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A bi-level optimization model for operation of distribution networks with micro-grids

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ABSTRACT

Active distribution grids (ADGs) consist of several distributed generations (DGs) and controllable loads (CLs). These resources are utilized in the form of several microgrids (MGs) which in turn facilitate managing of ADGs. Therefore, the problem of distribution company (DISCO) and MGs operation requires a hierarchical decision-making framework. An attempt is made in this paper to model such framework as a bi-level optimization problem. In the proposed bi-level model, the objective of the upper level (leader) problem is to maximize the profit of DISCO, and the objective of the lower level (follower) problems is to minimize the cost of MGs. The resulting model is a nonlinear bi-level problem which is transformed into a linear single-level problem through Karush–Kuhn–Tucker (KKT) conditions and dual theory. Since the proposed model creates a retail electricity market in distribution grid, two frameworks are considered for this market: various and uniform retail electricity prices. To illustrate the effectiveness of the model, a hypothetical distribution grid is considered as the case study. The impacts of the market price and various demand levels of MGs on the results are investigated in two scenarios.

Introduction

Motivation

Traditional power systems suffer from several crudities such as fossil fuel shortage, environmental issues of fossil fuel resources, and low energy efficiency in power delivery sector [1]. Moreover, electric power consumption is increasing due to the population growth, industrialization, and especially urban developments. Therefore, power system administrators try to supply the increasing demand in an efficient way. For this purpose, distributed generations (DGs) are penetrated in distribution grids to serve the load locally [1].

While conventional distribution grids, known as passive distribution grids, do not contribute in power generation, their main function is to provide consumers with the power from transmission grid. In the most passive distribution grids, distribution companies (DISCOs) purchase the required electrical energy from wholesale electricity market and sell it to the consumers with specified prices. Emerging DGs in distribution grids, referred to as active distribution grids (ADGs), enables the grids to produce power locally. In ADGs, due to presence of DGs, DISCOs have complex operation problems in comparison with passive distribution grids.

Generally, integration of DGs with local loads introduces a new concept called microgrids (MGs) [1]. From the technical point of view, a MG is a system with at least one DG and one demand which can be operated in both grid connected and standalone modes [2]. Usually, there are several MGs in ADG which may have independent operations. These MGs can be coordinated by the DISCO.

In ADGs, the decision making on the operation problem should be done in two levels. The early one is the DISCO and the latter is MGs as the upper and the lower levels decision makers, respectively. To operate the ADG in an optimal manner, these decision makers should be able to optimize their respective objective functions independently, and cooperate with each other simultaneously. Therefore, the operation problem requires a hierarchical decision-making framework. In this way, there are multiple decision makers with different objectives which the decision process has a structure on the order of levels [3]. As mentioned above, there are two levels of decision making in distribution grid for which the operation problem can be modeled as a bi-level optimization problem [4]. In a bi-level optimization problem, the decision makers optimize their respective objective functions independently. However, their decisions effect on the decision space of each other.

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Nomenclature

Indices

j index of MGs

Parameters

C_{DG}^{j} generation cost of DG ($/MW h)
C_{IL}^{j} load curtailment cost ($/MW h)
P_{demand}^{j} power demand (MW)
$P_{DG,\max}^{j}$ maximum DG capacity limit (MW)
$P_{DG,\min}^{j}$ minimum DG capacity limit (MW)
$P_{IL,\max}^{j}$ maximum load curtailment limit (MW)
$P_{\max}^{T_{up}}$ maximum limit for purchasing power from market (MW)
$P_{\max}^{T_j}$ maximum limit for power exchange between DISCO and MG j (MW)
ρ^{M} market price ($/MW h)
$\rho^{\max}$ maximum limit for price of power exchange between DISCO and MGs ($/MW h)

Variables

P^{M} power purchased from market (MW)
P_{D}^{j} power exchange between DISCO and MG j (MW)
P_{DG}^{j} power generation of DG (MW)
P_{IL}^{j} the amount of load curtailment (MW)
U_{i}^{j} binary variable used for linearization of the complementary slackness conditions for constraint i and MG j
λ_{i}^{j} dual variable for constraint i and MG j ($/MW h)
ρ_{D}^{j} price of power exchange between DISCO and MG j in framework 1 ($/MW h)
ρ^{D} price of power exchange between DISCO and all MGs in framework 2 ($/MW h)

Others

L^{j} lagrangian function

Literature review

The operation problem of DISCO, including DGs and controllable loads (CLs), is investigated in the literature. In [5], a two-stage hierarchical framework for operation of DISCO in day-ahead and real-time electricity markets is presented. In [6], the previous article is extended with consideration of uncertainty on real-time electricity prices and loads. Day-ahead scheduling of DISCO considering energy and reserve markets is addressed in [7,8].

The economic, environmental, and technical effects of MGs penetration on ADG are studied in different articles. In [9], multi criteria decision aid (MCDA) techniques are used for evaluating the impact of MGs on the ADG. To this end, five criteria including installation and operation costs, investment deferral, active power losses, environmental impact, and improving the distribution grid reliability are considered. In [10], three criteria including economic operation, active power losses, and environmental benefits are investigated to show the impacts of MGs on the ADG.

In some studies, distribution grid is considered as coupled MGs and operation of these MGs and cooperation among them is investigated. In [11], an optimal control algorithm for coupled microgrids is presented. In [12], energy resource scheduling of several microgrids in isolated distribution grid is investigated using multi-agent systems. In [13], energy consumption scheduling in distribution grids with coupled microgrids considering demand uncertainty is analyzed.

The operation problem of DISCO and MGs in ADG is modeled using system of systems framework in [14,15]. Although appropriate framework is addressed in these papers, the operation problem of DISCO and MGs is not modeled simultaneously. In other words, at first, the optimization problem is solved for MGs and then the problem is solved for DISCO. In each iteration, the required data for optimization is exchanged between these two systems. This iterative process is continued as long as the converge conditions are satisfied.

