The Role of “Frequency” in Low-Inertia Power Systems

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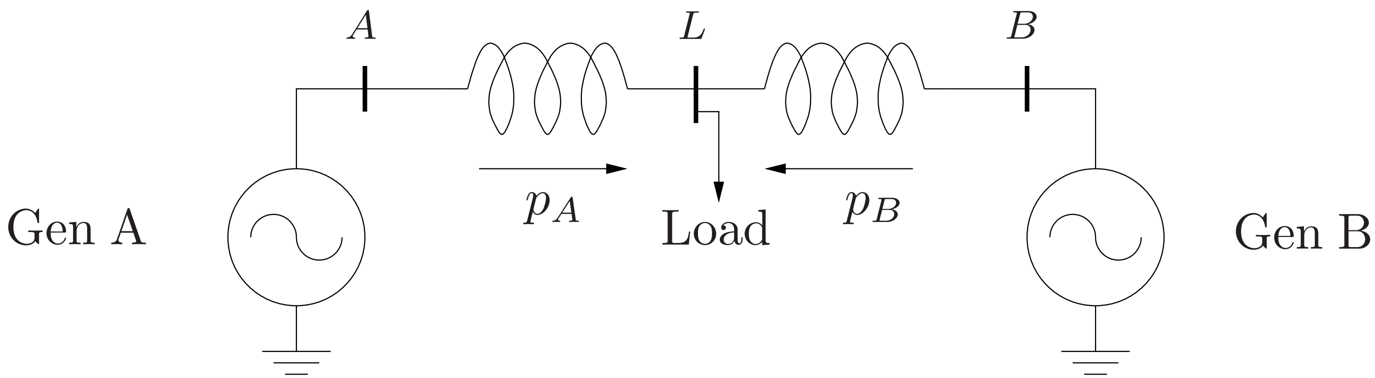
Contents

Motivations

Preamble

- Let's consider the two-machine system:



- Conventional Model:

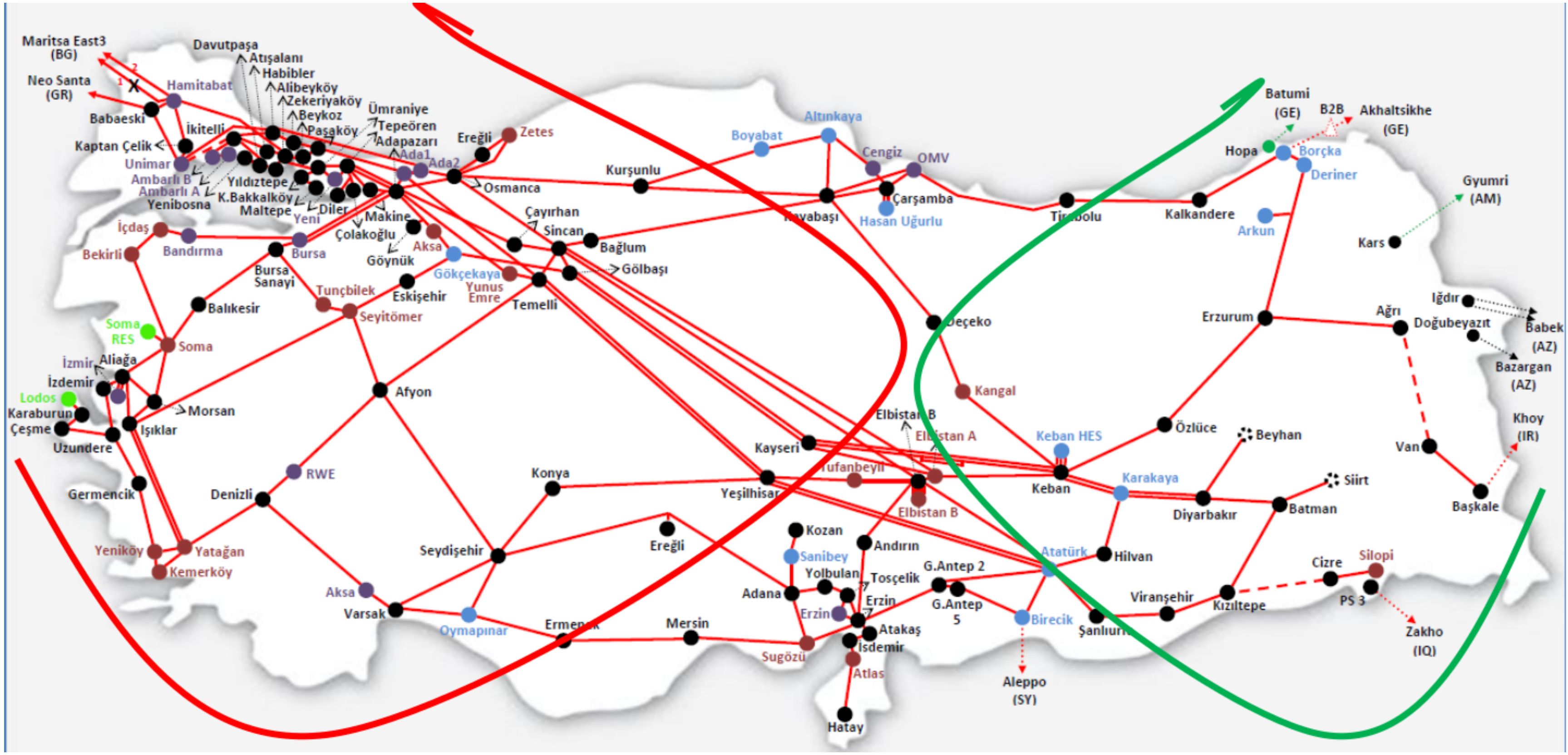
$$\frac{d\delta_A}{dt} = \omega_n(\omega_A - \omega_s) \qquad \frac{d\delta_B}{dt} = \omega_n(\omega_B - \omega_s)$$

$$\frac{d\omega_A}{dt} = \frac{1}{M_A}(p_{mA} - p_A) \qquad \frac{d\omega_B}{dt} = \frac{1}{M_B}(p_{mB} - p_B)$$

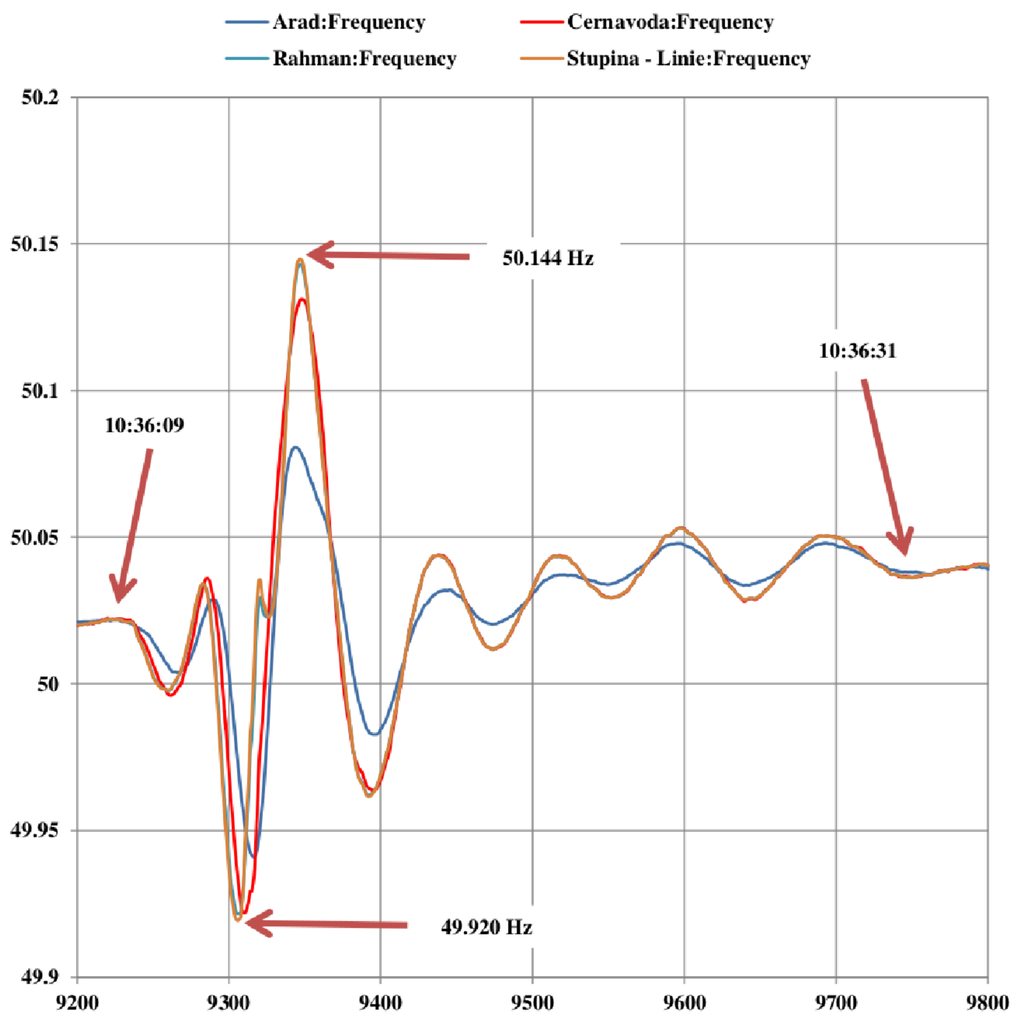
$$p_A = \frac{e'_A v_L}{x'_{dA} + x_{\mathrm{AL}}} \sin(\delta_A - \delta_L) \qquad p_B = \frac{e'_B v_L}{x'_{dA} + x_{\mathrm{BL}}} \sin(\delta_B - \delta_L)$$

Turkey Blackout on 31st of March 2015 – I

- The blackout in Turkey led to the outage of 32 GW.



Turkey Blackout on 31st of March 2015 – II



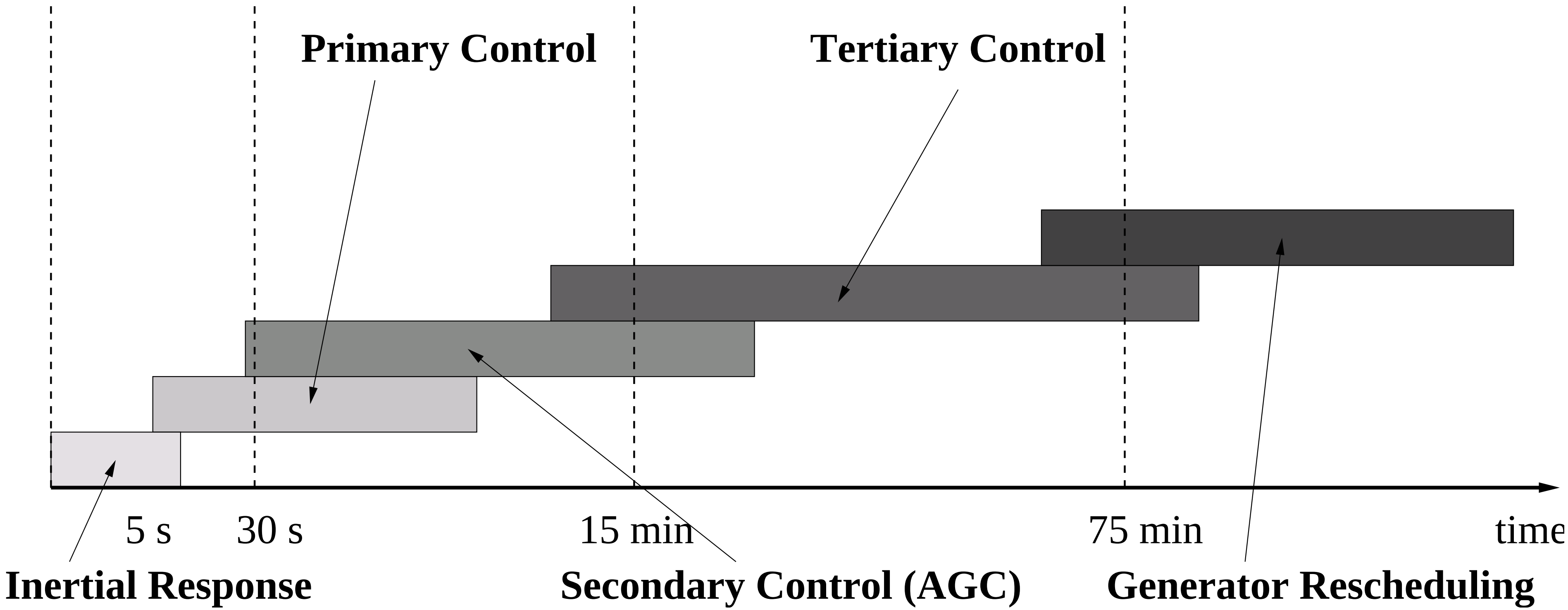
- As a consequence of the line outages and the blackout in Turkey, the Romanian system experimented severe frequency oscillations.
- Bigger oscillations were measured at locations geographically closer to Turkey.

Challenges

- The electric power system is currently undergoing a period of unprecedented changes
- This transition involves the major challenge of substituting synchronous machines with power electronics-interfaced generation (CIG)
- The regulation and interaction with the rest of the system of CIG is yet to be fully understood!

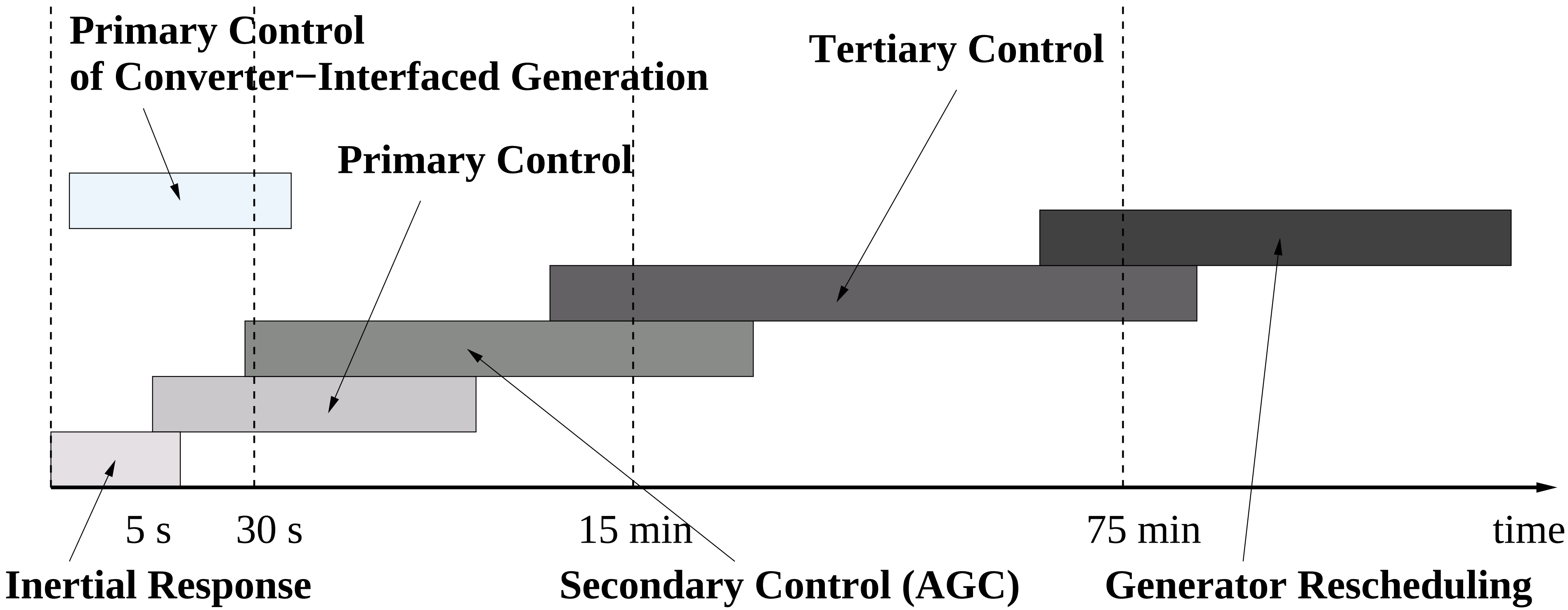
Time scales

- Typical time scales related to inertia and frequency control



Time Scales of a Conventional Power System & CIG

- CIG controllers can be fast (is this good?)



