Robust Control Systems

An Introduction on Robust Control Systems

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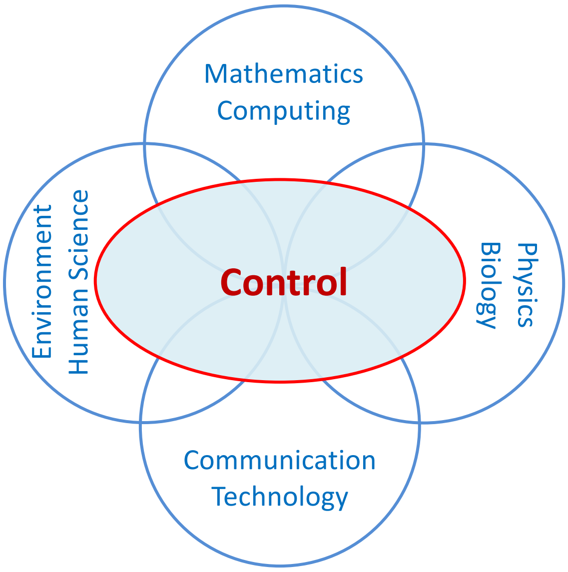
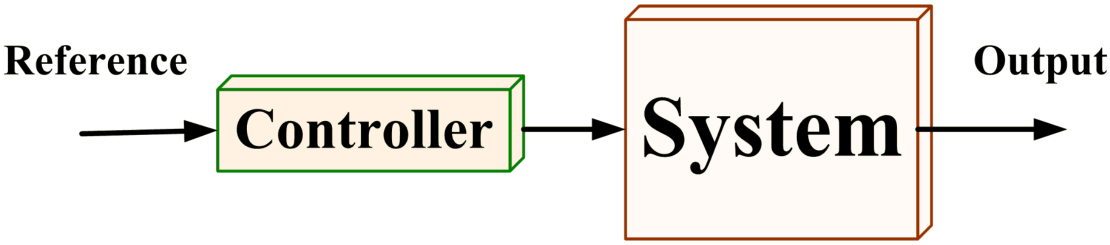
Control Engineering

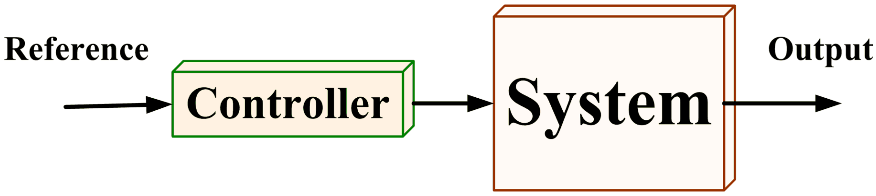


Control System

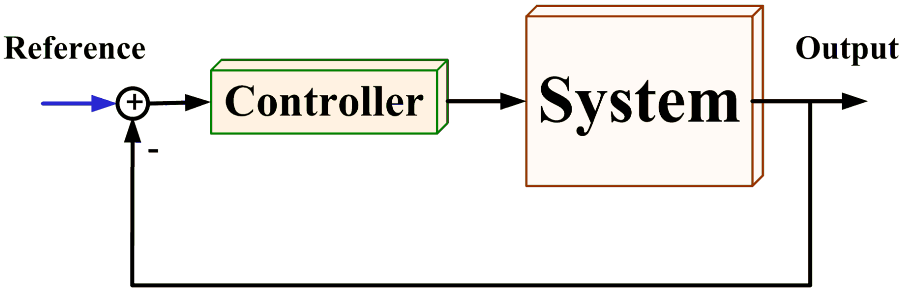


Control Systems

- Open-loop Control (Feedforward Control)

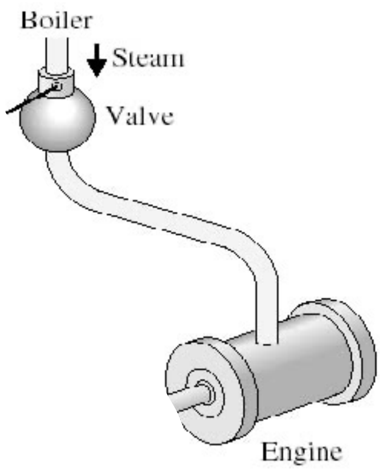


- Closed-loop Control (Feedback Control)



Feedback Control Example

- Watt’s centrifugal speed regulator (1976)