To model the optimal behavior of decision makers in two-level decision making, bi-level optimization model is proposed in literature. In [16], the bi-level optimization is used to model a demand response aggregator's behavior in offering strategy of a wind power producer. In the proposed model the wind power producer and demand response aggregator are considered as leader and follower, respectively. The proposed model in [16] is extended to model the day-ahead market clearing price in the optimal decision making of wind power producer in which wind power producer acts as leader and demand response aggregator's decision beside day-ahead market clearing process are considered as the followers [17]. In [18] a two-stage two-level model is proposed to model the optimal behavior of retailers in wholesale and retail electricity markets considering demand response. The optimal behavior of retailers in demand response market environment is modeled as bi-level optimization problem in [19] in which retailers and consumers are considered as leader and followers, respectively. Therefore, for modeling the operation problem of two-level of decision makers, i.e., Disco and MGs, a bi-level optimization approach is employed in this paper. In the proposed approach, the price and the amount of power exchange between DISCO and MGs are considered as two decision variables that link DISCO and MGs to each other. The proposed bi-level optimization problem is transformed into a single-level problem according to the Karush–Kuhn–Tucker (KKT) conditions. Then, nonlinear expression in the model is replaced with linear ones based on the dual theory.

Moreover, when the operation problem of DISCO and MGs is modeled as a bi-level optimization problem, a retail electricity market can be created in distribution grid that clears the electricity prices between DISCO and MGs. In this paper, two frameworks are considered for this market. In framework 1 (FM1), the price of power exchange between DISCO and each MG is determined. In framework 2 (FM2), the price of power exchange between DISCO and all MGs is determined as a uniform price. These two frameworks are compared with each other and remarkable differences are highlighted.

Although two frameworks are considered for retail electricity market, the proposed bi-level optimization models for these two frameworks are similar with small differences. Further details are discussed in the problem formulation section.

Contribution

The main contributions of this paper are categorized as follows:

(1) Proposing a bi-level optimization model that provides a hierarchical decision-making framework in which DISCO and MGs optimize their respective objective functions independently and in cooperation with each other.
(2) Transforming the nonlinear bi-level optimization model with the linear single-level optimization model using KKT conditions and the dual theory.
(3) Proposing a retail electricity market in distribution grid considering DISCO and MGs.
(4) Comparison between two different frameworks considered for the proposed retail electricity market.

Paper organization

The rest of this paper is organized as follows. The modeling framework is presented in Section "The modeling framework". Formulation of the bi-level optimization problem is described in Section "Problem formulation". Section "Solution methodology" describes the solution methodology of the problem. Numerical studies are done in Section "Numerical results". Section "Conclusi on" concludes the paper. Finally, mathematical backgrounds to convert the nonlinear bi-level optimization problem to the linear single-level optimization problem are explained in Appendices A and B.

The modeling framework

In this paper, the operation of DISCO and MGs problem is modeled as a bi-level optimization problem. Generally, bi-level optimization problems possess a two-level structure with two decision makers in which the decision maker in the upper level is called as the leader and the decision maker in the lower level is called as the follower. In the proposed model, DISCO and MGs are considered as the leader and the followers, respectively. The amount and the price of power exchange are considered as the interaction decision variables between the leader and the followers. Moreover, when the operation problem of DISCO and MGs is modeled as a bi-level optimization problem, a retail electricity market is created in distribution grid level as illustrated in Fig. 1. In this paper, this market is simple and its participants are the DISCO and MGs.

Two frameworks are considered to illustrate the structure of this market. In FM1, at first, the leader (DISCO) decides on the price

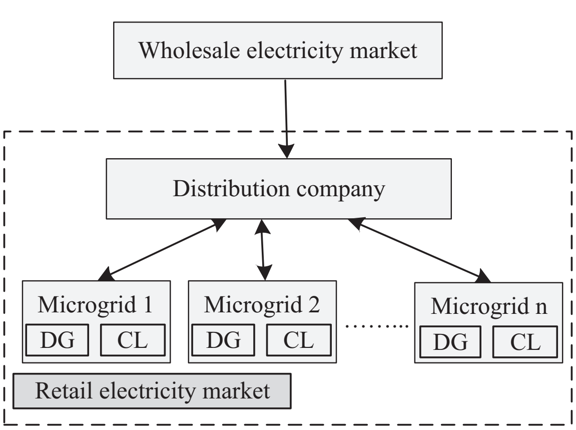


Fig. 1. Retail electricity market created in distribution grid level.

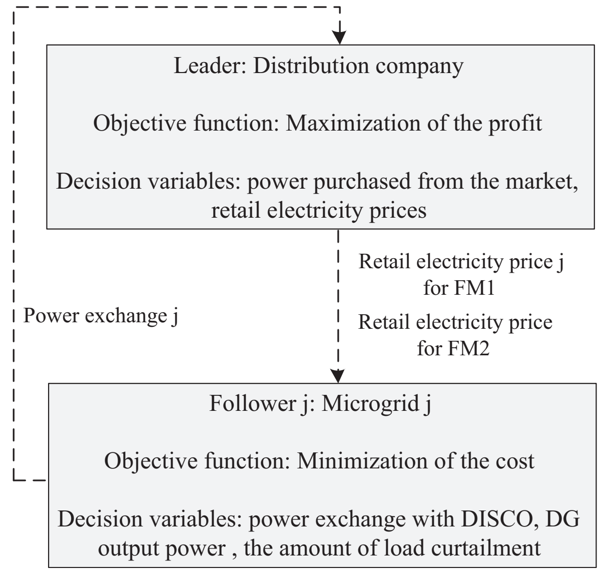


Fig. 2. Bi-level decision-making structure for two frameworks.

of power exchange with each MG, then each MG receives this price signal and solves its optimization problem so that it decides on its DG output power, the amount of load curtailment as well as the amount of power exchange with DISCO. In FM2, the leader (DISCO) decides on the price of power exchange with all MGs, then MGs receive this price signal and solve their optimization problems. These frameworks are illustrated in Fig. 2.