Electro-mechanical Dynamics – I

- Neglecting network topology, a conventional system where generation is attained with synchronous generation can be represented as

$$M\dot{\omega}_{\text{COI}}(t) = p_{\mathrm{s}}(t) - p_{\mathrm{l}}(t) - p_{\mathrm{j}}(t)\,,$$

  where

  - M is the total inertia of the synchronous machines
  - $\omega_{\text{COI}}(t)$ is the average frequency of the system
  - $\dot{\omega}(t)$ is called Rate of Change of Frequency (RoCoF)
  - p_{s} is the power of synchronous machines
  - $p_{\mathrm{l}} + p_{\mathrm{j}}$ are load demand and losses respectively.

Center of Inertia

- The *center of inertia* (COI) is a weighted arithmetic average of the rotor speeds of synchronous machines that are connected to a transmission system:

$$\omega_{\text{COI}} = \frac{\sum_{j \in \mathcal{G}} H_j \omega_j}{\sum_{j \in \mathcal{G}} H_j}$$

  where ω_j and H_j are the rotor speed and the inertia constant, respectively, of the synchronous machine j and $\mathcal{G}$ is the set of synchronous machines belonging to a given cluster.

Electro-mechanical Dynamics – II

- A system where generation is attained with synchronous as well as non-synchronous generation can be represented as

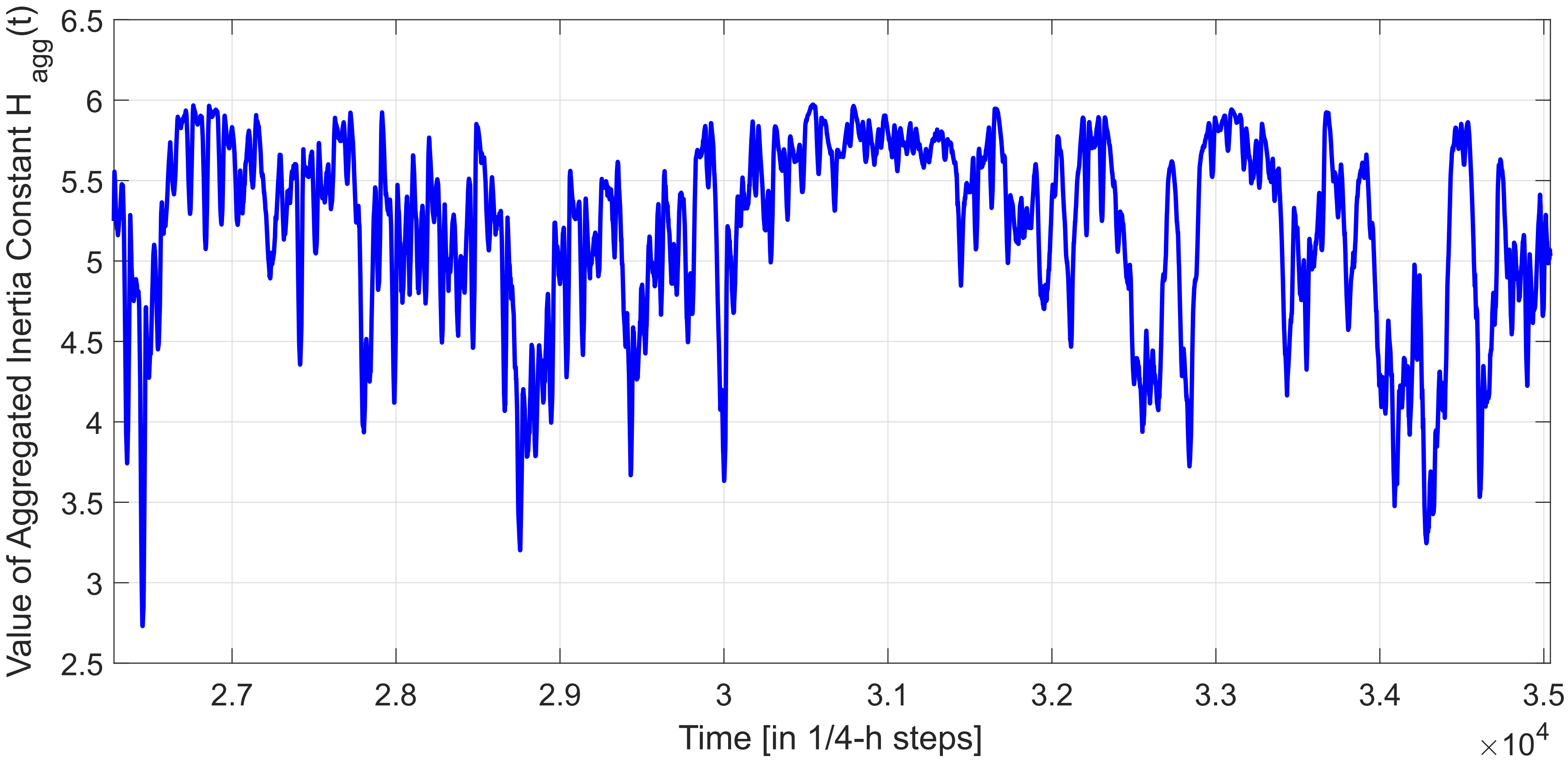
$$\tilde{M}\dot{\omega}_{\text{COI}}(t) = p_{\rm s}(t) + p_{\rm ns}(t) - p_{\rm l}(t) - p_{\rm j}(t) \, ,$$

where

  - $\tilde{M}$ is the total inertia of the synchronous machines, with $\tilde{M} < M$ or, in certain periods and certain systems, $\tilde{M} \ll M$
  - $p_{\rm ns}$ is the powers provided by CIG

Volatility of the inertia

- $\tilde{M}$ is a function of time!



Acknowledgment: Thanks to A. Ulbig and G. Andersson for data and script to generate figure

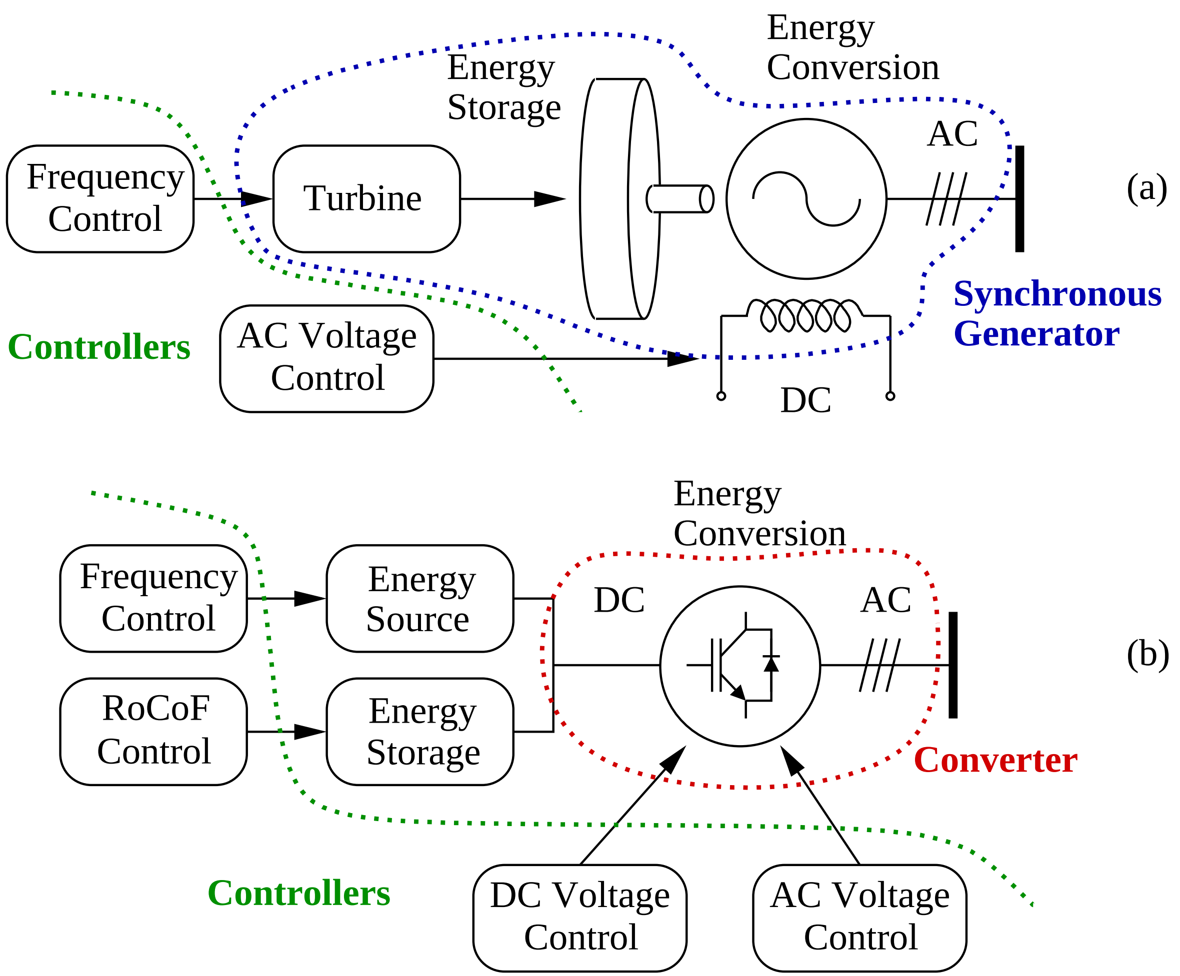
Extreme Case

- In a hypothetical system where there are no synchronous machines at all, $M \approx 0$ and the frequency is completely decoupled from the power balance of the system:

$$0 = p_{\mathrm{ns}}(t) - p_{\mathrm{l}}(t) - p_{\mathrm{j}}(t)$$

- This opertaing condition has never really happened in large networks (only in microgrids and small islanded systems)
- In this case, is still the frequency meaningful?

Analogy between Synchronous Machine and CIG

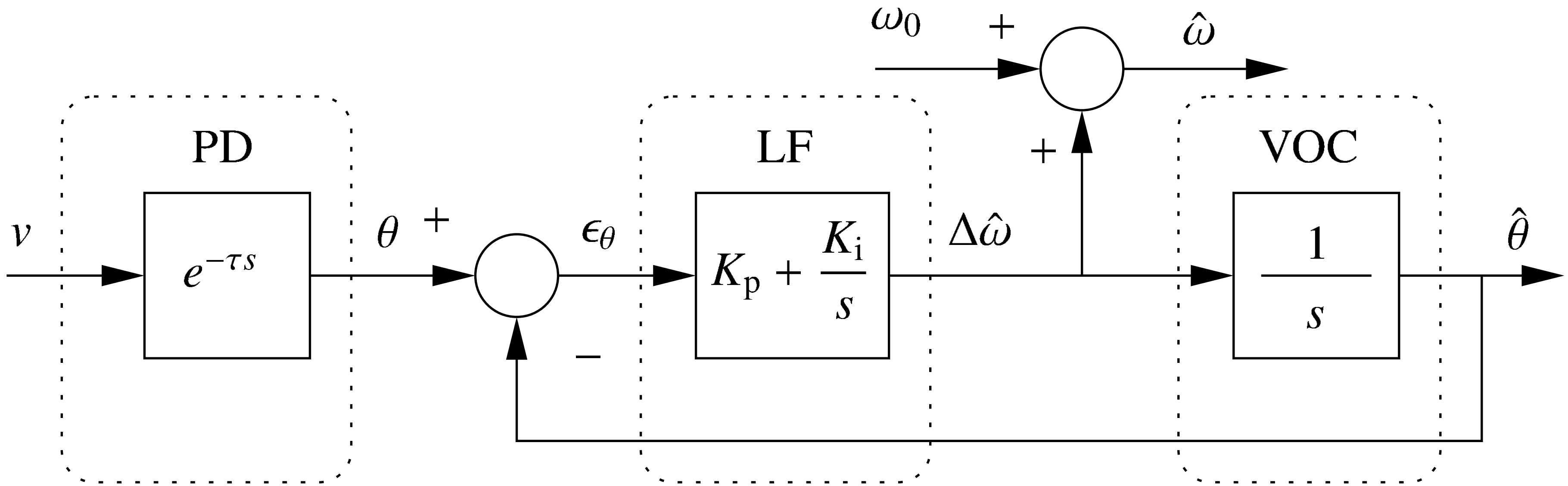


Drawbacks of CIG

- Reduce the inertia
- The local frequency must be measured (and properly defined) first!
- Often introduce volatility and uncertainty (e.g., wind and solar power plants)
- Often do not provide primary and/or secondary frequency control
- Since it is based on converter, its control can be potentially very fast

Phase-Locked Loop

- The phase-locked loop (PLL) is the most common device utilized in power converters to track the phase of the ac voltage at the bus where the converter is connected.
- It is composed of a phase detector (PD); a loop filter (LF); and and a voltage oscillator control (VOC).



Advantages of CIG

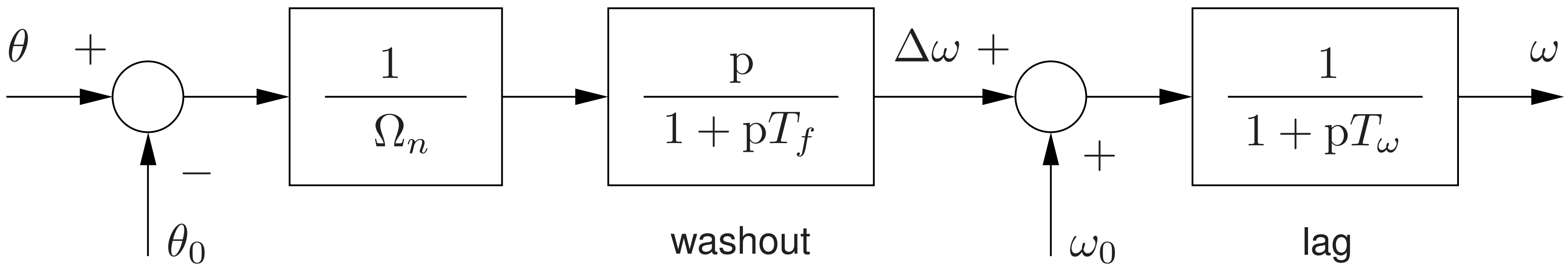
- Can provide primary and secondary control (if the resources are properly handled and/or storage is included)
- Quantities other than the frequency can be utilized (voltage?)
- Since it is based on converter, its control can be potentially very fast

Modelling Issues – I

- The conventional power system model for transient stability analysis is based on the assumption of quasi-steady-state phasors for voltages and currents.
- The crucial hypothesis on which such a model is defined is that the frequency required to define all phasors and system parameters is constant and equal to its nominal value.
- This model is appropriate as long as only the rotor speed variations of synchronous machines is needed to regulate the system frequency through standard primary and secondary frequency regulators.

Derivative of the Bus Voltage Phase Angle (θ)

- The frequency estimation is obtained by means of a washout and a low-pass filter.
- The washout filter approximates the derivative of the input signal.
- $T_f = 3/\Omega_n$ s and $T_\omega = 0.05$ s are used as default values for all simulations.



Modelling Issues – II

- In recent years, however, an increasing number of devices other than synchronous machines are expected to provide frequency regulation.
- These include, among others:
  - distributed energy resources, e.g., wind and solar generation
  - flexible loads providing load demand response
  - HVDC transmission systems
  - energy storage devices
- However, these devices do not impose the frequency at their connection point with the grid.
- There is thus the need to define with accuracy the local frequency at every bus of the network.