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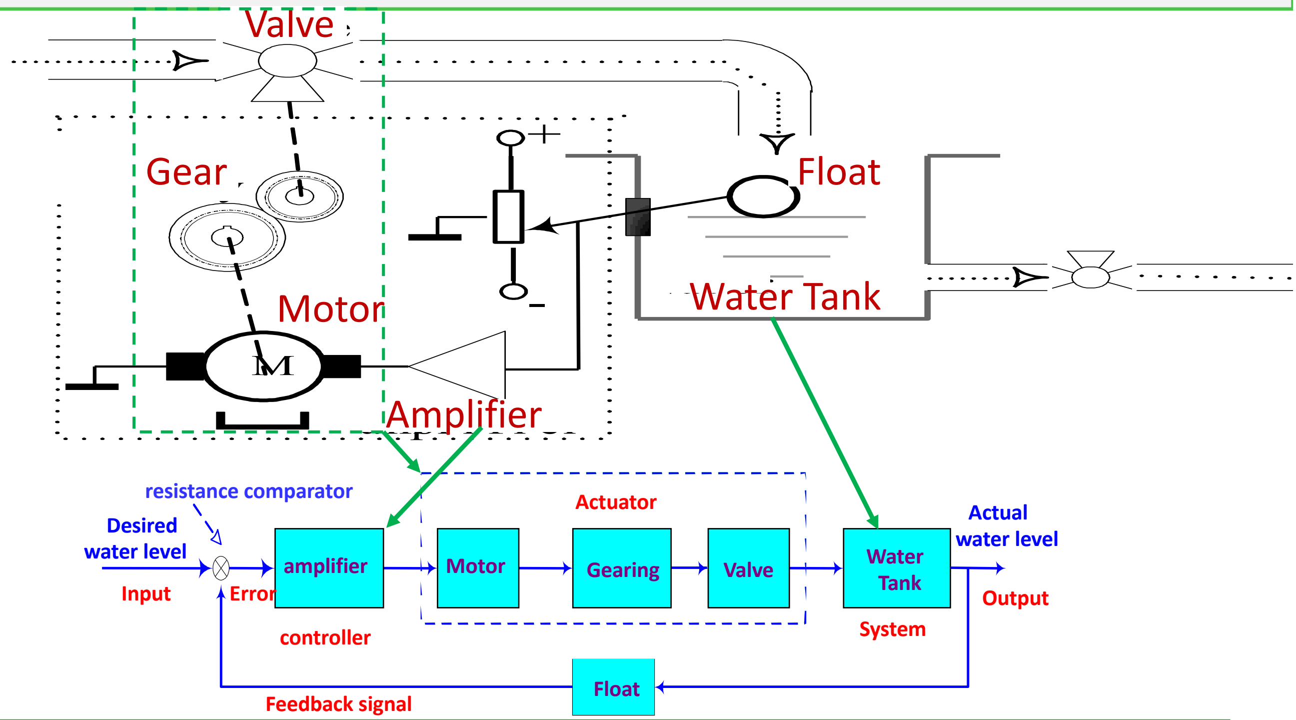
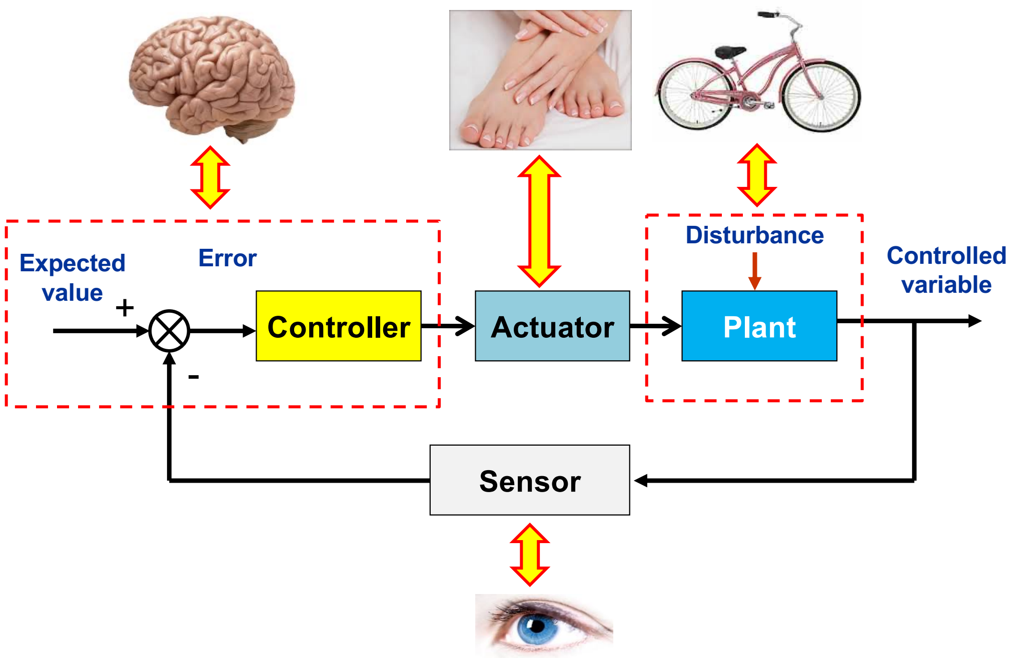