Problem formulation

The operation of DISCO and MGs problem is formulated as the following bi-level optimization problem:

$$\underset{P^M,\rho_D^j,P_D^j,P_{DG}^j,P_{IL}^j}{\text{Maximize}} \quad \sum_j(\rho_D^j P_D^j) - \rho^M P^M \tag{1}$$

Subject to

$$0 \leqslant \rho_D^j \leqslant \rho^{max} \quad \forall j \tag{2}$$

$$P^M \leqslant P_{max}^{T_{up}} \tag{3}$$

$$\sum_j P_D^j = P^M \tag{4}$$

where

$$P_{DG}^j, P_{IL}^j, P_D^j \in \arg\{\text{Minimize } \rho_D^j P_D^j + C_{DG}^j P_{DG}^j + C_{IL}^j P_{IL}^j \tag{5}$$

subject to

$$-P_{max}^{T_j} \leqslant P_D^j \leqslant P_{max}^{T_j} : \quad \lambda_1^j, \lambda_2^j \tag{6}$$

$$P_{DG,min}^j \leqslant P_{DG}^j \leqslant P_{DG,max}^j : \quad \lambda_3^j, \lambda_4^j \tag{7}$$

$$0 \leqslant P_{IL}^j \leqslant P_{IL,max}^j : \quad \lambda_5^j, \lambda_6^j \tag{8}$$

$$P_{DG}^j + P_D^j + P_{IL}^j = P_{demand}^j : \quad \lambda_7^j\}, \forall j \tag{9}$$

In the proposed model, Eqs. (1)–(4) represent the leader problem and Eqs. (5)–(9) denote the follower problems. In leader level, the operation problem of DISCO is solved in which the power purchased from the market and the price offered to each MG is determined. In the follower level, each MG determines its decision variables with notice to the price offered by DISCO, i.e., DG output power, the amount of load curtailment, and the power exchange with DISCO.

The objective function of the leader is to maximize its profit described by Eq. (1) including two terms. The first term is the revenue from power exchange with MGs. If $P_D^j > 0$, DISCO is selling power to MG j and if $P_D^j < 0$, DISCO is purchasing power from MG *j* and also if $P_D^j = 0$, there is not any power exchange between DISCO and MG *j*. The second term is the cost of power purchased from the market. Eq. (2) limits the price of power exchange between DISCO and MG *j*. Eq. (3) limits the power purchased from the market. Eq. (4) is the power balance constraint for DISCO. It shows that the sum of power exchanges with MGs is equal to the power purchased from the market.

Eqs. (5)–(9) model the operation problem of each MG and its reaction to the price offered by DISCO. Eq. (5) shows the objective function of each MG including the cost of power exchange with DISCO, the power generation cost of DG and the cost of load curtailment, respectively. Eq. (6) limits the amount of power exchange between DISCO and each MG. Eq. (7) limits the DG output power for each MG. Eq. (8) limits the amount of load curtailment for each MG. Eq. (9) shows the power balance constraint for each MG in which the sum of power exchange with DISCO, DG output power, and the amount of load curtailment is equal to the demand of each MG. The load curtailment is modeled as proposed in [20]. λ_1^j and λ_2^j are the dual variables (lagrangian multipliers) for the lower and upper limit of P_D^j. λ_3^j and λ_4^j are the dual variables for the lower and upper limit of P_{DG}^j. λ_5^j and λ_6^j are the dual variables for the lower and upper limit of P_{IL}^j and λ_7^j is the dual variable for the power balance constraint.

As in Eq. (1), two variables ρ_D^j and P_D^j are multiplied, the optimization problem (1)–(9) is a nonlinear bi-level optimization problem. In the next section, the solution methodology is presented to solve this problem.

In framework 2 (FM2), the term ρ^D is replaced with ρ_D^j in related equations and the other equations are the same as described for FM1. A series of assumptions are considered in the model as follows:

- The model is considered for one time step. This assumption is also considered in [14,15,21]. So, the terminology of power is used instead of energy in related equations.
- The power losses in distribution grid are neglected. In fact, the distribution grid is not modeled. This assumption is also considered in [11,21,22].

Solution methodology

There are many approaches to solve the bi-level optimization problems [4]. One of the common and exact solutions of these problems is to replace the set of the follower problems with their KKT conditions [4]. In bi-level optimization problems, the decision variables of the leader are considered as parameters in the follower problem. Hence, the price of power exchange between DISCO and MGs (ρ_D^j in FM1 and ρ^D in FM2), which is the leader variable, is considered as parameter in the follower problem. Since the follower problem for each MG is linear and continuous and thus convex, the follower problem can be replaced with its KKT conditions. So, the nonlinear bi-level problem is transformed into a nonlinear single-level problem through KKT conditions. Then, the nonlinear expression in the model is replaced with linear expressions using the dual theory [23]. Therefore, the solution methodology for the proposed nonlinear bi-level optimization problem (1)–(9) is as follows:

- Firstly, each follower problem is replaced with its KKT conditions. KKT conditions include nonlinear expressions in complementary slackness section. These nonlinear expressions is replaced with two linear constraints. Details of this step are illustrated in Appendix A.
- Secondly, the nonlinear expression in Eq. (1) is replaced with linear expressions. This replacement is achieved using the dual theory. Details of this linearization are described in Appendix B.