Frequency Divider Formula

Derivation of the FDF

- The very starting point is the augmented admittance matrix, with inclusion of synchronous machine internal impedances as it is commonly defined for fault analysis.
- System currents and voltages are linked as follows:

$$\begin{bmatrix} \bar{\boldsymbol{i}}_{\mathrm{G}} \\ \bar{\boldsymbol{i}}_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{Y}}_{\mathrm{GG}} & \bar{\mathbf{Y}}_{\mathrm{GB}} \\ \bar{\mathbf{Y}}_{\mathrm{BG}} & \bar{\mathbf{Y}}_{\mathrm{BB}} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{e}}_{\mathrm{G}} \\ \bar{\boldsymbol{v}}_{\mathrm{B}} \end{bmatrix} \tag{1}$$

where $\bar{\boldsymbol{v}}_{\mathrm{B}}$ and $\bar{\boldsymbol{i}}_{\mathrm{B}}$ are bus voltages and current injections, respectively, at network buses; $\bar{\boldsymbol{i}}_{\mathrm{G}}$ are generator current injections; $\boldsymbol{e}_{\mathrm{G}}$ are generator emfs behind the internal generator impedance; $\bar{\mathbf{Y}}_{\mathrm{BB}}$ is the standard network admittance matrix plus a diagonal matrix that accounts for the internal impedances of the synchronous machines at generator buses, i.e., $\bar{\mathbf{Y}}_{\mathrm{BB}} = \bar{\mathbf{Y}}_{\mathrm{bus}} + \bar{\mathbf{Y}}_{\mathrm{G0}}$; $\bar{\mathbf{Y}}_{\mathrm{GG}}$, $\bar{\mathbf{Y}}_{\mathrm{GB}}$ and $\bar{\mathbf{Y}}_{\mathrm{BG}}$ are admittance matrices obtained using the internal impedances of the synchronous machines.

Nodal Equations

- To further elaborate on (1), let us assume that load current injections $\bar{\boldsymbol{i}}_{\mathrm{B}}$ can be neglected in (1).
- Let's rewrite (1) as follows:

$$\begin{bmatrix} \bar{\boldsymbol{i}}_{\mathrm{G}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{Y}}_{\mathrm{GG}} & \bar{\mathbf{Y}}_{\mathrm{GB}} \\ \bar{\mathbf{Y}}_{\mathrm{BG}} & \bar{\mathbf{Y}}_{\mathrm{BB}} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{e}}_{\mathrm{G}} \\ \bar{\boldsymbol{v}}_{\mathrm{B}} \end{bmatrix} \tag{2}$$

- Bus voltages $\bar{\boldsymbol{v}}_{\mathrm{B}}$ are thus a function of generator emfs and can be computed explicitly:

$$\bar{\boldsymbol{v}}_{\mathrm{B}} = -\bar{\mathbf{Y}}_{\mathrm{BB}}^{-1} \bar{\mathbf{Y}}_{\mathrm{BG}} \bar{\boldsymbol{e}}_{\mathrm{G}} = \bar{\mathbf{D}}\, \bar{\boldsymbol{e}}_{\mathrm{G}} \tag{3}$$

Time Derivative in the dq-frame – I

- Let's consider the time derivative – indicated with the functional $\mathrm{p}(\cdot)$ – of the bus voltage phasors in a dq-frame rotating with frequency ω_0:

$$\mathrm{p}\bar{v}_{\mathrm{dq},h} = \frac{d}{dt}\bar{v}_{\mathrm{dq},h} + j\omega_0\bar{v}_{\mathrm{dq},h} \tag{4}$$

where $\bar{v}_{\mathrm{dq},h} = v_{\mathrm{d,h}} + jv_{\mathrm{q,h}}$.

- Assuming "slow" electromechanical transient, (4) can be approximated as:

$$\mathrm{p}\,\bar{v}_h \approx j\,\omega_h\,\bar{v}_h \tag{5}$$

where $\Delta\omega_h = \omega_0 + \Delta\omega_h$ is the frequency at bus h.

Other Approximations

- The following approximations and assumptions are applied:
  - $\bar{\boldsymbol{v}}_{\mathrm{B}} \approx \mathbf{1}$ pu and $\bar{\boldsymbol{e}}_{\mathrm{G}} \approx \mathbf{1}$ pu;
  - The conductances of the elements of all admittance matrices utilized to compute $\bar{\mathbf{D}}$ are negligible, e.g., $\bar{\mathbf{Y}}_{\mathrm{BB}} \approx j\mathbf{B}_{\mathrm{BB}}$;
- Finally, let us define bus abd generator frequency variations as:

$$
\begin{aligned}
\Delta\boldsymbol{\omega}_{\mathrm{B}} &= \boldsymbol{\omega}_{\mathrm{B}} - \omega_0 \cdot \mathbf{1} \\
\Delta\boldsymbol{\omega}_{\mathrm{G}} &= \boldsymbol{\omega}_{\mathrm{G}} - \omega_0 \cdot \mathbf{1}
\end{aligned}
\tag{6}
$$

where, usually, $\omega_0 = 1$ pu.

Frequency Divider Formula

- After applying all approximations above, we obtain again the **frequency divider formula**:

$$\boxed{\mathbf{B}_{\rm BB}\,\Delta\boldsymbol{\omega}_{\rm B} = -\mathbf{B}_{\rm BG}\Delta\boldsymbol{\omega}_{\rm G}} \tag{7}$$

  or, alternatively:

$$\boxed{\Delta\boldsymbol{\omega}_{\rm B} = \mathbf{D}\,\Delta\boldsymbol{\omega}_{\rm G}} \tag{8}$$

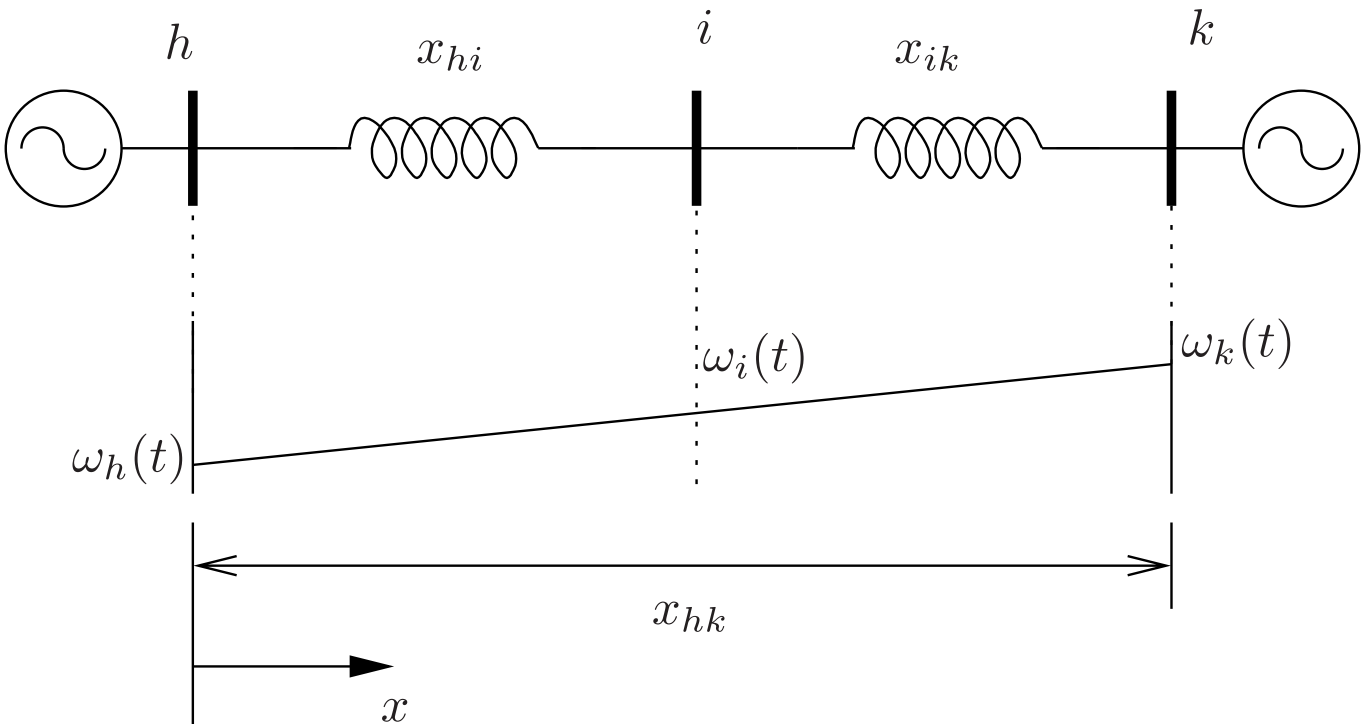
  were $\mathbf{D} = -\mathbf{B}_{\rm BB}^{-1}\mathbf{B}_{\rm BG}$.

- The latter formula has the same formal structure of voltage dividers in resitive dc circuits, hence the proposed name.

Illustrative Example

Radial System – I

- Let assume a lossless connection, with total reactance $x_{hk} = x_{hi} + x_{ik}$.
- The frequencies at buses h and k, say ω_h and ω_k, respectively, are the rotor speeds of the synchronous generators.



Radial System – II

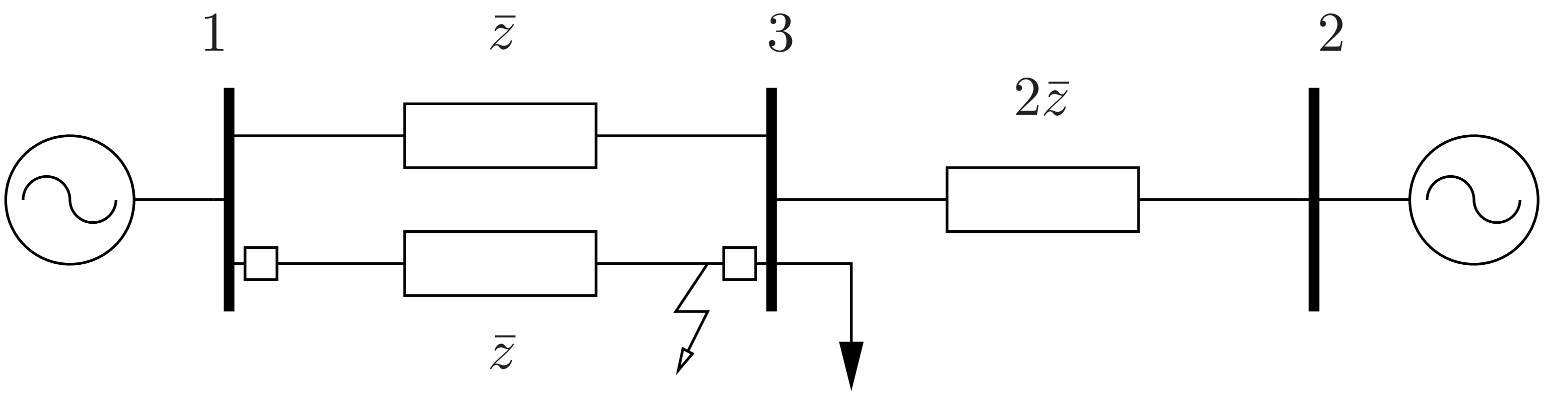
- Applying the frequency divider formula (8), we obtain:

$$\begin{aligned}\omega_i(t) &= \mathbf{D} \cdot \begin{bmatrix}\omega_h(t) \\ \omega_k(t)\end{bmatrix} = -\mathbf{B}_{\text{BB}}^{-1}\mathbf{B}_{\text{BG}} \cdot \begin{bmatrix}\omega_h(t) \\ \omega_k(t)\end{bmatrix} \\ &= \left[\frac{1}{x_{hi}} + \frac{1}{x_{ik}}\right]^{-1} \begin{bmatrix}\dfrac{1}{x_{hi}} & \dfrac{1}{x_{ki}}\end{bmatrix} \cdot \begin{bmatrix}\omega_h(t) \\ \omega_k(t)\end{bmatrix} \\ &= \frac{x_{ik}}{x_{hk}} \cdot \omega_h(t) + \frac{x_{hi}}{x_{hk}} \cdot \omega_k(t)\end{aligned} \tag{9}$$

- The instantaneous frequency $\omega_i(t)$ at a generic point i between the boundaries h and k is a linear interpolation between $\omega_h(t)$ and $\omega_k(t)$.

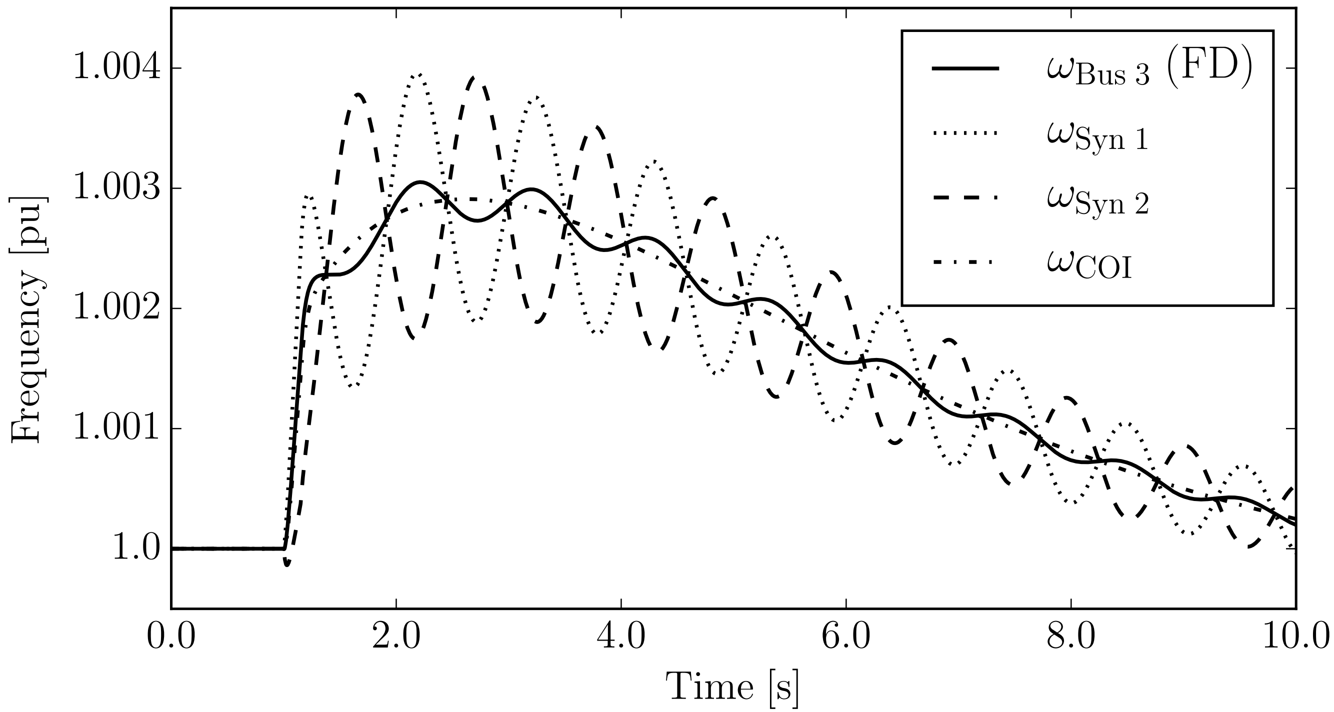
Example – I

- Let's first consider a standard model for transient stability analysis where transmission lines are lumped and modeled as constant impedances and generator flux dynamics are neglected.
- Generators are equal and are modeled as a 6$^{\text{th}}$ order synchronous machine with AVRs and turbine governors.
- The load is modeled as a constant admittance. The disturbance is a three-phase fault that occurs at bus 3 at $t = 1$ s and is cleared after 150 ms by opening one of the two lines connecting buses 1 and 3.