Block Diagram of a Feedback Control System

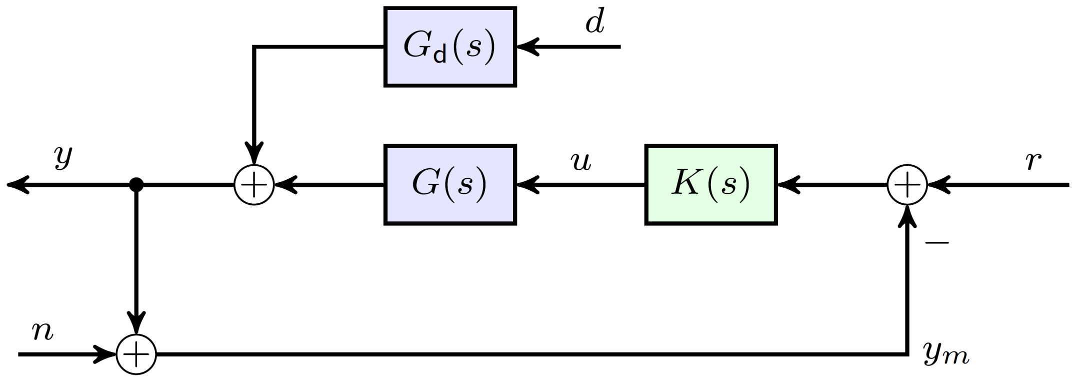


Power of Feedback

- ✓ **Stabilize unstable systems**
- ✓ **Good performance from poor components**
- ✓ **Attenuate disturbance impacts**
- ✓ **Provide degrees of freedom**
- ✓ **Attenuate parameter variation impacts**
- ✓ **Shape behavior**

- **Main drawback**

Feedback Control



Objectives:

- Closed-loop stability
- Reference tracking
- Disturbance rejection
- Noise response

Difficulties:

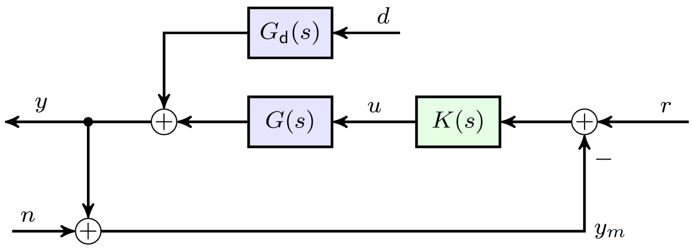
- Model errors
- Fundamental limits on controllability of $G(s)$
- Actuation constraints

Transfer Functions

Loop transfer function	$L(s) = G(s)K(s)$
Sensitivity function	$S(s) = \dfrac{1}{1+G(s)K(s)} = \dfrac{1}{1+L(s)}$
Complementary sensitivity	$T(s) = \dfrac{G(s)K(s)}{1+G(s)K(s)} = \dfrac{L(s)}{1+L(s)}$
Output response	$y = T(s)\,r \;+\; S(s)G_{\mathsf{d}}(s)\,d \;-\; T(s)n$
Error response	$e = r - y$ $= S(s)\,r \;-\; S(s)G_{\mathsf{d}}(s)\,d \;+\; T(s)n$

Conflicting Objectives

$$y = T(s)\,r \;+\; S(s)G_{\mathsf{d}}(s)\,d \;-\; T(s)n$$



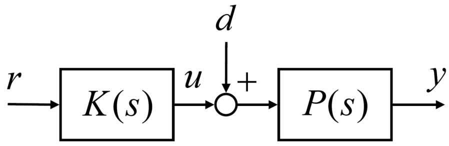
- **Performance Requirements**

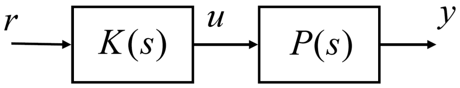
Reference tracking	$T(s) \approx 1$
Noise rejection	$T(s) \ll 1$
Disturbance rejection	$S(s)G_{\mathsf{d}}(s) \ll 1$
Low closed-loop plant sensitivity	$S(s) \ll 1$