So, the nonlinear bi-level optimization problem is transformed into the single-level mixed-integer linear problem (MILP) as follows:

$$\underset{\substack{P^M,\rho_D^j,P_D^j,\\P_{DG}^j,P_{IL}^j}}{\text{Maximize}} \sum_j \begin{pmatrix} -C_{DG}^j P_{DG}^j - C_{IL}^j P_{IL}^j - P_{\max}^{T_j}(\lambda_1^j+\lambda_2^j) + P_{DG,\min}^j \lambda_3^j \\ -P_{DG,\max}^j \lambda_4^j - P_{IL,\max}^j \lambda_6^j + P_{demand}^j \lambda_7^j \end{pmatrix} - \rho^M P^M \tag{10}$$

Subject to

$$0 \leqslant \rho_D^j \leqslant \rho^{\max} \quad \forall j \tag{11}$$

$$P^M \leqslant P_{\max}^{T_{up}} \tag{12}$$

$$\sum_j P_D^j = P^M \tag{13}$$

$$\rho_D^j - \lambda_1^j + \lambda_2^j - \lambda_7^j = 0 \quad \forall j \tag{14}$$

$$C_{DG}^j - \lambda_3^j + \lambda_4^j - \lambda_7^j = 0 \quad \forall j \tag{15}$$

$$C_{IL}^j - \lambda_5^j + \lambda_6^j - \lambda_7^j = 0 \quad \forall j \tag{16}$$

$$P_{DG}^j + P_D^j + P_{IL}^j = P_{demand}^j \quad \forall j \tag{17}$$

$$C_i^j \geqslant 0 \quad \forall i = 1,2,\ldots,6 \quad \forall j \tag{18}$$

$$\lambda_i^j \geqslant 0 \quad \forall i = 1,2,\ldots,6 \quad \forall j \tag{19}$$

$$\lambda_7^j \quad \forall j \quad \text{Unrestricted} \tag{20}$$

$$C_i^j \leqslant MU_i^j \quad \forall i = 1,2,\ldots,6 \quad \forall j \tag{21}$$

$$\lambda_i^j \leqslant M(1-U_i^j) \quad \forall i = 1,2,\ldots,6 \quad \forall j \tag{22}$$

Eq. (10) is the same objective function of DISCO described in Eq. (1). The procedure of obtaining the first term of this equation is described in Appendix B. Eqs. (11)–(13) are the same as Eqs. (2)–(4). Other equations are described in Appendix A.

For FM2, the single-level MILP is similar to Eqs. (10)–(22) and only the variable ρ^D is replaced with ρ_D^j in related equations and the other equations are the same as described for FM1.

Table 1
Characteristics of DG units of MGs.

MG (*j*)	$P_{DG,\min}^j$ (MW)	$P_{DG,\max}^j$ (MW)	C_{DG}^j (\$/MW h)
1	0	4	37
2	0	5	40
3	0	5.5	35
4	0	7	45

Table 2
Characteristics of each scenario.

Scenario	ρ^M (\$/MW h)	P^1_{demand} (MW)	P^2_{demand} (MW)	P^3_{demand} (MW)	P^4_{demand} (MW)
1	(34–46)	5	5	6	5.5
2	43	(2–8)	(2–8)	(2–8)	(2–8)