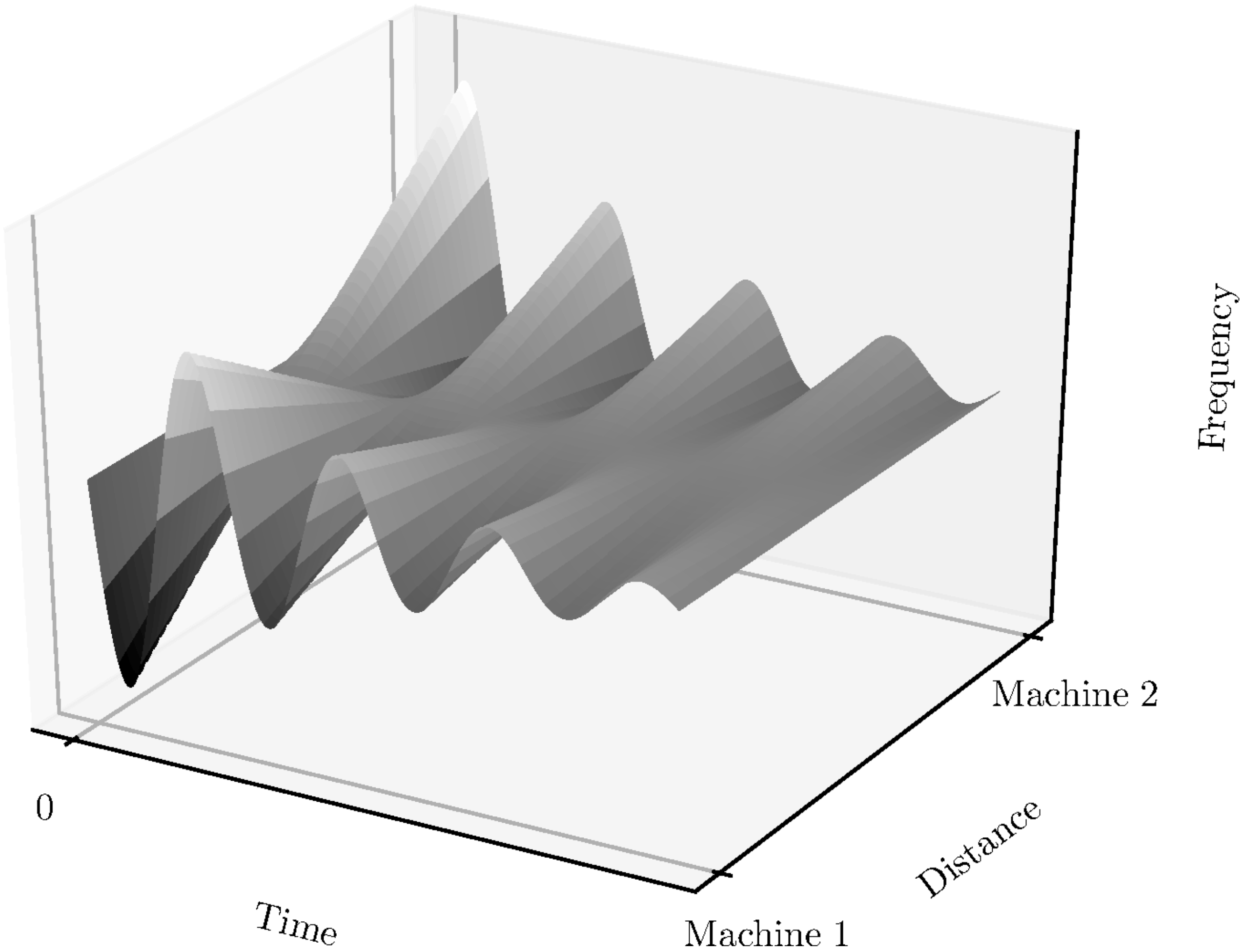
Example – II

- Transient behavior of synchronous machine rotor speeds, the frequency of the COI (ω_{COI}), and the estimated frequency at the load bus using the proposed frequency divider approach.



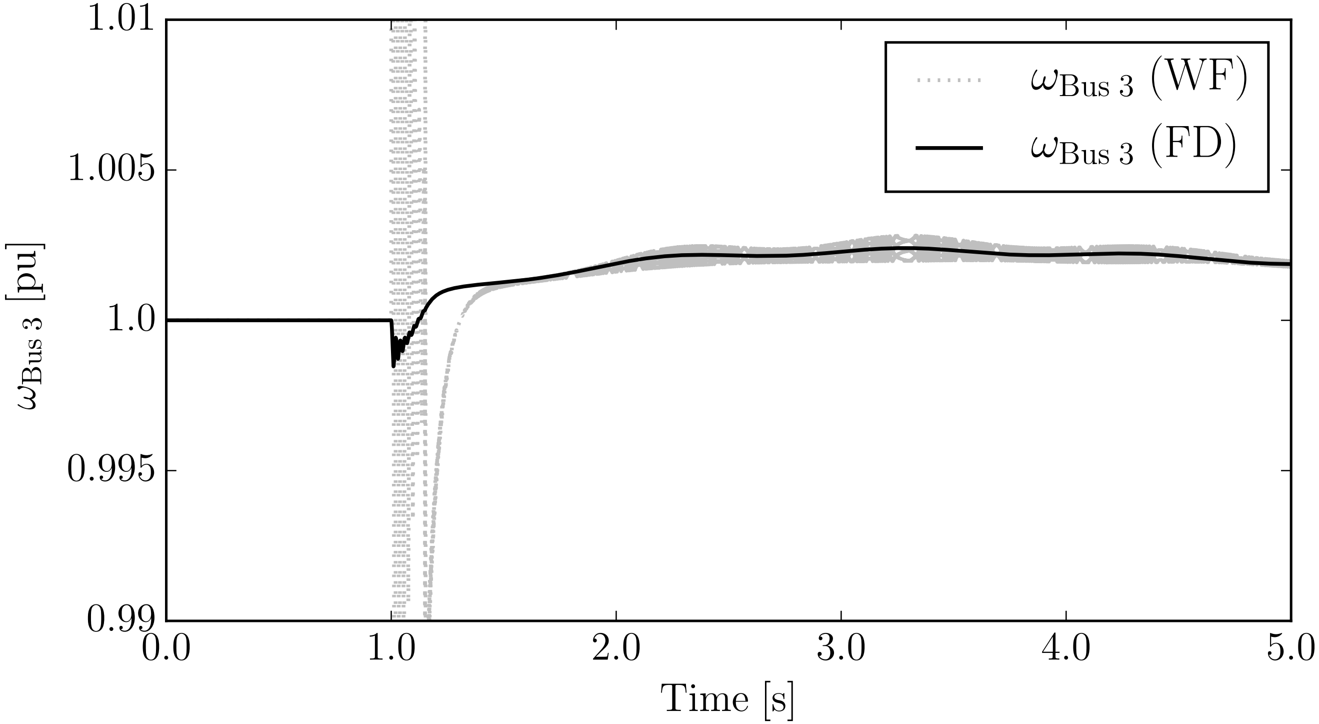
Example – III

- Graphical representation of the frequency divider.



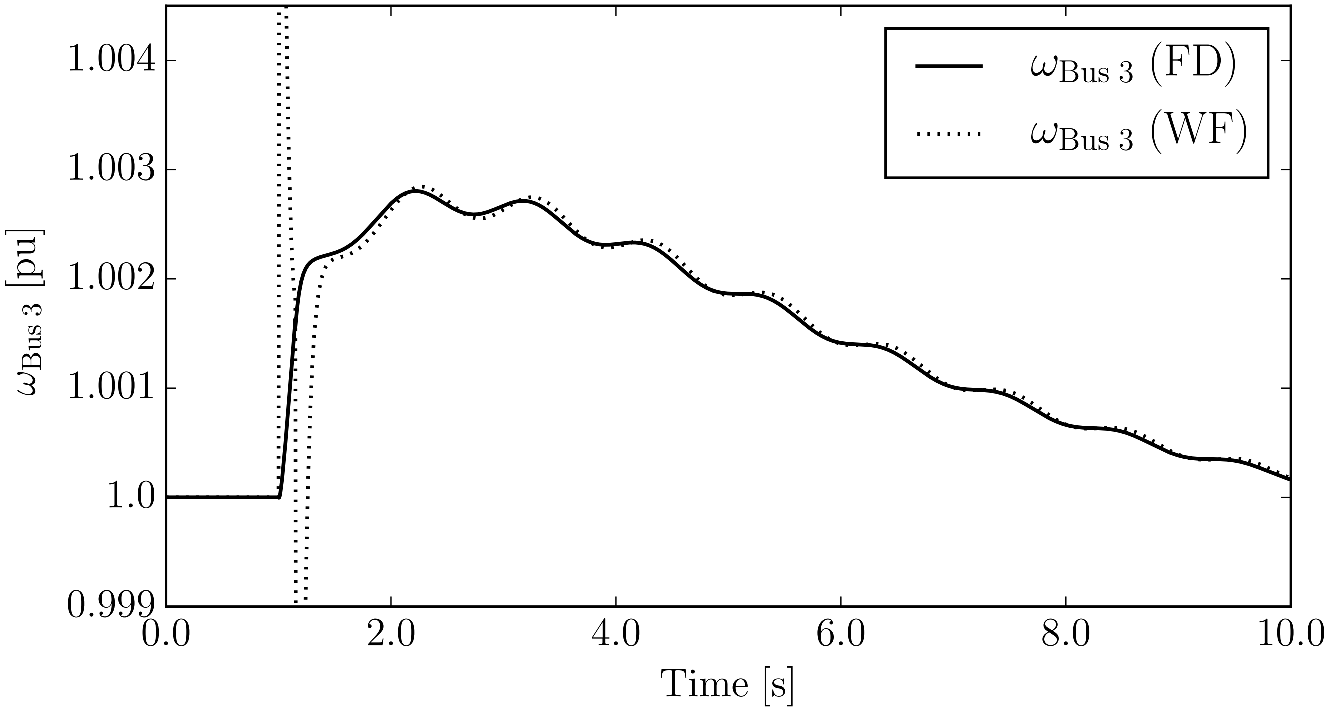
Example – IV

- Frequency at bus 3 estimated with the frequency divider (FD) and the conventional washout filter (WF). The system is simulated using the fully-fledged dq-axis model.



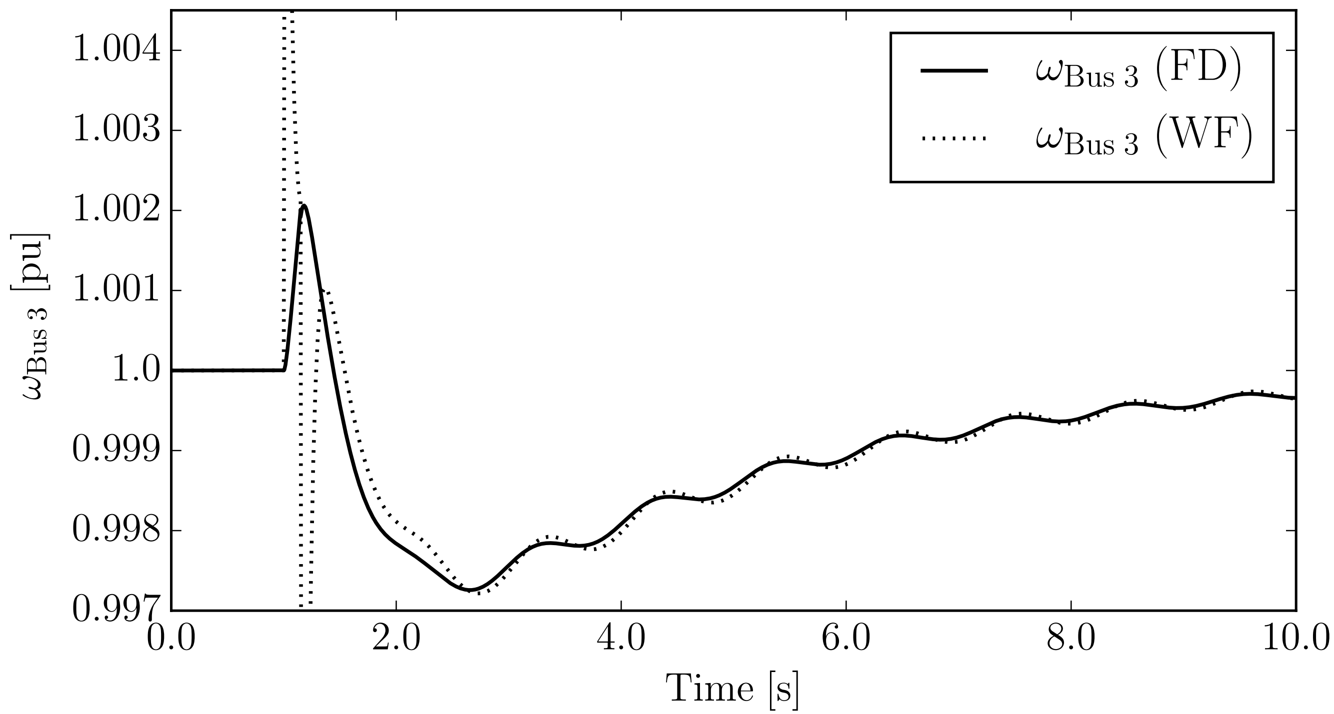
Example – V

- Frequency at bus 3 estimated with the frequency divider (FD) and the conventional washout filter (WF). The load is modelled as a frequency-dependent load representing an aluminum plant



Example – VI

- Frequency at bus 3 estimated with the frequency divider (FD) and the conventional washout filter (WF). The load is a squirrel cage induction motor with a 5th-order dq-axis model.



Properties of Matrix $\mathbb{D}$

Physical Meaning of Matrix D – I

- Let's look again at the FDF:

$$\Delta\boldsymbol{\omega}_{\mathrm{B}} = \mathbf{D}\Delta\boldsymbol{\omega}_{\mathrm{G}}$$

where

$$\Delta\boldsymbol{\omega}_{\mathrm{B}} = \boldsymbol{\omega}_{\mathrm{B}} - \mathbf{1}_n$$
$$\Delta\boldsymbol{\omega}_{\mathrm{G}} = \boldsymbol{\omega}_{\mathrm{G}} - \mathbf{1}_m$$

- We can think of the element $D_{i,j}$ matrix $\mathbf{D}$ as the participation factor of rotor speed ω_{Gj} to bus frequency ω_{Bi}.

Physical Meaning of Matrix D – II

- We know that $\mathbf{D}$ is dense . . .
  - Let d_{D} be the density index of matrix $\mathbf{D}$, such that:

$$d_{\mathrm{D}} = 100 \cdot \frac{\mathrm{NNZ}(\mathbf{D})}{(m \cdot n)}$$

    where $\mathrm{NNZ}(\mathbf{D})$ is the number of non-zero elements of $\mathbf{D}$.
  - Since $\mathbf{D}$ is dense, $d_{\mathrm{D}} \approx 100\%$.
- ... hence, in principle, all machine rotor speeds participate to all bus frequencies.
- This conclusion is mathematically correct but . . .
- . . . it clahses with common sense: is it possible that a machine in Poland affects the frequency of a bus in Spain?

Relevant Property of Matrix $\mathbf{D}$

- It can be shown that the sum of the elements of each row of $\mathbf{D}$ is:

$$\sum D_i \approx 1, \quad \forall i = 1, \ldots, n$$

- The key point is that not all elements of each row weight in the same way.
- The intuition would suggest that a generator in Poland participate *more* to the frequency deviations of buses in Poland, and *less* to the buses in Spain.

$\mathbf{D}_{\mathrm{r}}$ – Reduced $\mathbf{D}$ – I

- Let's sort the elements of each row i of $\mathbf{D}$ in descending order.
- The first, and thus the biggest k_i elements of each row of the sorted matrix $\tilde{\mathbf{D}}$ are summed such that:

$$\sum_{h=1}^{k_i} \tilde{D}_{i,h} < \alpha_{\mathrm{D}} \sigma_{\mathrm{D},i}$$

  where $\alpha_{\mathrm{D}} \in [0, 1]$ is a given threshold.
- Finally, the reduced matrix $\mathbf{D}_{\mathrm{r}}$ is obtained by setting to zero all elements $\tilde{D}_{i,\mathcal{H}}$ with $\mathcal{H} = k_i + 1, \ldots, n$, and rearranging $\tilde{D}_{i,h\cup\mathcal{H}}$ according to their original positions before the sorting, i.e., $D_{i,j}$.