- **Constraint** $S(s) + T(s) = 1$ for all s

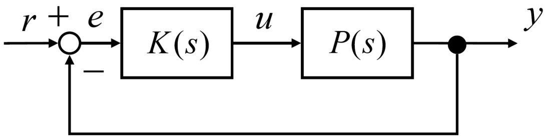
Feedback vs Feedforward

$$P(s)=\frac{A}{\tau s+1} \qquad K(s)=K \qquad (d=0)$$





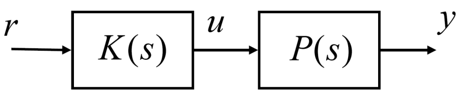
$$y(s)=P(s)K(s)r(s)$$
$$=\frac{A}{\tau s+1}\cdot K\cdot r(s)$$
$$=\frac{AK}{\tau s+1}r(s)$$



$$\begin{cases} y(s)=P(s)K(s)e(s) \\ e(s)=r(s)-y(s) \end{cases}$$

$$\left(1+P(s)K(s)\right)y(s)$$
$$=P(s)K(s)r(s)$$
$$y(s)=\frac{P(s)K(s)}{1+P(s)K(s)}r(s)$$
$$=\frac{AK}{\tau s+1+AK}r(s)$$

Feedback vs Feedforward



$$K=\frac{1}{A}$$

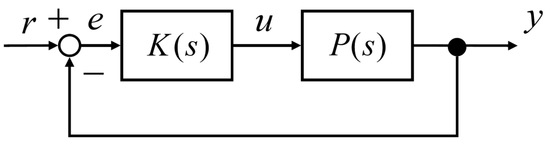
$$y(s)=\frac{AK}{\tau s+1}r(s)=\frac{1}{\tau s+1}r(s)$$

