_∞ control for uncertain switched linear systems," *Int. J. Syst. Sci.*, vol. 48, no. 15, pp. 3172–3185, 2017.

[28] F. Li, L. Gao, G. Dou, and B. Zheng, "Dual-side event-triggered output feedback H_∞ control for NCS with communication delays," *Int. J. Control Autom. Syst.*, vol. 16, no. 1, pp. 108–119, 2018.

[29] Q. Zhang, D. Zhao, and Y. Zhu, "Event-triggered H_∞ control for continuous-time nonlinear system via concurrent learning," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 7, pp. 1071–1081, Jul. 2017.

[30] X.-M. Zhang and Q.-L. Han, "Event-triggered H_∞ control for a class of nonlinear networked control systems using novel integral inequalities," *Int. J. Robust Nonlin. Control*, vol. 27, no. 4, pp. 679–700, 2017.

[31] M. L. Corradini and G. Orlando, "A discrete adaptive variable-structure controller for MIMO systems, and its application to an underwater ROV," *IEEE Trans. Control Syst. Technol.*, vol. 5, no. 3, pp. 349–359, May 1997.

[32] A. Shaw, A. Al-Shamma'a, S. R. Wylie, and D. Toal, "Experimental investigations of electromagnetic wave propagation in seawater," in *Proc. 36th Eur. Microw. Conf.*, 2006, pp. 572–575.

[33] D. R. Blidberg, "The development of autonomous underwater vehicles (AUV); a brief summary," in *Proc. IEEE ICRA*, vol. 4, 2001, pp. 1–12.

[34] T. Baser and P. Bernhard, *H_∞ Optimal Control and Related Minimax Design Problems: A Dynamic Game Approach*, 2nd ed. Boston, MA, USA: Birkhäuser, 1995.

[35] H. Modares, F. L. Lewis, and Z.-P. Jiang, "H_∞ tracking control of completely unknown continuous-time systems via off-policy reinforcement learning," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 10, pp. 2550–2562, Oct. 2015.

[36] M. Souza and J. C. Geromel, "H_2 dynamic output feedback for local sensor-remote actuator networks," *IMA J. Math. Control Informat.*, vol. 33, no. 2, pp. 239–256, 2014.

[37] Y. Batmani, "H_∞ suboptimal tracking controller design for a class of nonlinear systems," *Int. J. Control Autom. Syst.*, vol. 15, no. 5, pp. 2080–2087, 2017.

[38] T. Cimen, "Survey of state-dependent Riccati equation in nonlinear optimal feedback control synthesis," *J. Guid. Control Dyn.*, vol. 35, no. 4, pp. 1025–1047, 2012.

[39] T. Cimen, "Systematic and effective design of nonlinear feedback controllers via the state-dependent Riccati equation (SDRE) method," *Annu. Rev. Control*, vol. 34, no. 1, pp. 32–51, 2010.

[40] A. J. Van Der Schaft, "L_2-gain analysis of nonlinear systems and nonlinear state-feedback H_∞ control," *IEEE Trans. Autom. Control*, vol. 37, no. 6, pp. 770–784, Jun. 1992.

[41] Y. Batmani, M. Davoodi, and N. Meskin, "Event-triggered suboptimal tracking controller design for a class of nonlinear discrete-time systems," *IEEE Trans. Ind. Electron.*, vol. 64, no. 10, pp. 8079–8087, Oct. 2017.

[42] X. Meng and T. Chen, "Event detection and control co-design of sampled-data systems," *Int. J. Control*, vol. 87, no. 4, pp. 777–786, 2014.

[43] M. S. Naik and S. N. Singh, "State-dependent Riccati equation-based robust dive plane control of AUV with control constraints," *Ocean Eng.*, vol. 34, nos. 11–12, pp. 1711–1723, 2007.

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