$\mathbf{D}_\mathrm{r}$ – Reduced $\mathbf{D}$ – II

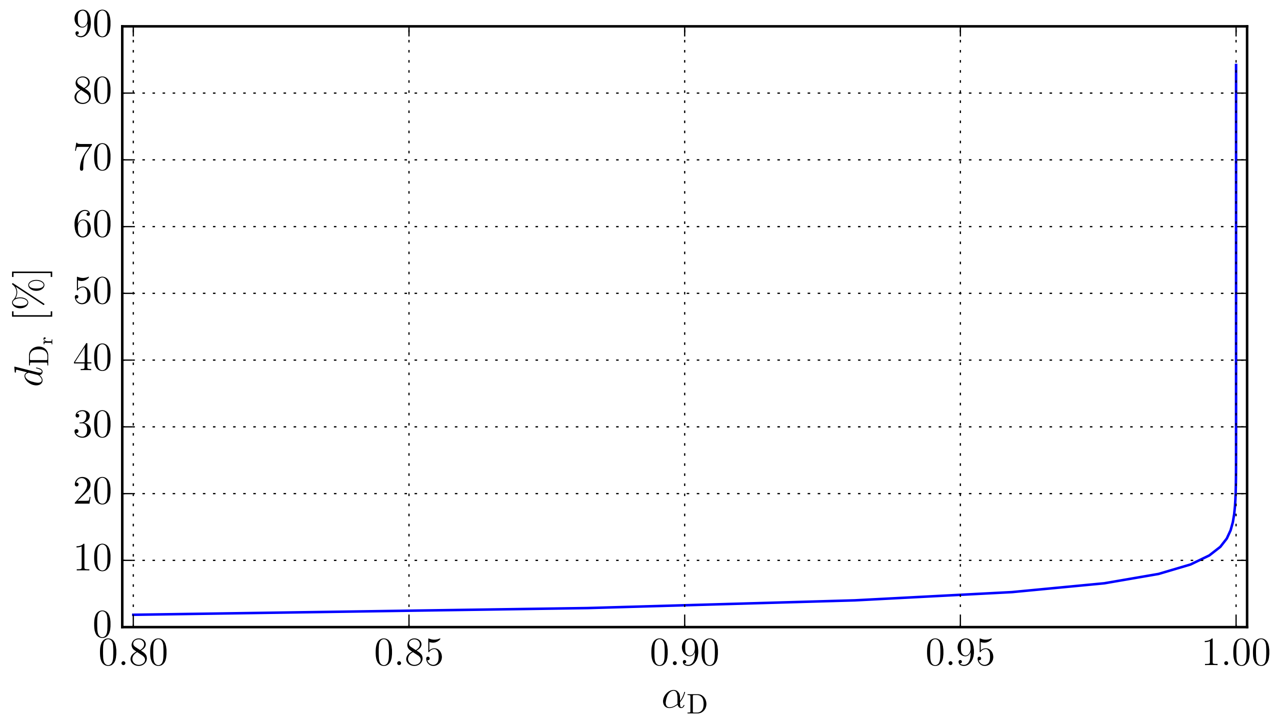
- Limits cases for $\mathbf{D}_\mathrm{r}$ are as follows:

$$\mathbf{D}_\mathrm{r} = \begin{cases} \mathbf{0}\,, & \text{if } \alpha_\mathrm{D} = 0\,; \\ \mathbf{D}\,, & \text{if } \alpha_\mathrm{D} = 1\,. \end{cases}$$

- The main property of $\mathbf{D}_\mathrm{r}$ is that it guarantees at least the specified accuracy α_D.
- However, sorting each row can be computationally demanding is the system is large.

Example 2: ENTSO-E System

- 21,177 buses, 30,968 branches, 15,756 loads, and 4,832 power plants.
- Density of matrix $\mathbf{D}_{\mathrm{r}}$ of the ENTSO-E transmission system for $\alpha_{\mathrm{D}} \in [0.8, 1]$.



Dynamic State Estimation

Dynamic State Estimation

- In practice, a very reduced number of rotor speeds are needed to estimate a given bus frequency.
- Typically, required rotor speeds are also those of the generators that are geographically close.
- A dynamic state estimation problem based on the FDF is presented in:

J. Zhao, L. Mili, F. Milano, “Robust Frequency Divider for Power System Online Monitoring and Control,” *IEEE Transactions on Power Systems*, accepted on December 2017, in press.

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- However, the problem is to have a good estimation of $\boldsymbol{\omega}_G \dots$

Dynamic State Estimation – Continued

- What the TSO really knows is not $\boldsymbol{\omega}_{\mathrm{G}}$, but $\boldsymbol{\omega}_{\mathrm{B}}$.
- In fact, the TSO can install PMUs on (virtually) every node of the system but generator rotor speed are not accessible (in general)
- Is it possible to use the FDF to estimate $\boldsymbol{\omega}_{\mathrm{G}}$ given $\boldsymbol{\omega}_{\mathrm{B}}$?
- Matrix $\mathbf{D}$ is not square, it is actually a $n \times m$, matrix, with $n \gg m$ (there are many more buses than generators).

Pseudo Inverse $\mathbf{D}^{+}$

- For this kind of problems we can use the Moore-Penrose pseudo-inverse:

$$\Delta\boldsymbol{\omega}^{*}_{\mathrm{G}} = (\mathbf{D}^{T}\mathbf{D})^{-1}\mathbf{D}^{T}\Delta\tilde{\boldsymbol{\omega}}_{\mathrm{B}} = \mathbf{D}^{+}\Delta\tilde{\boldsymbol{\omega}}_{\mathrm{B}} \,, \tag{10}$$

  where $\Delta\tilde{\boldsymbol{\omega}}_{\mathrm{B}}$ are the measures of the bus frequency deviations and $\Delta\boldsymbol{\omega}^{*}_{\mathrm{G}}$ the estimated rotor speed deviations.

- $\mathbf{D}^{+}$ is unique if $\mathbf{D}$ has rank m.

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Pseudo Inverse $\mathbf{D}^+$

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- It turns out that (12) is the solution of a weighted least square problem (with independent measures).
- So this is actually a classical (and linear) state estimation problem!

Pseudo Inverse $\mathbf{D}^+$

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- $\mathbf{D}^+$ is unique if $\mathbf{D}$ has rank m.
- It turns out that (12) is the solution of a weighted least square problem (with independent measures).
- So this is actually a classical (and linear) state estimation problem!
- $\Delta\boldsymbol{\omega}^*_{\mathrm{G}}$ is also the *optimal* value of the rotor speed estimations.

Density of $\mathbb{D}^+$

- At this point, one may ask whether $\mathbb{D}^+$ is dense or not.

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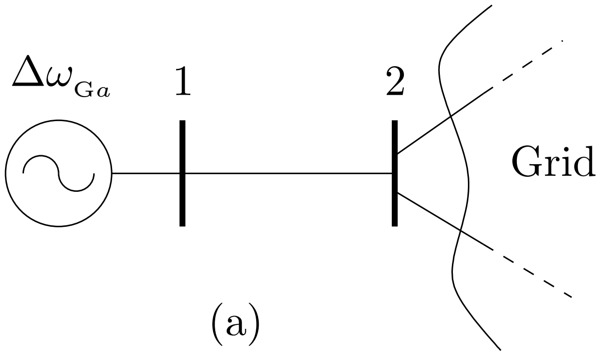
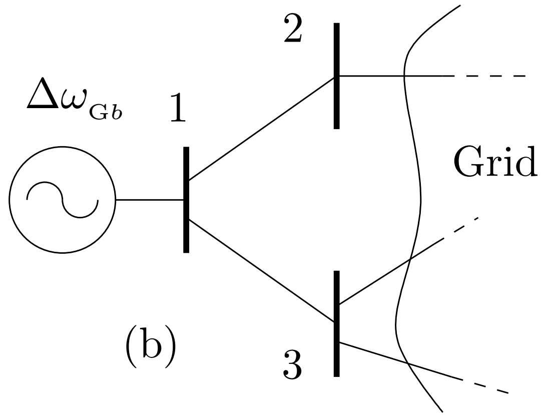
- At this point, one may ask whether $\mathbf{D}^+$ is dense or not.
- If it is dense, in fact, the state estimation problem (12) would be useless ...
- Surprisingly, even if $\mathbf{D}$ is dense, $\mathbf{D}^+$ is extremely sparse!

Examples



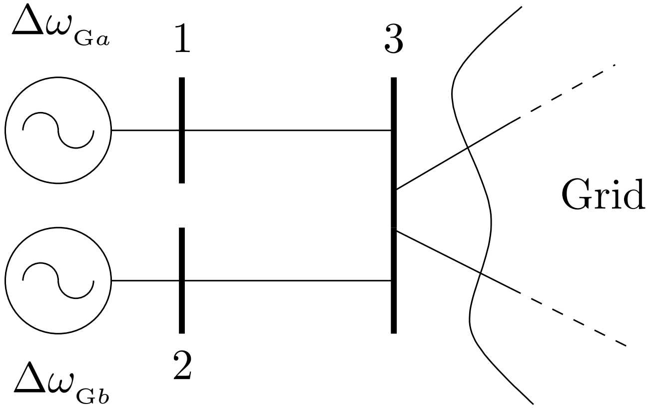
(a)

$$\Delta\omega^*_{\mathrm{G}a} = d^+_{a1}\Delta\tilde{\omega}_{\mathrm{B1}} + d^+_{a2}\Delta\tilde{\omega}_{\mathrm{B2}}$$



(b)

$$\Delta\omega^*_{\mathrm{G}b} = d^+_{b1}\Delta\tilde{\omega}_{\mathrm{B1}} + d^+_{b2}\Delta\tilde{\omega}_{\mathrm{B2}} + d^+_{b3}\Delta\tilde{\omega}_{\mathrm{B3}}$$



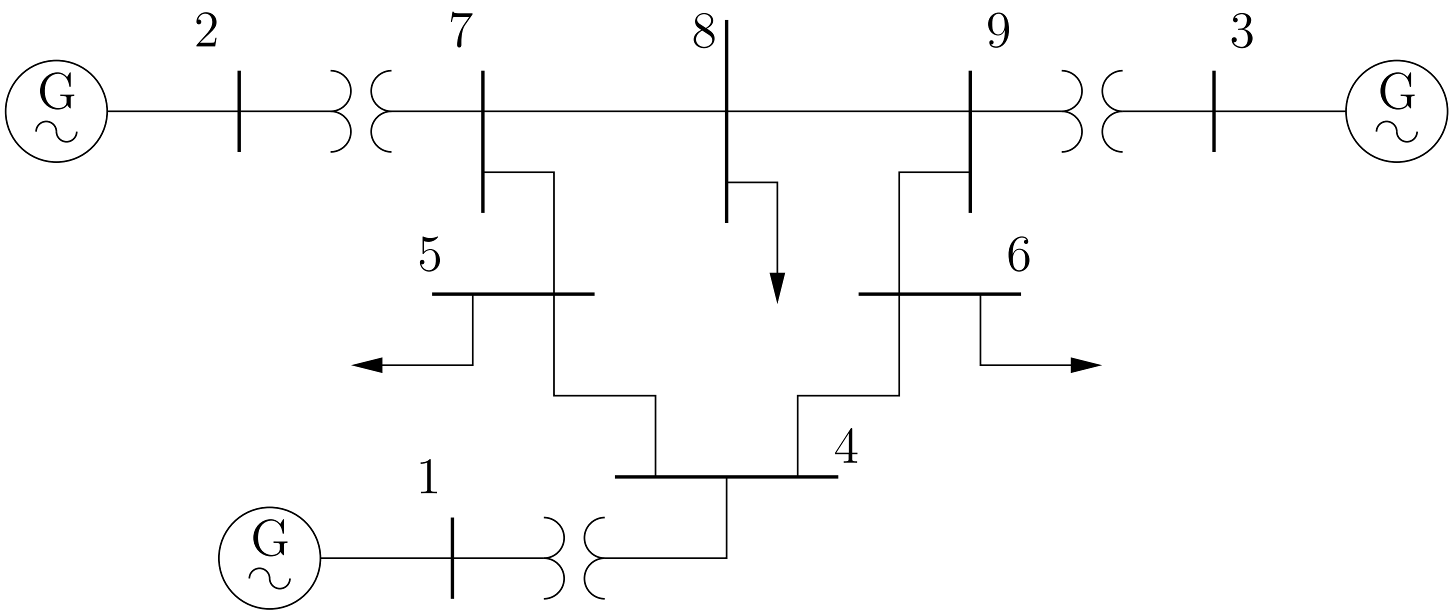
$$\Delta\omega^*_{\mathrm{G}a} = d^+_{a1}\Delta\tilde{\omega}_{\mathrm{B1}} + d^+_{a3}\Delta\tilde{\omega}_{\mathrm{B3}}$$

$$\Delta\omega^*_{\mathrm{G}b} = d^+_{b2}\Delta\tilde{\omega}_{\mathrm{B2}} + d^+_{b3}\Delta\tilde{\omega}_{\mathrm{B3}}$$

(c)

Example: WSCC 9-bus System – Topology

- 9 buses, 9 branches, 3 loads, and 3 machines



Example: WSCC 9-bus System – Matrix B_{BB}

Bus	Bus #								
#	1	2	3	4	5	6	7	8	9
1	-30.04	0	0	17.36	0	0	0	0	0
2	0	-22.32	0	0	0	0	16.00	0	0
3	0	0	-21.70	0	0	0	0	0	17.06
4	17.36	0	0	-39.31	11.60	10.51	0	0	0
5	0	0	0	11.60	-17.34	0	5.975	0	0
6	0	0	0	10.51	0	-15.84	0	0	5.588
7	0	16.00	0	0	5.975	0	-35.45	13.70	0
8	0	0	0	0	0	0	13.70	-23.30	9.784
9	0	0	17.06	0	0	5.588	0	9.784	-32.15

Example: WSCC 9-bus System – Matrix $\mathrm{B}_{\mathrm{BG}}^{T}$

Gen.	Bus #								
#	1	2	3	4	5	6	7	8	9
1	12.682	0	0	0	0	0	0	0	0
2	0	6.315	0	0	0	0	0	0	0
3	0	0	4.637	0	0	0	0	0	0

Example: WSCC 9-bus System – Matrix D^T

Gen.	Bus #								
#	1	2	3	4	5	6	7	8	9
1	0.8225	0.2510	0.2847	0.6928	0.5843	0.5874	0.3500	0.3578	0.3620
2	0.1249	0.6499	0.2327	0.2163	0.3211	0.2479	0.5118	0.4251	0.2959
3	0.1041	0.1708	0.5668	0.1801	0.2027	0.2780	0.2383	0.3287	0.4492

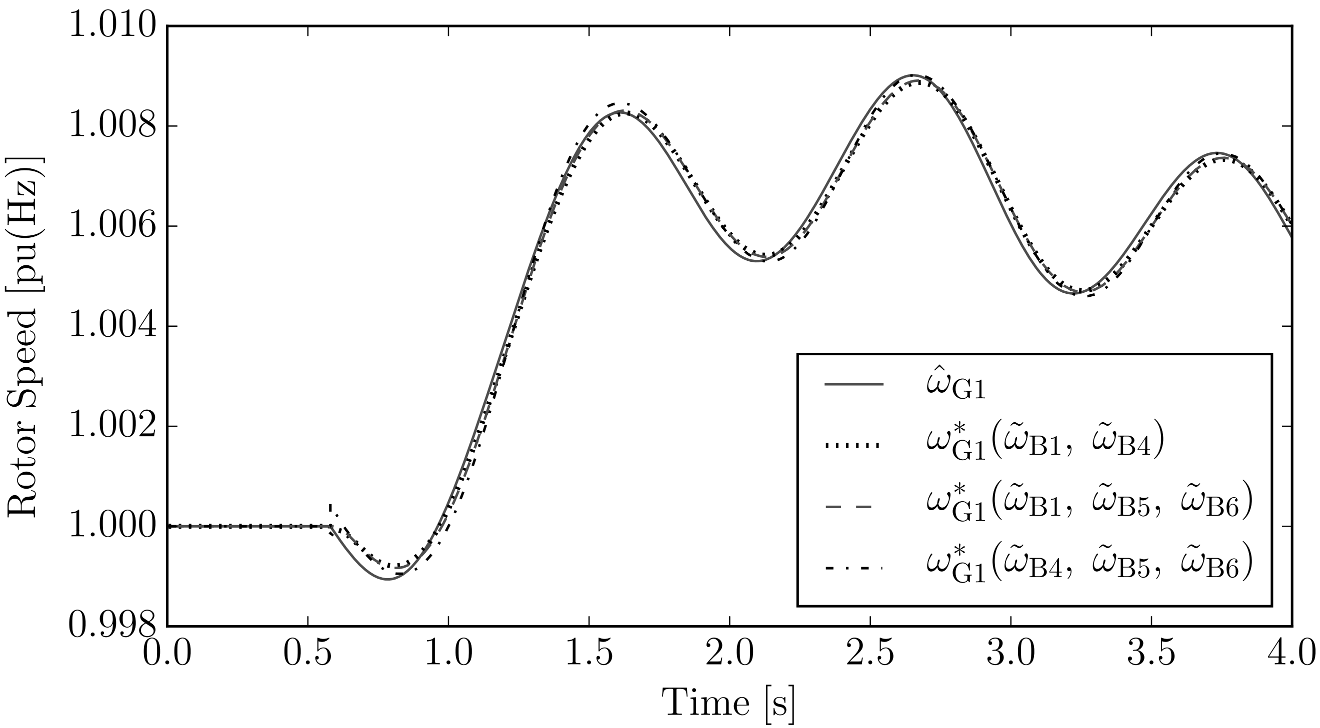
Example: WSCC 9-bus System – Matrix $\mathbf{D}^+$

$\partial\omega_{Gi}$	$\partial\tilde{\omega}_{Bj}$								
	1	2	3	4	5	6	7	8	9
1	2.369	0	0	-1.369	0	0	0	0	0
2	0	3.534	0	0	0	0	-2.534	0	0
3	0	0	4.680	0	0	0	0	0	-3.680

- Note that $\mathbf{D}^+$ can be also viewed as the matrix of the sensitivities or participation factors $\partial\omega_{Gi}/\partial\tilde{\omega}_{Bj}$.