$$y(t)\approx r(t) \quad (t\to\infty)$$

$$\tilde{A}=1.4A$$

$$\tilde{y}(s)=\frac{\tilde{A}K}{\tau s+1}r(s)=\frac{1.4}{\tau s+1}r(s)$$

$$\tilde{y}(t)\approx 1.4r(t)=(1.4y(t))$$



$$y(s)=\frac{AK}{\tau s+1+AK}r(s)$$

$$K\to\infty$$

$$\frac{AK}{\tau s+1+AK}\approx\frac{AK}{AK}=1$$

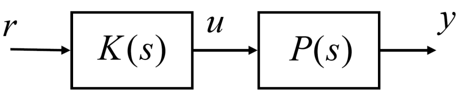
$$\therefore \quad y(t)\approx r(t)$$

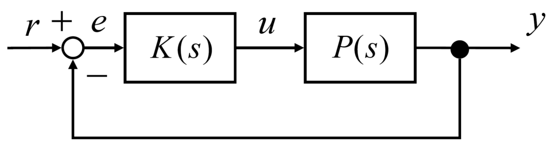
Feedback vs Feedforward

$$P(s)=\frac{A}{\tau s+1}\qquad K(s)=K$$

$\tau=1,\ A=5$

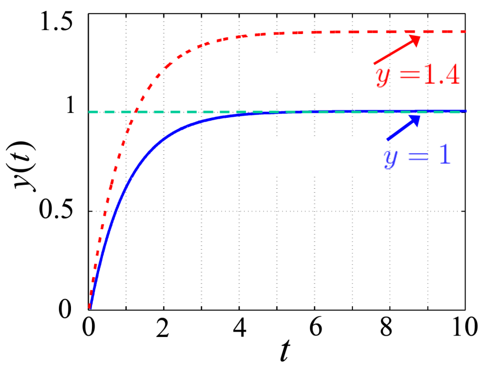
$A\rightarrow\tilde{A}=7$

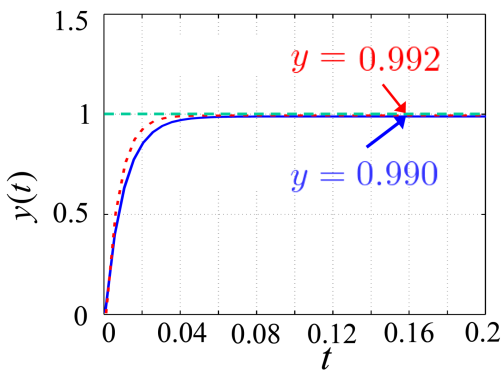




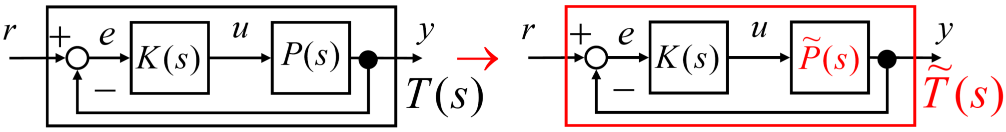
$$K=\frac{1}{A}=0.2$$

$$K=20$$





Sensitivity



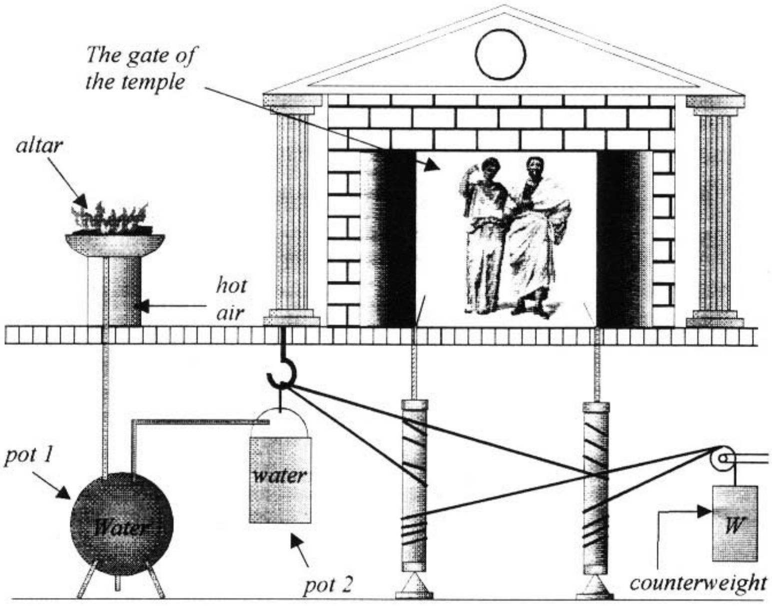
$$T(s)=\frac{P(s)K(s)}{1+P(s)K(s)}\qquad P(s)\rightarrow\tilde{P}(s)\qquad T(s)\rightarrow\tilde{T}(s)$$

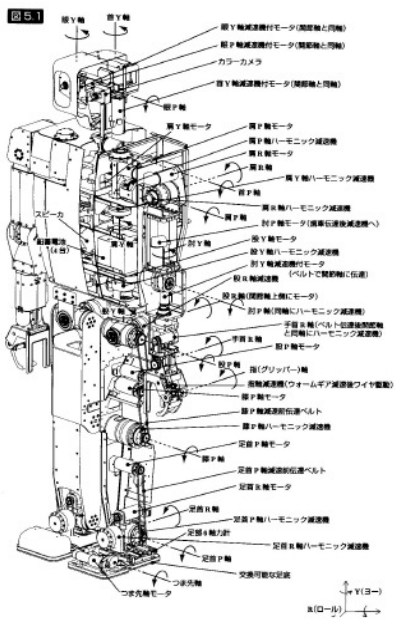
$$\Delta_P(s)=\frac{P(s)-\tilde{P}(s)}{\tilde{P}(s)}\qquad \Delta_T(s)=\frac{T(s)-\tilde{T}(s)}{\tilde{T}(s)}$$

$$\Delta_T(s)=\frac{\frac{PK}{1+PK}-\frac{\tilde{P}K}{1+\tilde{P}K}}{\frac{\tilde{P}K}{1+\tilde{P}K}}=\frac{PK(1+\tilde{P}K)-\tilde{P}K(1+PK)}{\tilde{P}K(1+PK)}$$

$$=\frac{(P-\tilde{P})K}{\tilde{P}K(1+PK)}=\frac{1}{1+P(s)K(s)}\Delta_P(s)\ \Rightarrow\ S(s)=\frac{1}{1+P(s)K(s)}$$

The regulator of Heron of Alexandria





Control History

- Control appeared in the industries that emerged in the 19th and 20th centuries: steam power, electric power, ships, aircrafts, chemicals, telecommunication.

- In the 1940s it appeared as a separate engineering discipline, and it has developed rapidly ever since. Academic positioning difficult since it fits poorly into the ME, EE, ChemE framework. Today applications everywhere.

Control History

- **1868** first article of control on governors (Maxwell)
- **1877** Routh stability criterion
- **1892** Lyapunov stability condition
- **1895** Hurwitz stability condition
- **1932** Nyquist
- **1945** Bode
- **1947** Nichols
- **1948** Root locus
- **1949** Wiener optimal control research
- **1955** Kalman filter and controlbility observability analysis

Control History

- **1956** Artificial Intelligence
- **1957** Bellman optimal and adaptive control
- **1962** Pontryagin optimal control
- **1965** Zadeh Fuzzy set
- **1972** Multi-variable optimal control and Robust control
- **1981** Doyle Robust control theory
- **1990** Neuro-Fuzzy
- **2000** More intelligent control
- **2010** Wide-area & distributed controls
- **2020** Data-driven & Intelligent controls