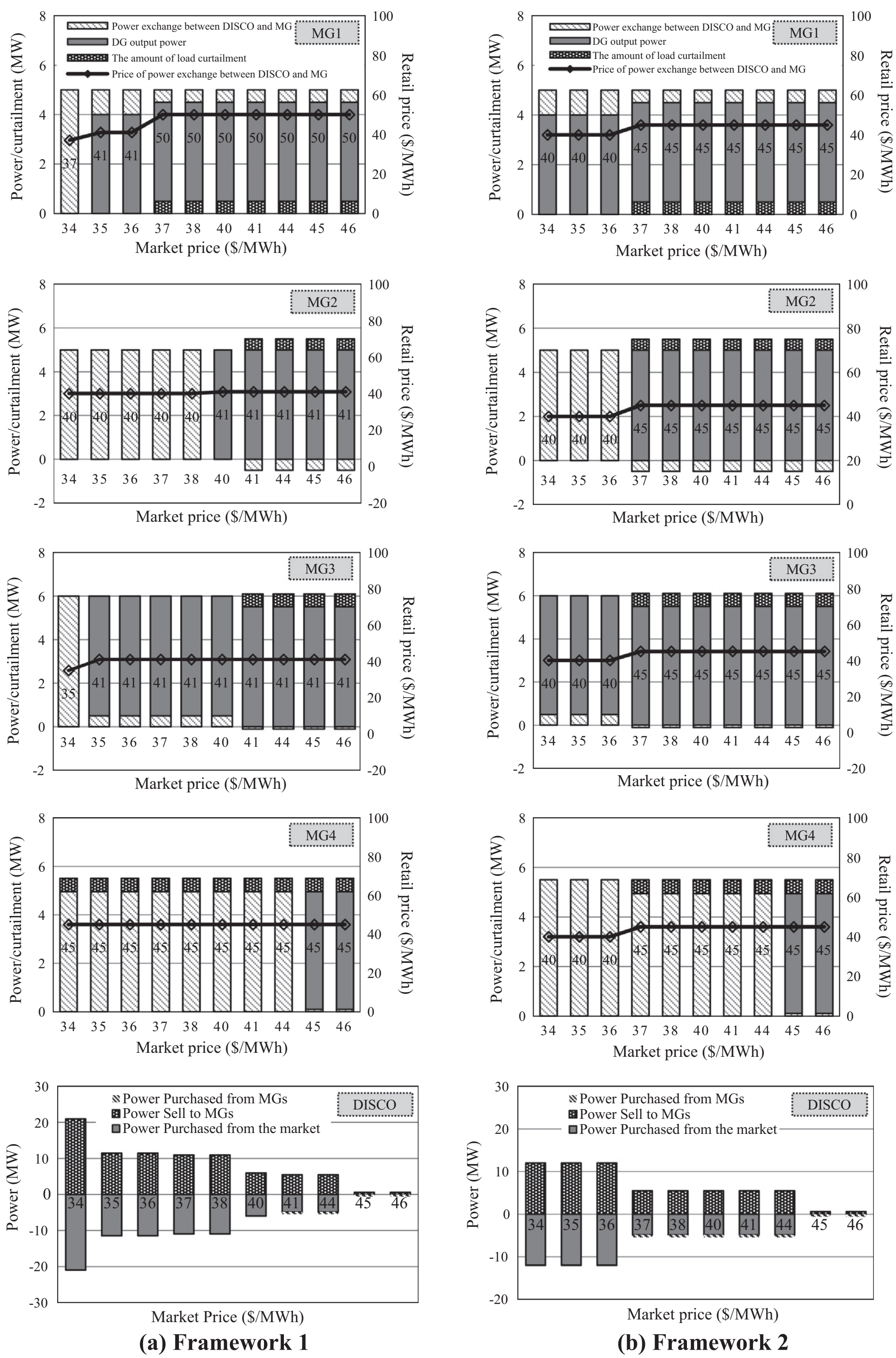


Fig. 3. Operation results in scenario 1 for MG1, MG2, MG3, MG4, and DISCO, respectively.

Table 3
Profit of DISCO and cost of each MG in scenario 1 for each framework ($).

Market price ($/MW h)	FM1					FM2				
	DISCO Profit	MG1 Cost	MG2 Cost	MG3 Cost	MG4 Cost	DISCO Profit	MG1 Cost	MG2 Cost	MG3 Cost	MG4 Cost
34	105.45	185	200	210	245.3	72	188	200	212.5	220
35	83.5	189	200	213	245.3	60	188	200	212.5	220
36	72.05	189	200	213	245.3	48	188	200	212.5	220
37	63.1	193.5	200	213	245.3	38.8	191	198	212.6	245.3
38	52.15	193.5	200	213	245.3	33.95	191	198	212.6	245.3
40	30.25	193.5	200	213	245.3	24.25	191	198	212.6	245.3
41	24.3	193.5	200	213	245.3	19.4	191	198	212.6	245.3
44	9.75	193.5	200	213	245.3	4.85	191	198	212.6	245.3
45	4.9	193.5	200	213	245.3	0	191	198	212.6	245.3
46	4.9	193.5	200	213	245.3	0	191	198	212.6	245.3

Numerical results

The proposed bi-level optimization models, related to the two frameworks, are applied to a hypothetical distribution grid including four MGs. Characteristics of DG units of MGs are listed in Table 1 [20]. The capacity of transformers between the upstream grid and DISCO (T_{up}) and between DISCO and each MG (T_j) are 40 and 8 MW, respectively. The maximum amount of the load curtailment is 10% of demand for each MG. The maximum limit for price of power exchange between DISCO and MGs is assumed to be 50 $/MW h.

There are three parameters including C_{IL}^j, P_{demand}^j, and ρ^M that have considered in the optimization process with different values. At first, the optimization is done for different values of C_{IL}^j and the results showed that C_{IL}^j has minor effect on the operation of DISCO and MGs due to its low value for each MG. So, C_{IL}^j is fixed at 41 $/MW h during the optimization process [20].

The impact of remaining parameters on the optimization results is investigated in this section. For this purpose, two scenarios are defined as presented in Table 2. In scenario 1, the demand of each MG is fixed and ρ^M has different values from 34 to 46 $/MW h, which are selected with notice to the generation cost of DG units and load curtailment cost. In scenario 2, $\rho^M = 43$ $/MW h and is fixed but the demand of each MG has different values from 2 to 8 MW.

Scenario 1

In this scenario, the impact of the market price on the optimization results is investigated. The operation results in scenario 1 for FM1 and FM2 are shown in Fig. 3a and b, respectively. The remarkable results from Fig. 3a. are as follows:

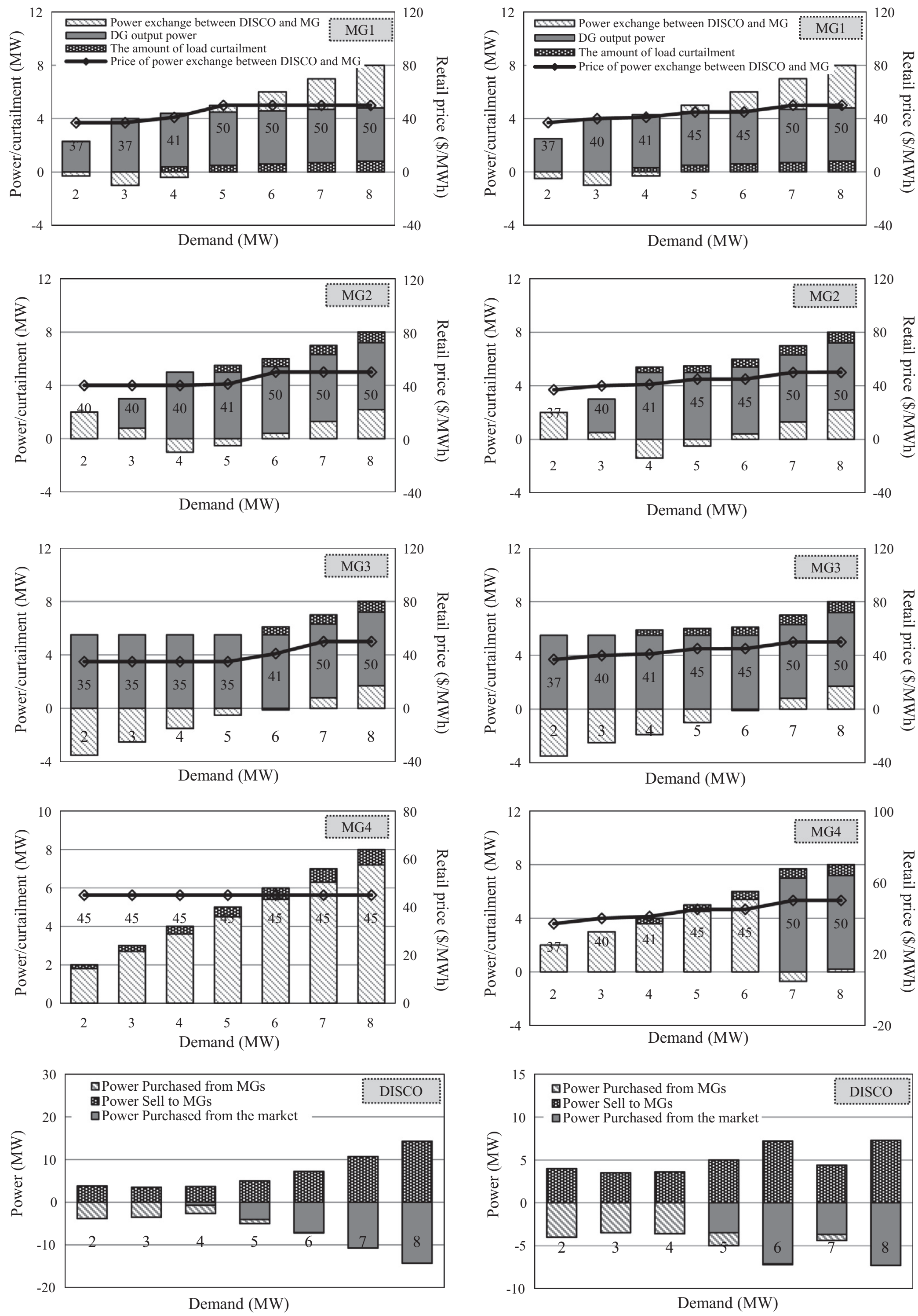
- When the price of power exchange between DISCO and each MG is equal to the generation cost of its DG, since for MG1, MG2, and MG3, these prices are lower than the load curtailment cost, they purchase the whole required demand from DISCO. However, since for MG4, the load curtailment cost is lower than the price of power exchange with DISCO, as long as the market price is lower than this price, MG4 curtails its load and then purchases the rest of demand from DISCO. In this manner, DISCO purchases power from market and sells it with higher prices to MGs.
- When prices of power exchange with MG1, MG2, and MG3 are equal to the load curtailment cost and for MG2 and MG3 the market price is lower than the load curtailment cost, these MGs dispatch their DGs and then purchase the remaining demand from DISCO.
- When the price of power exchange with MG1 is greater than the load curtailment cost and is 50 $/MW h, MG1 dispatches its DG, curtails its load, and purchases the remaining demand from DISCO, respectively.
- When the market price is greater than or equal to the load curtailment cost, MG2 and MG3 sell their extra power to DISCO. Also, when the market price is greater than or equal to 45 $/MW h, MG4 curtails its load, dispatches its DG and then purchases the remaining demand from DISCO. In this manner, DISCO will not purchase any power from the market.
- When the market price is increased, the power purchased from the market by DISCO is decreased. Also, when the market price is greater than or equal to the load curtailment cost, DISCO purchases power from MGs.

So, in FM1 different retail electricity prices for each market price are determined between DISCO and MGs regarding to the market price, generation cost of DGs and the load curtailment cost.

Fig. 3b. shows the operation results for DISCO and each MG for FM2 wherein the price of power exchange between DISCO and MGs is uniform for each market price. The remarkable results from Fig. 3b. are as follows:

- Two values are determined for the retail electricity price between DISCO and MGs. When this price is equal to 40 $/MW h, since this price is greater than the generation cost of DGs for MG1 and MG3, these MGs dispatch their DGs and then purchase the remaining demand from DISCO. Also, since this price is lower than or equal to the generation cost of DGs for MG2 and MG4, these MGs purchase the whole required demand from DISCO. Moreover, MGs will not curtail their demands because the load curtailment cost is greater than the retail electricity price.
- When the retail electricity price is equal to 45 $/MW h, since this price is greater than the load curtailment cost and the generation cost of DGs for MG1, MG2, and MG3, these MGs dispatch their DGs, curtail their demands, and then exchange power with DISCO so that MG1 purchases power from DISCO and also, MG2 and MG3 sell their extra power to DISCO. In this case, the behavior of MG4 is similar to FM1.
- In FM2, DISCO purchases less power from the market and more power from MGs in comparison with FM1.

The profit of DISCO and the cost of each MG in scenario 1 for each framework are listed in Table 3. Since in FM2, the DISCO exchanges power with MGs with uniform price for each value of market price, its profit is decreased in comparison with FM1 in which DISCO exchanges power with MGs with different prices.



(a) Framework 1 **(b) Framework 2**

Fig. 4. Operation results in scenario 2 for MG1, MG2, MG3, MG4, and DISCO, respectively.

Also, when the market price is increased, the profit of DISCO is decreased in each framework because of two reasons. First, when the market price is increased, DISCO earns a lower profit from purchasing power from the market and selling it to MGs. Second, when the market price is increased, the retail electricity prices are also increased and so, MGs prefer to dispatch their DGs and curtail their demands and then purchasing their rest demand from the DISCO. Therefore, the DISCO sells lower power to MGs.