Example: WSCC 9-bus System – Simulation

- There is more than one way to estimate a rotor speed



Estimation of the Frequency of the COI

- A relevant byproduct of the estimation of the vector of $\boldsymbol{\omega}_{\rm G}$ is that we can effectively estimate the freqeuncy of the COI if we know the inertia of the machines.
- One has:

$$\omega_{\rm COI} = \mathbf{h}^T \boldsymbol{\omega}_{\rm G} \ ,$$

- where $\mathbf{h}$ is the vector of weights, $h_i = H_i / \sum_i^m H_j$.
- Then:

$$\begin{aligned} \omega_{\rm COI} - 1 &= -\mathbf{h}^T \mathbf{B}_{\rm BG}^{+} (\mathbf{B}_{\rm BB} + \mathbf{B}_{\rm BS})(\boldsymbol{\omega}_{\rm B} - \mathbf{1}_{n,1}) \\ &= \boldsymbol{\xi}^T (\boldsymbol{\omega}_{\rm B} - \mathbf{1}_{n,1}) \end{aligned}$$

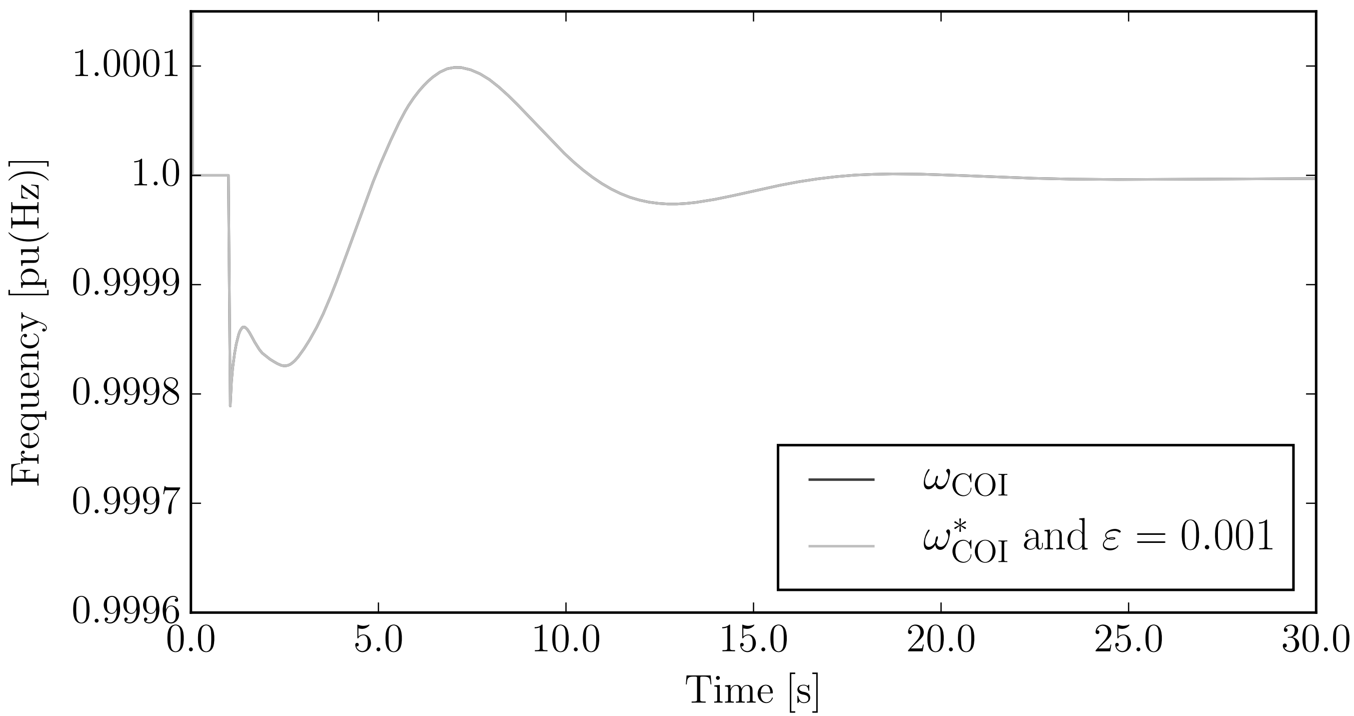
and, finally:

$$\boxed{\omega_{\rm COI}^{*} = \boldsymbol{\xi}^T \boldsymbol{\omega}_{\rm B} + \boldsymbol{\alpha}}$$

where $\boldsymbol{\alpha} = 1 - \boldsymbol{\xi}^T \mathbf{1}_{n,1}$ is an offset, with $|\boldsymbol{\alpha}| \ll 1$.

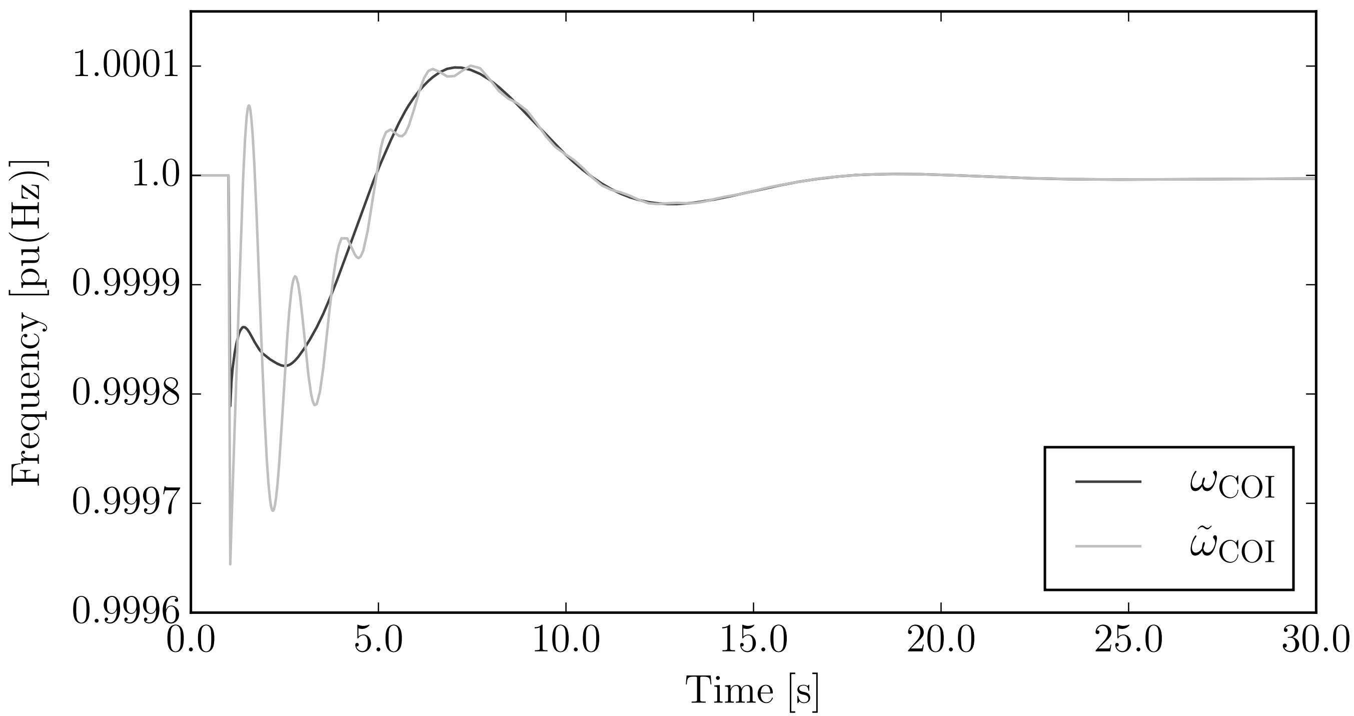
Example: All-island Irish System – Estimation of the COI

- Simulated and estimated COI for the all-island Irish system. Only 42 PMU measures are needed (over 1,479 buses).



Example: All-island Irish System – Estimation of the COI

- If we used only the bus frequencies at the generator buses (22 measurements), the estimation of the COI would not be so good.



Frequency Influencers

(Rate of Change of Power)

Power Flow Equations – I

- Let's start from the well-knonw power flow equations:

$$\bar{\boldsymbol{s}}_{\mathrm{B}}(t) = \boldsymbol{p}_{\mathrm{B}}(t) + j\boldsymbol{q}_{\mathrm{B}}(t) = \bar{\boldsymbol{v}}_{\mathrm{B}}(t) \circ \left[\bar{\mathbf{Y}}_{\mathrm{bus}}\, \bar{\boldsymbol{v}}_{\mathrm{B}}(t)\right] \,, \tag{13}$$

- For the sake of the derivation, it is convenient to rewrite (13) in an element-wise notation and extract the active power:

$$\begin{aligned} p_{\mathrm{B},h}(t) &= v_{\mathrm{B},h}(t) \sum_{k \in \mathbb{B}} v_{\mathrm{B},k}(t)\, G_{\mathrm{bus}}^{hk} \cos\theta_{\mathrm{B},hk}(t) \\ &\quad + v_{\mathrm{B},h}(t) \sum_{k \in \mathbb{B}} v_{\mathrm{B},k}(t)\, B_{\mathrm{bus}}^{hk} \sin\theta_{\mathrm{B},hk}(t) \,, \end{aligned} \tag{14}$$

Power Flow Equations – II

- Let us differentiate (15) and write the active power injections as the sum of two components:

$$
\begin{aligned}
dp_{\mathrm{B},h} &= \sum_{k \in \mathbb{B}} \frac{\partial p_{\mathrm{B},h}}{\partial \theta_{\mathrm{B},k}} \, d\theta_{\mathrm{B},k} + \sum_{k \in \mathbb{B}} \frac{\partial p_{\mathrm{B},h}}{\partial v_{\mathrm{B},k}} \, dv_{\mathrm{B},k} \\
&= dp'_{\mathrm{B},h} + dp''_{\mathrm{B},h} \, ,
\end{aligned}
\tag{15}
$$

- In (15), $dp_{\mathrm{B},h}$ is the total variation of power at bus h, while $dp'_{\mathrm{B},h}$ is what, in the following, we will call "regulating active power"
- The other component, $dp''_{\mathrm{B},h}$, is the "passive" component of the active power.

Simplifications

- The exact expression of the regulating power is thus:

$$dp'_{\mathrm{B},h} = \sum_{k \in \mathbb{B}} \frac{\partial p_{\mathrm{B},h}}{\partial \theta_{\mathrm{B},k}} \, d\theta_{\mathrm{B},k} \tag{16}$$

- This expression can be conveniently simplified by assuming that, in $\frac{\partial p_{\mathrm{B},h}}{\partial \theta_{\mathrm{B},k}}$:
  - voltage magnitudes are ≈ 1;
  - line resistances are negligible; and
  - $\cos(\theta_{\mathrm{B},h} - \theta_{\mathrm{B},k}) \approx 1$.

Link between Regulating Power and Frequency

- Recovering the vector notation, equation (16) can be thus approximated (without loss of accuracy) as:

$$\boldsymbol{p}'_{\mathrm{B}}(t) = -\mathbf{B}_{\mathrm{bus}}\boldsymbol{\theta}_{\mathrm{B}}(t)\,, \tag{17}$$

- Then, differentiating (17) with respect to time gives the most important equation of this presentation, namely:

$$\boxed{\dot{\boldsymbol{p}}'_{\mathrm{B}}(t) = -\Omega_b\mathbf{B}_{\mathrm{bus}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t) = -\hat{\mathbf{B}}_{\mathrm{bus}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)} \tag{18}$$

- In (18), $\dot{\boldsymbol{p}}'_{\mathrm{B}}(t)$ is the rate of change of (regulating) power or RoCoP, and $\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)$ are the variations of frequency at network buses.

Applications?

- So, the first question that one may ask is: Why would we even need the definition of the RoCoP?
- The answer relies on the ability to define, for each component of the grid, an expression of $\dot{p}'_{\mathrm{B},h}$.
- Fortunately, there are some very relevant cases for which one can determine $\dot{p}'_{\mathrm{B},h}$ analytically (we will discuss these cases next).
- In all other cases, we need to rely on measurements of bus frequencies to deduce $\dot{p}'_{\mathrm{B},h}$ (we will see an application at the end of this presentation).

Two (Very) Relevant Cases – I

- **Constant Admittance Loads**: for this kind of loads, one can show that:

$$\dot{p}'_{\mathrm{B},h} \equiv 0\,, \tag{19}$$

  which, incidentally, implies $\dot{p}_{\mathrm{B},h} = \dot{p}''_{\mathrm{B},h}$ (fully passive device!) or, equivalently, **constant admittances do not modify the frequency**.

- Note that, for all other types of loads, $\dot{p}'_{\mathrm{B},h} \neq 0$, which means that loads that are not pure admittances **do** modify the freqeuncy at their point of connection.

- In other words, non-constant admittance loads imply some sort of *regulation*.

Two (Very) Relevant Cases – II

- **Synchronous Machines**: In this case it is possible to show that:

$$\dot{\boldsymbol{p}}'_{\mathrm{B}}(t) \approx \Omega_b[\mathbf{B}_{\mathrm{BG}}\Delta\boldsymbol{\omega}_{\mathrm{G}}(t) + \mathbf{B}_{\mathrm{G}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)] \,, \tag{20}$$

where $\mathbf{B}_{\mathrm{BG}}$ and $\mathbf{B}_{\mathrm{G}}$ and the incidence susceptance matrices formed with the internal reactances of the machines and $\Delta\boldsymbol{\omega}_{\mathrm{G}}(t)$ are the machine rotor speeds.