Historic Turning Points

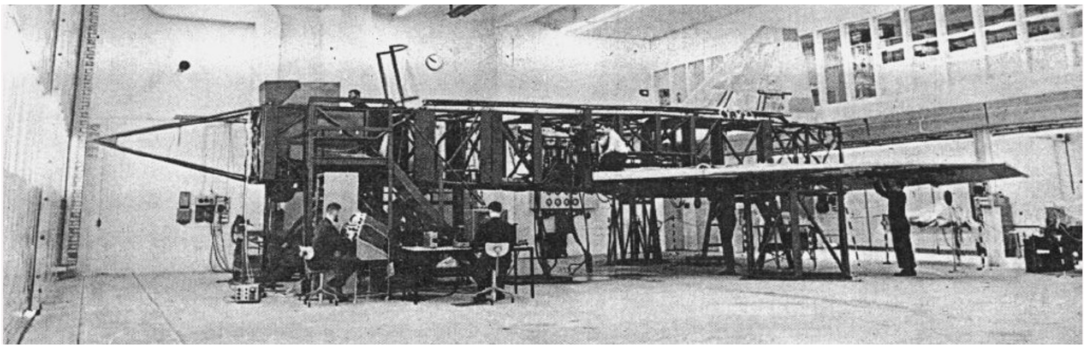
- **1945:** Drivers (gun control, radar), Modeling block diagram, transfer functions, simulation and Theorems
- **1965:** Computational tools, Kalman filter, Nonlinear and stochastic, LQG and H_∞ (optimal control)
- **1985:** Digital control, Robust control
- **2010:** Wide-area control, Distributed control systems
- **2020:** Data-driven control, AI Control

Historical Control Example

- **Flight Control**

Problem: How to fly in a stabilized condition?

Solution: Stabilization using Feedback



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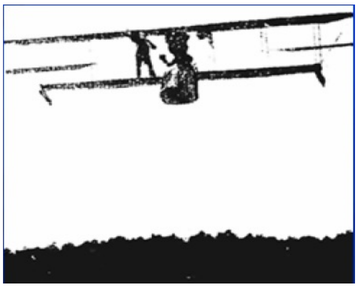
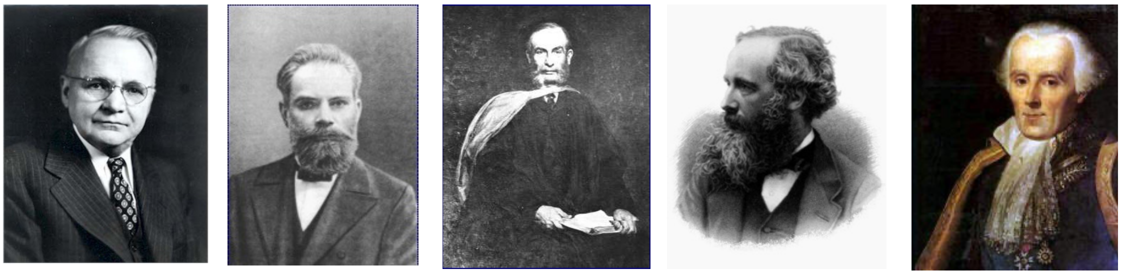
1. The Wright Brothers1903
2. Sperry's Autopilot 1912
3. V1 and V2 1942
4. Robert E. 1947
5. Sputnik1957
6. Apollo 1969
7. Mars Pathfinder 1997
8. UAVs 2020