When MG2 and MG3 sell power to DISCO, they earn greater revenues in FM2 in comparison with FM1. When the market price is 34 $/MW h and the retail electricity price is 40 $/MW h in FM2, the cost of MG1 and MG3 are increased because they purchase

Table 4
Profit of DISCO and cost of each MG in scenario 2 for each framework ($).

Demand (MW)	FM1					FM2				
	DISCO Profit	MG1 Cost	MG2 Cost	MG3 Cost	MG4 Cost	DISCO Profit	MG1 Cost	MG2 Cost	MG3 Cost	MG4 Cost
2	27.4	74	80	70	89.2	0	74	74	63	74
3	29	111	120	105	133.8	0	108	120	92.5	120
4	23	148	160	140	178.4	0	148	159	131	164
5	17.5	193.5	200	175	223	7	191	198	168	223
6	23.6	242.6	244.6	213	267.6	14.2	235.6	242.6	212.6	267.6
7	43.4	291.7	293.7	261.2	312.2	25.9	291.7	293.7	261.2	308.7
8	64.1	340.8	342.8	310.3	356.8	51.1	340.8	342.8	310.3	357.8

power from DISCO with higher price in comparison with FM1. When the market price is between 34 and 36 $/MW h and the retail electricity price is equal to 40 $/MW h in FM2, the cost of MG4 is decreased because it purchases power from DISCO with lower price in comparison with FM1.

Scenario 2

The impact of different demand levels of MGs on the operation of DISCO and each MG for two frameworks is investigated in scenario 2. The operation results for DISCO and each MG for FM1 and FM2 are illustrated in Fig. 4a and b, respectively. The remarkable results from Fig. 4 are as follows:

- As long as the sum of DG output power and the amount of load curtailed of MG1, MG2, and MG3 are greater than or equal to their demand, they sell their extra power to DISCO with prices that are lower than the market price. In FM1, these power exchanges has benefit for DISCO because it purchases power from some MGs with prices lower than the market price and then sells it with higher prices to other MGs. Also, MG1, MG2, and MG3 earn higher revenues from selling power to DISCO in FM2 in comparison with FM1.
- When the demand of MG1, MG2, and MG3 are increased, they purchase the remaining demand from DISCO.
- When the demand of MGs is between 2 and 4 MW, MG4 purchases power from DISCO with lower price in FM2 in comparison with FM1. Moreover, in FM2, MG4 sells power to DISCO due to the high retail electricity price when the demand of all MGs is 7 MW.
- In FM2, DISCO purchases less power from the market and more power from MGs in comparison with FM1.

Table 4 shows the profit of DISCO and the cost of each MG in scenario 2 for each framework. Since in FM2, DISCO exchanges power with MGs with uniform price for each demand level of MGs, its profit is decreased in comparison with FM1 in which DISCO exchanges power with MGs with different prices. In FM2, when the demand of MGs are between 2 and 4 MW, DISCO will not purchase any power from the market and so it exchanges power with MGs with uniform retail electricity price so that DISCO purchases power from some MGs with retail price and then sell the same amount of the power with the same price to other MGs. So, DISCO earns no profit from these exchanges. Also, the cost of each MG decreases in this situation in comparison with FM1.

Conclusion

Since the problem of DISCO and MGs operation requires a hierarchical decision-making framework, this framework is modeled as a bi-level optimization problem in this paper. Furthermore, two frameworks are presented to illustrate the structure of retail electricity market created between DISCO and MGs in distribution grid. The resulting models are the nonlinear bi-level optimization problems transformed into the single-level linear optimization problems using KKT conditions and the dual theory. A hypothetical distribution grid including four MGs is considered as the case study. The impact of the market price and various demand levels of MGs on the results have been investigated in each framework through two scenarios. The results revealed that the proposed bi-level optimization has the ability to model the decision-making framework of DISCO and MGs in distribution grid appropriately. Moreover, the remarkable conclusions from the mentioned models and the two frameworks are as follows:

- The results showed that the structure of the market has important impacts on the profit of DISCO and the cost of MGs. The profit of DISCO and the cost of MGs are decreased in FM2 in comparison with FM1. In fact, FM1 and FM2 are economically suitable for DISCO and MGs, respectively.
- Power purchased from MGs by DISCO is increased in FM2 in comparison with FM1 due to incentive prices offered to MGs.
- In scenario 1, when the retail electricity price is lower than or equal to the generation costs of DGs and the load curtailment cost, MGs prefer to purchase their required demand from the DISCO. However, when the retail electricity price is increased, MGs prefer to dispatch their DGs, curtail their loads and then exchange power with DISCO.

Appendix A

To transform the bi-level optimization problem into the single-level optimization problem, the follower problem can be replaced with its KKT conditions as the follower problem is continuous and linear and thus is convex. At first, constraints of the follower problem (except power balance constraint) are rewritten as greater than or equal to zero constraints as illustrated in the following:

$$C_1^j = P_D^j + P_{\max}^{T_j} \geqslant 0: \quad \lambda_1^j, \quad \forall j \tag{A1}$$

$$C_2^j = P_{\max}^{T_j} - P_D^j \geqslant 0: \quad \lambda_2^j, \quad \forall j \tag{A2}$$

$$C_3^j = P_{DG}^j - P_{DG,\min}^j \geqslant 0: \quad \lambda_3^j, \quad \forall j \tag{A3}$$

$$C_4^j = P_{DG,\max}^j - P_{DG}^j \geqslant 0: \quad \lambda_4^j \quad \forall j \tag{A4}$$

$$C_5^j = P_{IL}^j \geqslant 0: \quad \lambda_5^j \quad \forall j \tag{A5}$$

$$C_6^j = P_{IL,\max}^j - P_{IL}^j \geqslant 0: \quad \lambda_6^j, \quad \forall j \tag{A6}$$

$$P_{DG}^j + P_D^j + P_{IL}^j = P_{demand}^j: \quad \lambda_7^j, \quad \forall j \tag{A7}$$