- Equation (20) can be rewritten as:

$$\dot{\boldsymbol{p}}'_{\mathrm{B}}(t) = \hat{\mathbf{B}}_{\mathrm{BG}}\left[\Delta\boldsymbol{\omega}_{\mathrm{G}}(t) - \Delta\boldsymbol{\omega}_{\mathrm{BG}}(t)\right] \,, \tag{21}$$

- Putting together (18) and (21), we obtain another very relevant expression:

$$\boxed{\mathbf{B}_{\mathrm{BG}}\Delta\boldsymbol{\omega}_{\mathrm{G}}(t) = -\mathbf{B}_{\mathrm{BB}}\Delta\boldsymbol{\omega}_{\mathrm{B}}(t)} \,, \tag{22}$$

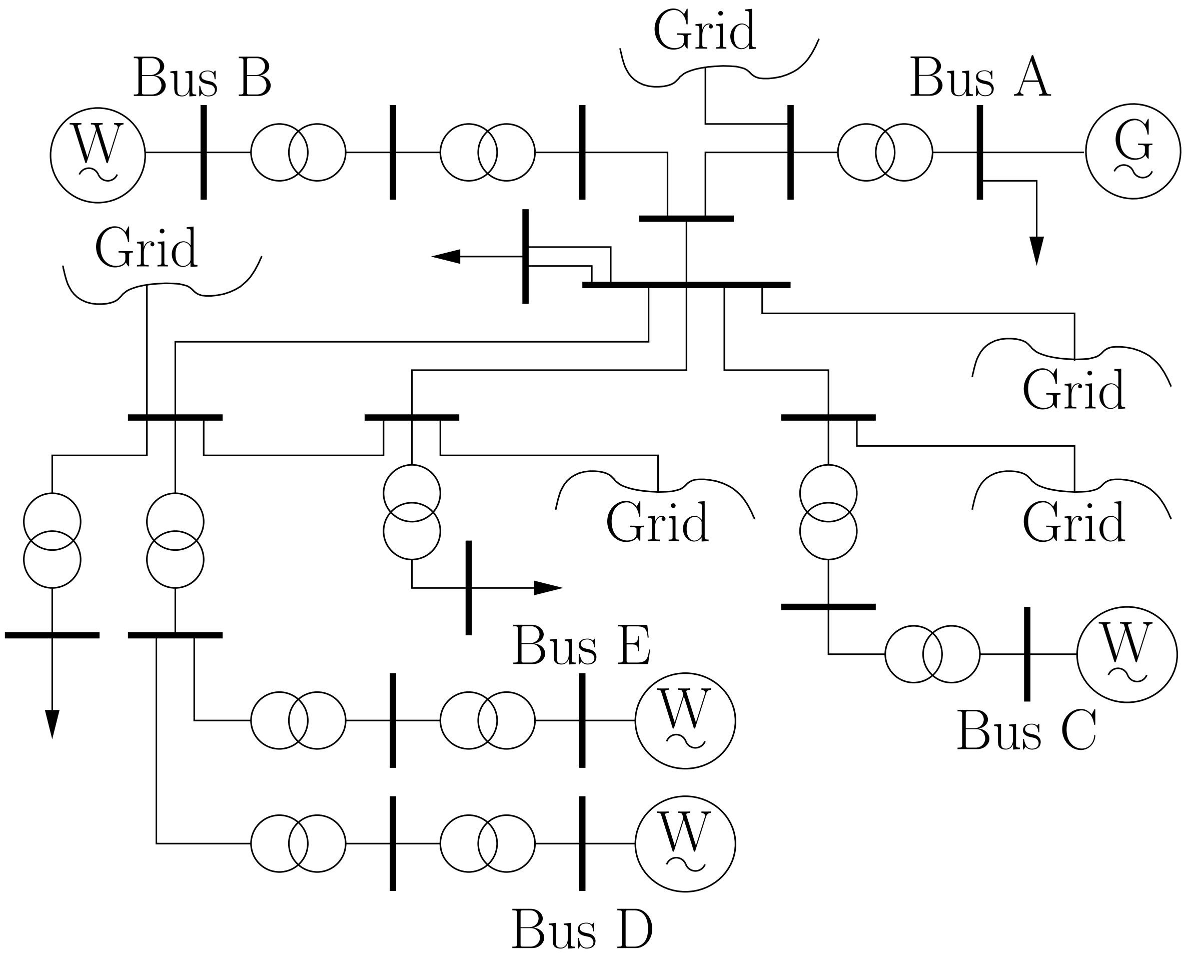
where $\mathbf{B}_{\mathrm{BB}} = \mathbf{B}_{\mathrm{bus}} + \mathbf{B}_{\mathrm{G}}$.

- We have just (re)obtained the **Frequency Divider Formula**!

Examples on the RoCoP

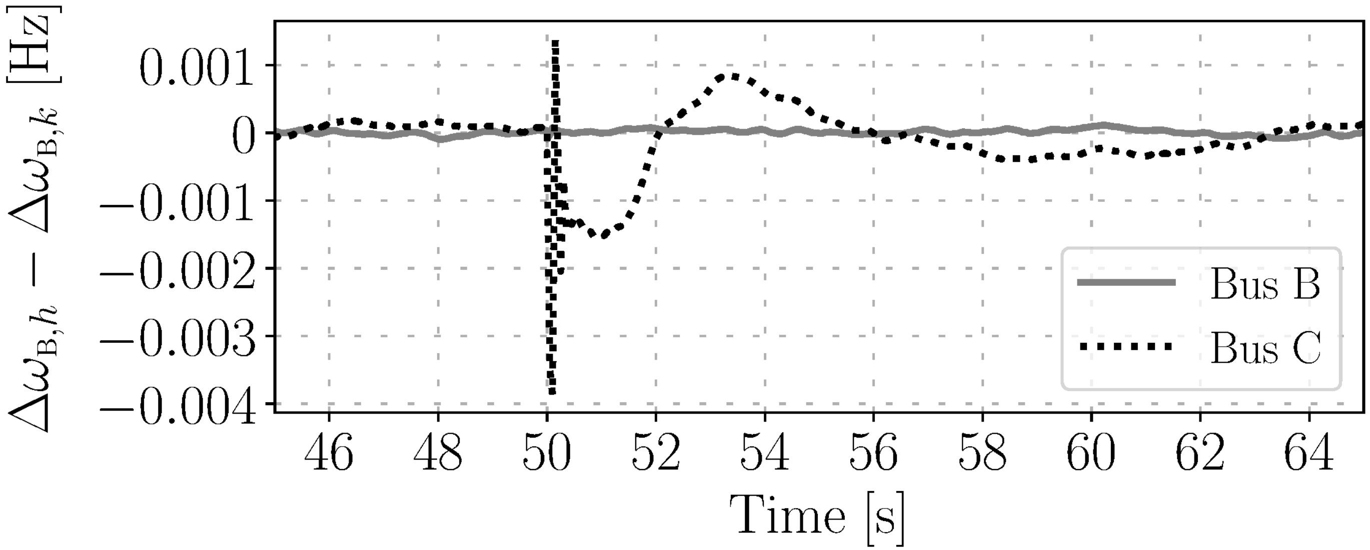
Section of the Irish System

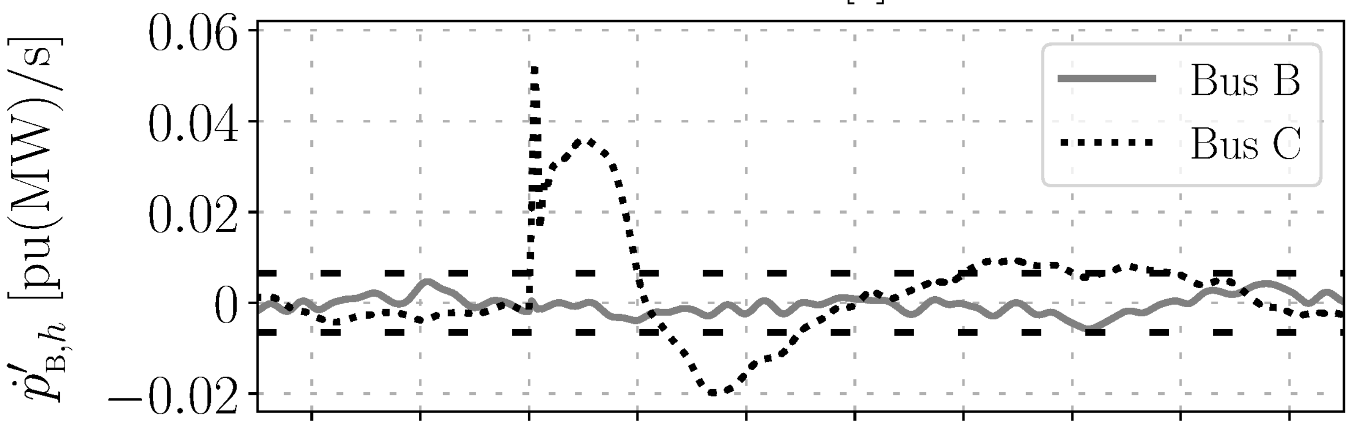
- Let consider a section of the Irish system with conventional generation and WECS.



RoCoP and Freqeuncy Variations

- The RoCoP is able to identify the WECS that provide fast frequency regulation.





Voltage Dependent Loads – I

- Voltage dependent loads can be modelled as:

$$p_{\mathrm{D}}(t) = p_{\mathrm{D},o}\, v_h^{\gamma_p}(t)\,, \qquad q_{\mathrm{D}}(t) = q_{\mathrm{D},o}\, v_h^{\gamma_q}(t)\,,$$

- Using the RoCoP theory, one ca find out that:

γ_p	$\dot{p}'_h$	$\dot{p}''_h$	$\dot{p}_h$
0	$-\dot{p}''_h$	$-\dot{p}'_h$	0
1	$\dot{p}_h$	0	$\dot{p}'_h$
2	0	$\dot{p}_h$	$\dot{p}''_h$

Voltage Dependent Loads – II

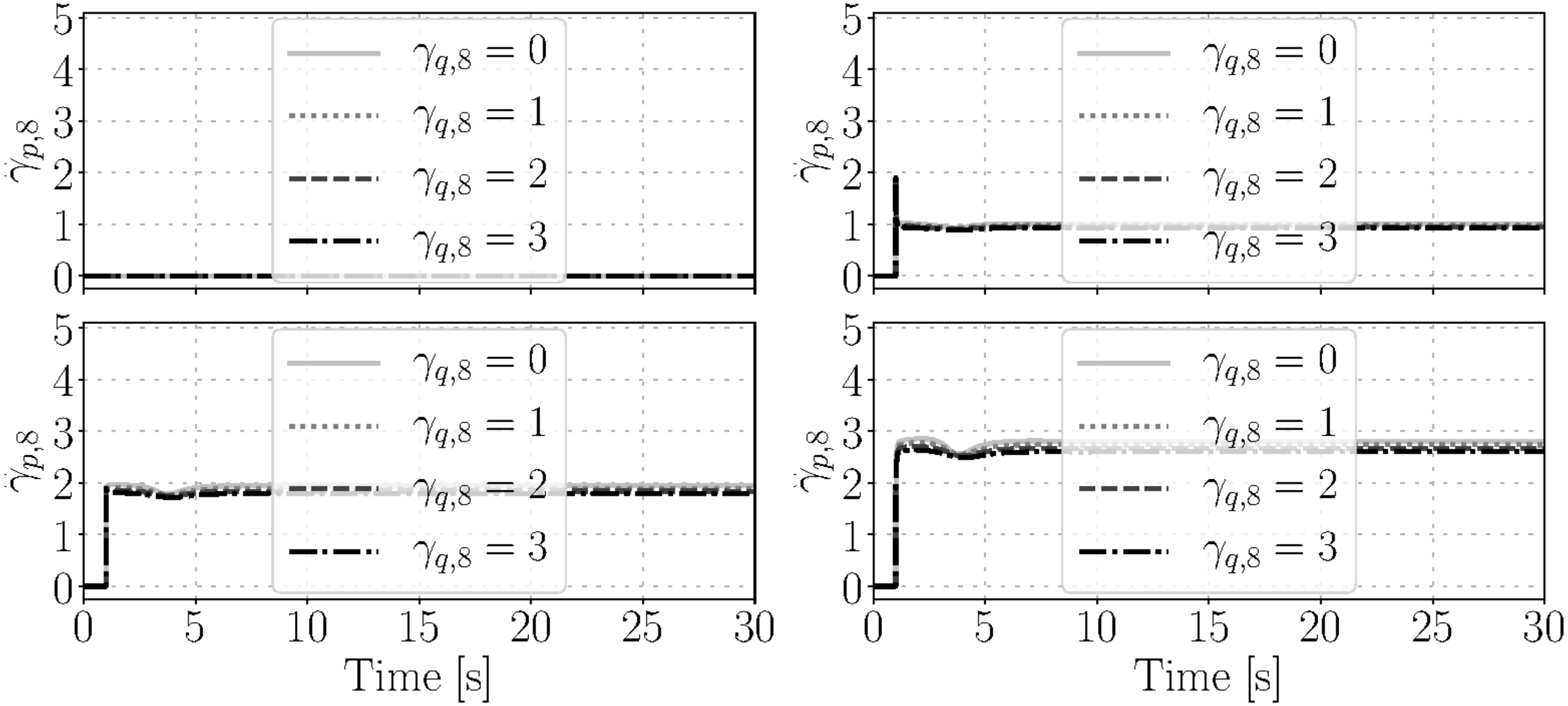
- The previous table leads to the following formula:

$$\breve{\gamma}_p(t) \approx 2\, \frac{\dot{p}_h(t)}{\dot{p}_h(t) + \dot{p}'_h(t)} \approx 2\, \frac{\Delta p_h(t)}{\Delta p_h(t) + \Delta p'_h(t)}\,,$$

- Hence, measuring Δp_h and estimating $\dot{p}'_h$ through frequency measurements, one can estimate the coefficients γ_p.
- A similar expression holds for γ_q
- Note that no voltage measurements are involved!

Example of Estimation of VDL

- Example of estimation of γ_p for different scenarios, i.e., different values of γ_q.



Estimation of the Inertia

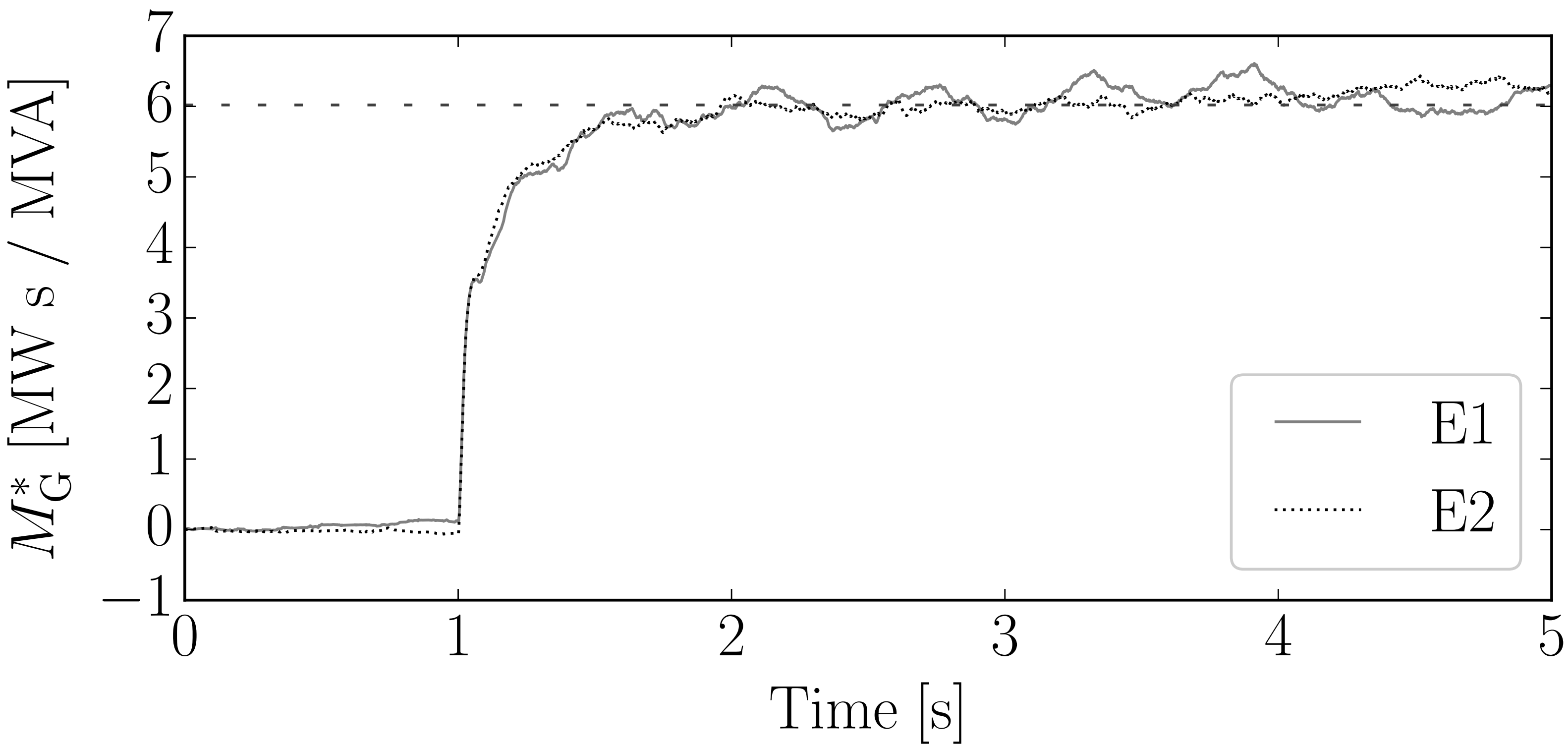
- The FDF allows estimating the rotor speed of synchronous machines.
- Using the well-known swing equation of the machine and estimating the value of the RoCoP, the inertia of the machine can be estimated as follows:

$$M_{\mathrm{G},h}(t) \approx \frac{-\dot{p}'_{\mathrm{B},h}(t)}{\frac{d^2}{dt^2}\left[\Delta\omega_{\mathrm{B},h}(t) - \hat{x}_{\mathrm{G},h}\dot{p}'_{\mathrm{B},h}(t)\right]}, \text{ for } t < t^* ,$$

where t^* is a suitable time interval (2 to 5 seconds).