UAV: Unmanned Aerial Vehicle

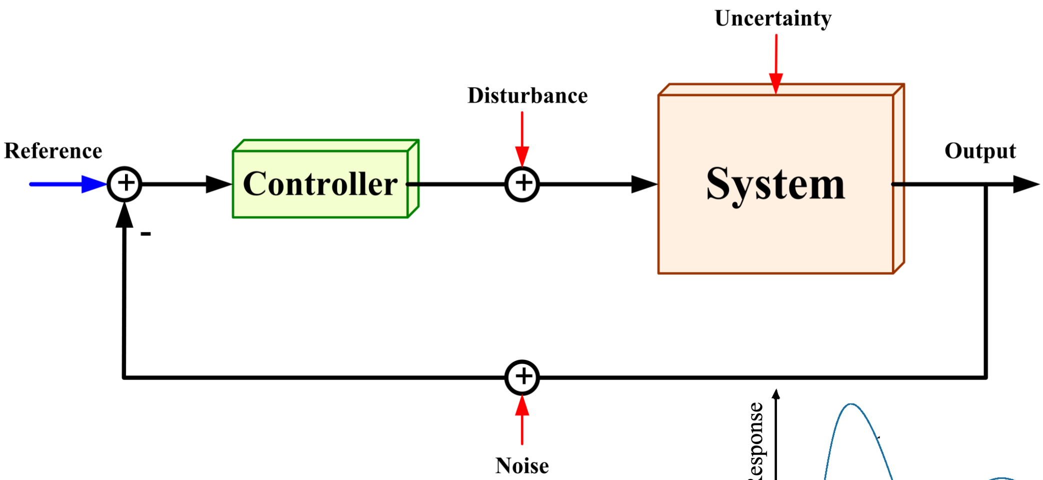
Control History

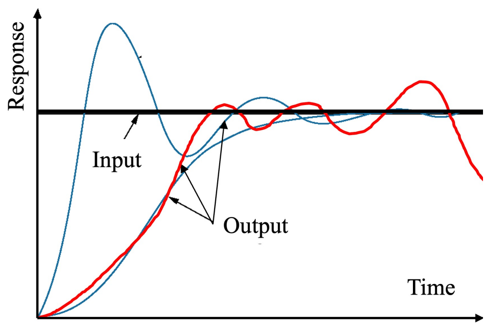


Nyquist Lyapunov Routh Maxwell Laplace

Kalman Zames Zadeh Doyle Nichols

Feedback Control System

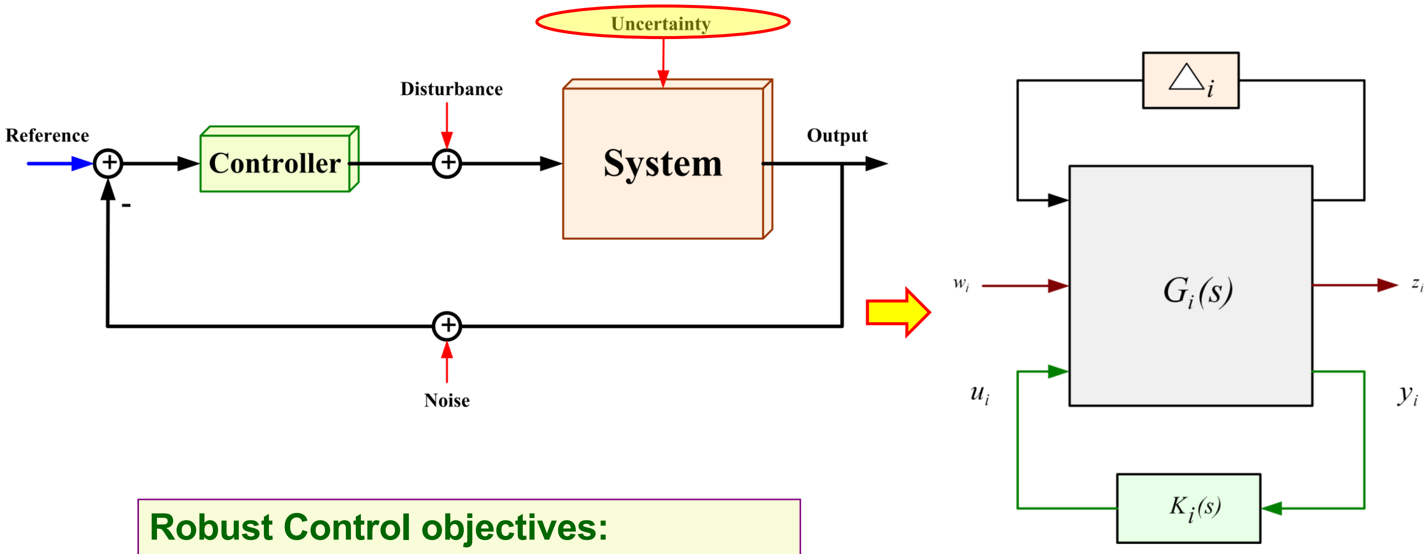




Control objectives:

- ○ **Stability**
- ○ **Performance** (reference tracking, disturbance/noise rejection, rapidness; etc.)

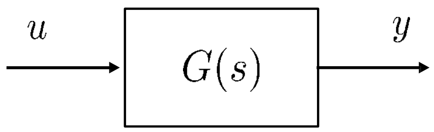
Robust Control?