For each follower problem (5)–(9) the lagrangian function is:

$$L^j = \rho_D^j P_D^j + C_{DG}^j P_{DG}^j + C_{IL}^j P_{IL}^j - \lambda_1^j (P_D^j + P_{\max}^{T_j}) - \lambda_2^j (P_{\max}^{T_j} - P_D^j) - \lambda_3^j (P_{DG}^j - P_{DG,\min}^j) - \lambda_4^j (P_{DG,\max}^j - P_{DG}^j) - \lambda_5^j (P_{IL}^j) - \lambda_6^j (P_{IL,\max}^j - P_{IL}^j) - \lambda_7^j (P_{DG}^j + P_D^j + P_{IL}^j - P_{demand}^j) \tag{A8}$$

It should be noted that for each follower problem, ρ_D^j is considered as a parameter. KKT conditions including 4 s described in the following:

Stationarity:

$$\frac{\partial L^j}{\partial P_D^j} = \rho_D^j - \lambda_1^j + \lambda_2^j - \lambda_7^j = 0 \tag{A9}$$

$$\frac{\partial L^j}{\partial P_{DG}^j} = C_{DG}^j - \lambda_3^j + \lambda_4^j - \lambda_7^j = 0 \tag{A10}$$

$$\frac{\partial L^j}{\partial P_{IL}^j} = C_{IL}^j - \lambda_5^j + \lambda_6^j - \lambda_7^j = 0 \tag{A11}$$

Primal feasibility:

Eqs. (A1)–(A7) represent the primal feasibility conditions.

Dual feasibility:

$$\lambda_i^j \geqslant 0 \quad \forall i = 1, 2, \ldots, 6 \quad \forall j \tag{A12}$$

$$\lambda_7^j \quad \forall j \quad \text{Unrestricted} \tag{A13}$$

As Eqs. (A1)–(A6) are greater than or equal to zero, the respective dual variables (lagrangian multipliers) are in the same form. Also, as Eq. (A7) is equal constraint, the respective dual variable is unrestricted in sign.

Complementary slackness:

$$C_i^j \lambda_i^j = 0 \quad \forall i = 1, 2, \ldots, 6 \quad \forall j \tag{A14}$$

As these constraints are nonlinear, they are replaced with two sets of the linear constraints as follows:

$$C_i^j \leqslant MU_i^j \quad \forall i = 1, 2, \ldots, 6 \quad \forall j \tag{A15}$$

$$\lambda_i^j \leqslant M(1 - U_i^j) \quad \forall i = 1, 2, \ldots, 6 \quad \forall j \tag{A16}$$

where M is sufficiently large positive constant.

Therefore, each follower problem (5)–(9) can be equivalently replaced by constraints (6)–(9), (A9)–(A13), (A15) and (A16).

For FM2, the transformation is similar to (A1)–(A16) and only the term ρ^D is replaced with ρ_j^D in related equations and the other equations are the same as described for FM1.

Appendix B

In this appendix, the nonlinear expression $\rho_D^j P_D^j$ in Eq. (1) is replaced with linear expressions. At first, according to the dual theory, the dual problem of each follower problem (5)–(9) is constructed as follows:

$$\text{Maximize} \quad -P_{\max}^{T_j}\lambda_1^j - P_{\max}^{T_j}\lambda_2^j + P_{DG,\min}^j\lambda_3^j - P_{DG,\max}^j\lambda_4^j - P_{IL,\max}^j\lambda_6^j + P_{demand}^j\lambda_7^j \tag{B1}$$

$$\lambda_1^j - \lambda_2^j + \lambda_7^j = \rho_D^j \quad \forall j \tag{B2}$$

$$\lambda_3^j - \lambda_4^j + \lambda_7^j \leqslant C_{DG}^j \quad \forall j \tag{B3}$$

$$\lambda_5^j - \lambda_6^j + \lambda_7^j \leqslant C_{IL}^j \quad \forall j \tag{B4}$$

Based on the strong duality theory $P_D^j, P_{DG}^j, P_{IL}^j$ are optimal solutions of the primal problem and $\lambda_1^j, \lambda_2^j, \lambda_3^j, \lambda_4^j, \lambda_5^j, \lambda_6^j, \lambda_7^j$ are optimal solutions of the dual problem if and only if:

$$\rho_D^j P_D^j + C_{DG}^j P_{DG}^j + C_{IL}^j P_{IL}^j = -P_{\max}^{T_j}\lambda_1^j - P_{\max}^{T_j}\lambda_2^j + P_{DG,\min}^j\lambda_3^j - P_{DG,\max}^j\lambda_4^j - P_{IL,\max}^j\lambda_6^j + P_{demand}^j\lambda_7^j \tag{B5}$$

Eq. (B5) can be rewritten as follows:

$$\rho_D^j P_D^j = -C_{DG}^j P_{DG}^j - C_{IL}^j P_{IL}^j - P_{\max}^{T_j}(\lambda_1^j + \lambda_2^j) + P_{DG,\min}^j\lambda_3^j - P_{DG,\max}^j\lambda_4^j - P_{IL,\max}^j\lambda_6^j + P_{demand}^j\lambda_7^j \tag{B6}$$

Therefore, the nonlinear expression $\rho_D^j P_D^j$ is replaced with linear ones. The right hand side of Eq. (B6) is replaced with $\rho_D^j P_D^j$ in Eq. (1) and is rewritten as Eq. (10).

For FM2, the linearization is similar to (B1)–(B6) and only the term ρ^D is replaced with ρ_D^j in related equations and the other equations are the same as described for FM1.

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