Example

- The previous formula is quite reliable, and can be improved with some filtering.

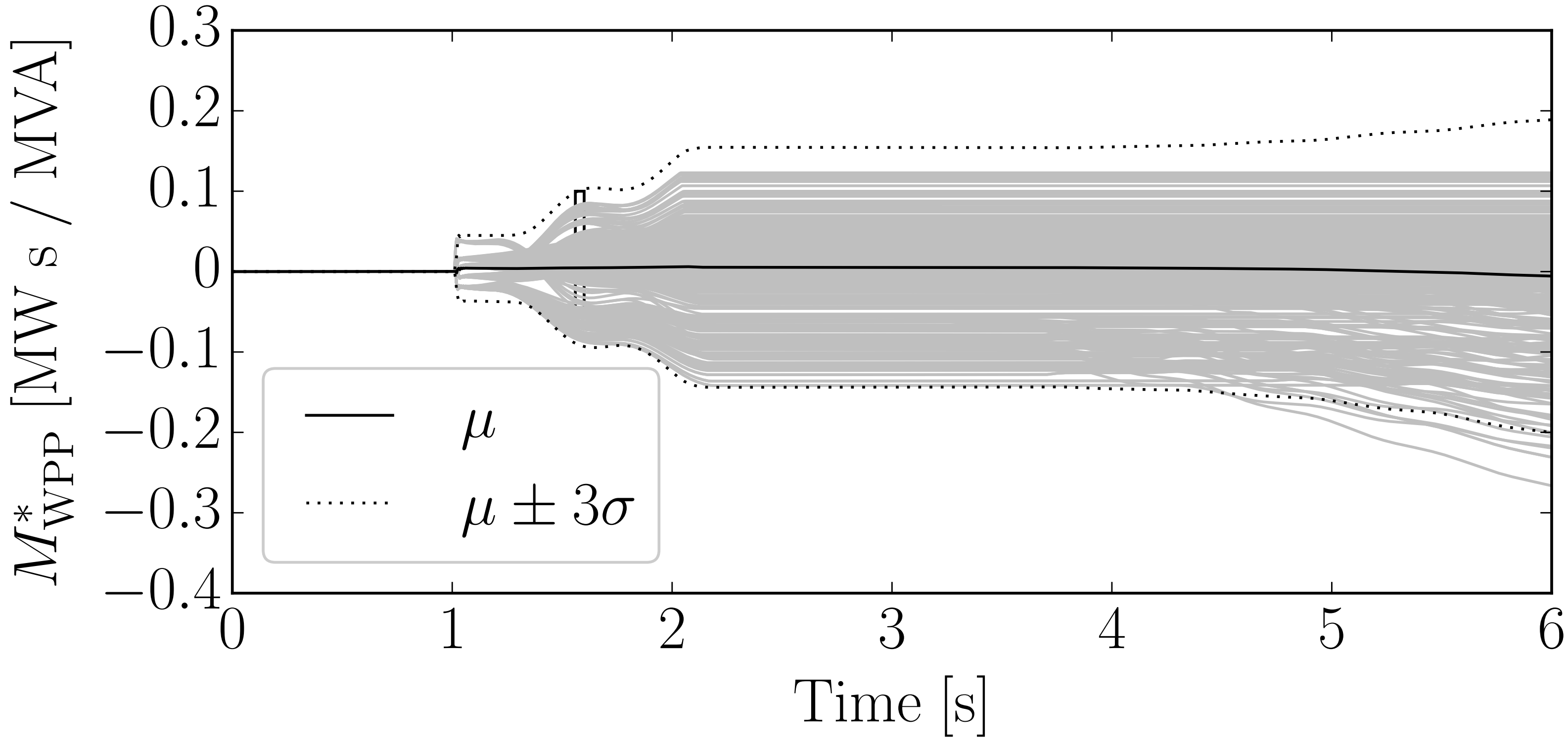


Estimation of the Virtual Inertia

- Interestingly, the previous formula can be utilized to estimate the inertia of non-synchronous devices.
- The only additional information is to assign a value to the reactance $\hat{x}_{\mathrm{G},h}$.

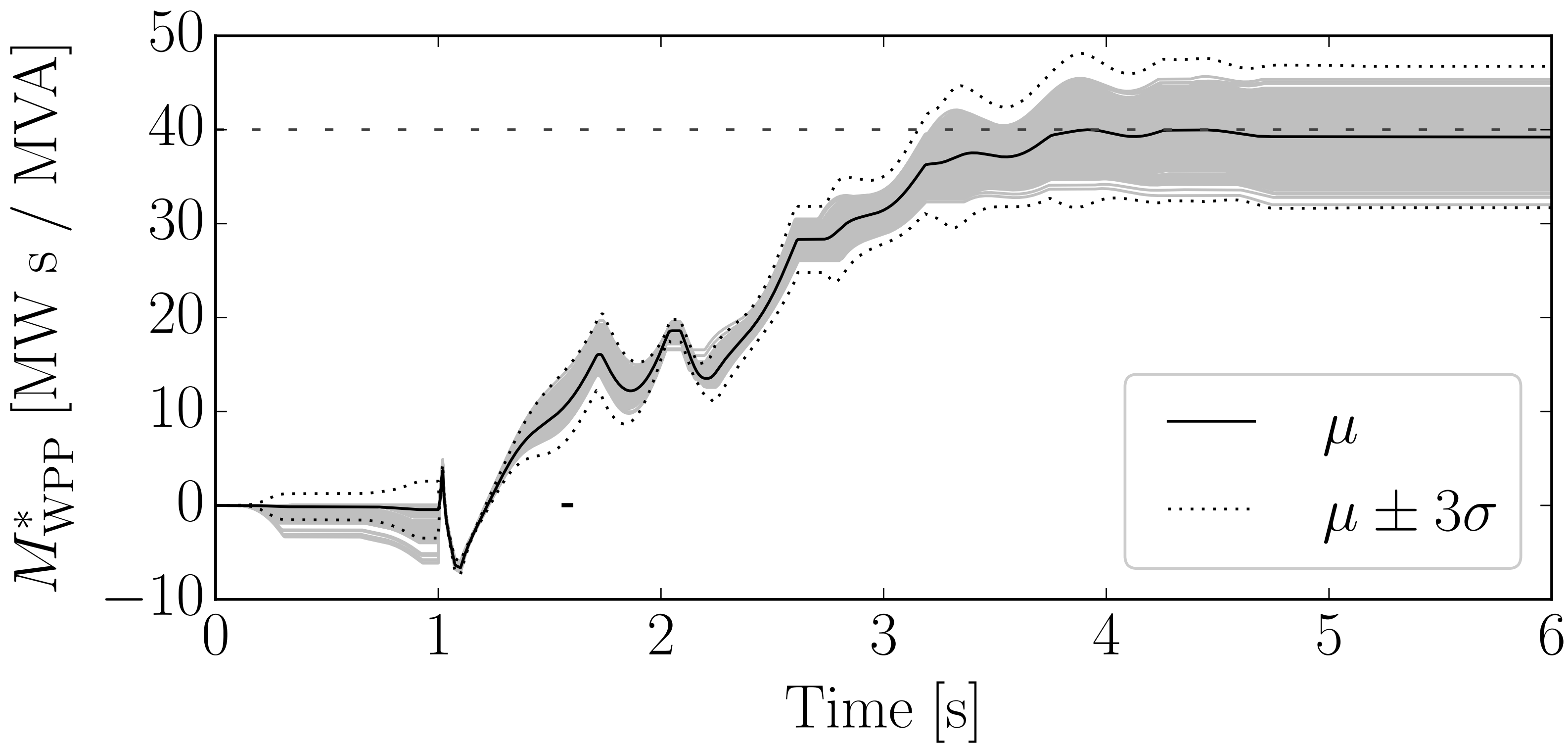
Example – I

- WECS without fast frequency control



Example – II

- WECS with an energy storage system that provides fast frequency control



Conclusions, Future Work

and References

Conclusions (for now . . .)

- A general but approximated expression to estimate frequency variations during the transient of electric power systems has been deduced.
- It appears that “frequency variations” in an AC system are strictly related to power variations, hence freqeuncy can be relevant also in low or no inertia systems.
- Simulation results show that the proposed formulas are quite accurate, numerically robust and computationally efficient.
- These formulas have relevant applications in dynamic state estimation and control.

Open Questions

- How to take into account fast flux transients and wave propagation?
- Effect of loads?
- Ho to derive $\dot{p}'_{\mathrm{B},h}$ in general, i.e., for any device connected to the grid?
- How to take into reactive power?

- Future work will focus on a *generalized* definition of “frequency” that is able to take into account complex power variations and does not require model simplifications.

References – I

- F. Milano, Á. Ortega, *Frequency Divider*, IEEE Transactions on Power Systems, Vol. 32, No. 2, pp. 1493-1501, March 2017.
- Á. Ortega, F. Milano, *Comparison of Bus Frequency Estimators for Power System Transient Stability Analysis*, IEEE PowerCon, Wollongong, Australia, September-October 2016.
- Á. Ortega, F. Milano, *Impact of Frequency Estimation for VSC-based Devices with Primary Frequency Control*, IEEE PES ISGT Europe 2017, Turin, Italy, 26-29 September 2017.
- F. Milano, *Rotor Speed-free Estimation of the Frequency of the Center of Inertia*, IEEE Transactions on Power Systems, Vol. 33, No. 1, pp. 1153-1155, January 2018.

References – II

- J. Zhao, L. Mili, F. Milano, *Robust Frequency Divider for Power System Online Monitoring and Control*, IEEE Transactions on Power Systems, Vol. 33, No. 4, pp. 4414-4423, July 2018.
- Á. Ortega, A. Musa, A. Monti, F. Milano, *Hardware-in-the-Loop Validation of the Frequency Divider Formula*, IEEE PES General Meeting, Portland, OR, August 2018.
- Á. Ortega, F. Milano, *Frequency Control of Distributed Energy Resources in Distribution Networks*, 10th Symposium on Control of Power and Energy Systems (IFAC CPES2018), Tokyo, Japan, September 2018.
- Á. Ortega, F. Milano, *Frequency Participation Factors, IEEE Transactions on Power Systems*, Vol. 33, No. 5, pp. 5563-5571, September 2018.

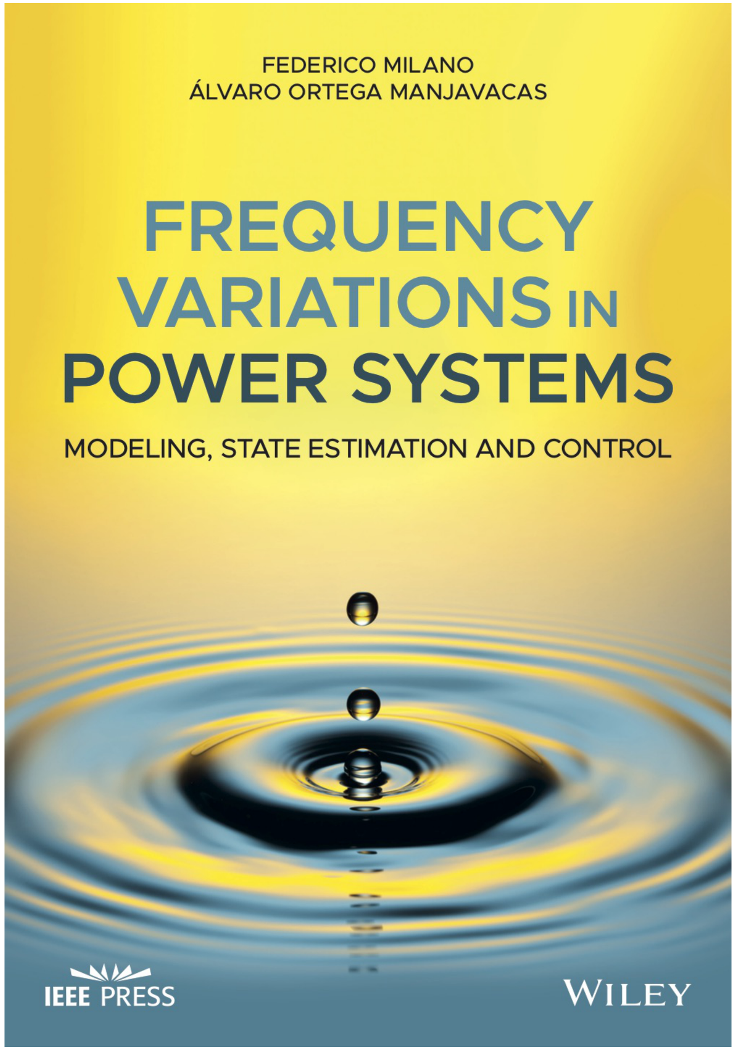
References – III

- F. Milano, Á. Ortega, A. J. Conejo, *Model-Agnostic Linear Estimation of Generator Rotor Speeds based on Phasor Measurement Units*, IEEE Transactions on Power Systems, Vol. 33, No. 6, pp. 7258-7268, November 2018.
- M. Liu, Á. Ortega, F. Milano, *PMU-based Estimation of the Frequency of the Center of Inertia and Generator Rotor Speeds*, IEEE PES General Meeting, Atlanta, GA, August 2019.
- F. Milano, Á. Ortega, *Frequency-dependent Model for Transient Stability Analysis*, IEEE Transactions on Power Systems, Vol. 34, No. 1, pp. 806-809, January 2019.

References – IV

- Á. Ortega, F. Milano, *Estimation of Voltage Dependent Load Models through Power and Frequency Measurements*, IEEE Transactions on Power Systems, Vol. 35, No. 4, pp. 3308-3311, July 2020.
- F. Milano and Á. Ortega, *A Method for Evaluating Frequency Regulation in an Electrical Grid Part I: Theory*, in IEEE Transactions on Power Systems, Vol. 36, No. 1, pp. 183-193, January 2021.
- Á. Ortega and F. Milano, *A Method for Evaluating Frequency Regulation in an Electrical Grid Part II: Applications to Non-Synchronous Devices*, in IEEE Transactions on Power Systems, Vol. 36, No. 1, pp. 194-203, January 2021.

Book on the FDF



Fondúireacht Eolaíochta Éireann Dá bhfuil romhainn

Thanks much for your attention!