Robust Control objectives:
Robust stability & Robust performance

Transfer Function and Step Response

- Transfer Function



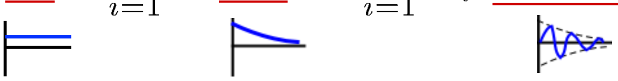
$$G(s) = \frac{N(s)}{D(s)}$$

- Step response (Laplace transform) From partial fraction decomposition

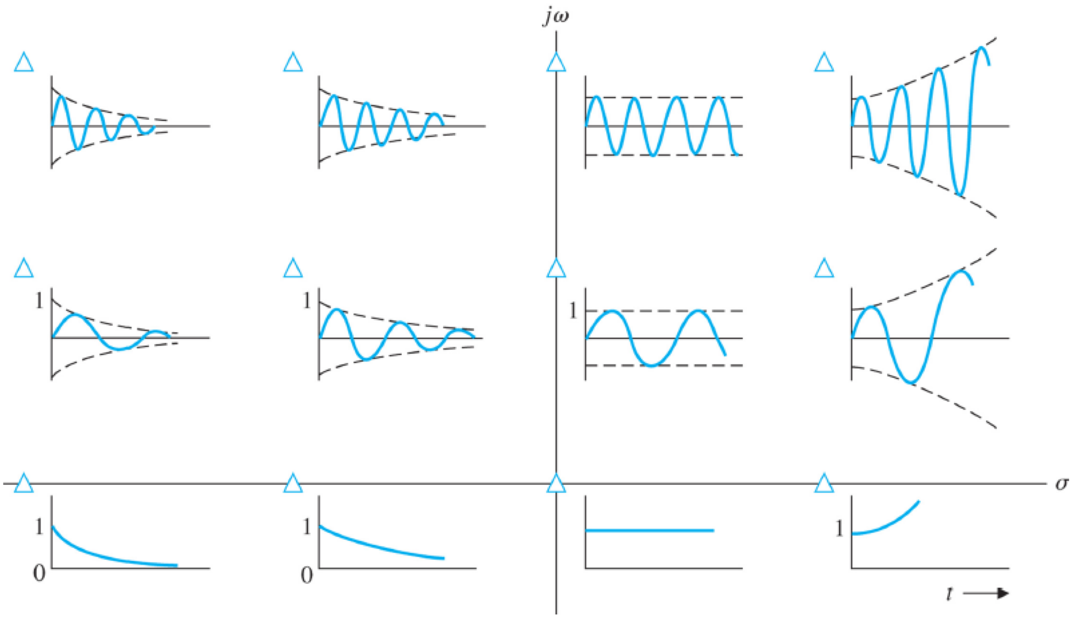
$$y(s) = G(s)\frac{1}{s} = \frac{A_0}{s} + \sum_{i=1}^{M} \frac{A_i}{s+\sigma_i} + \sum_{i=1}^{N} \frac{B_i}{(s+\alpha_i)^2 + \omega_i^2}$$

- Step response

$$y(t) = A_0 + \sum_{i=1}^{M} A_i e^{-\sigma_i t} + \sum_{i=1}^{N} \frac{B_i}{\omega_i} e^{-\alpha_i t} \sin \omega_i t$$

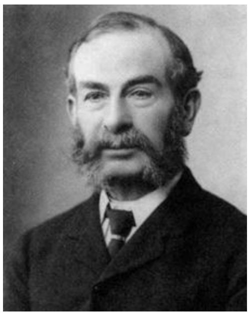


Stability and Performance

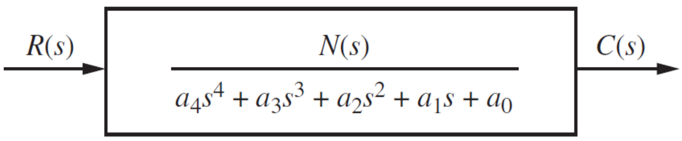


Stability Criteria (Routh Method)

- Using the "Routh Table" (1905).
- Then it is generalized to Routh-Hurwitz method



Example:

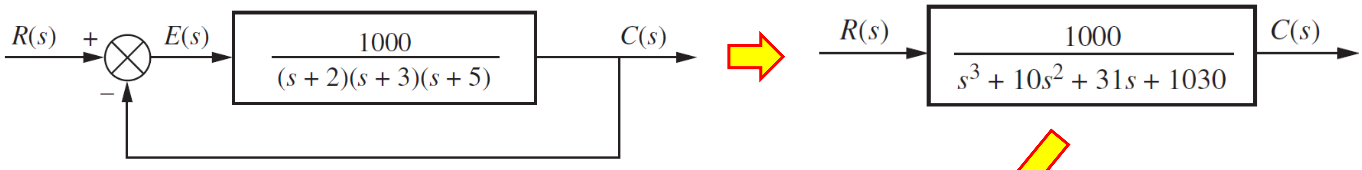


$$G(s) = \frac{N(s)}{D(s)}$$

Routh Table:

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

Example 1

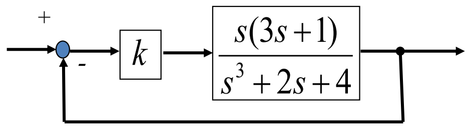


s^3	1	31	0
s^2	~~10~~ 1	~~1030~~ 103	0
s^1	$\frac{-\begin{vmatrix}1 & 31\\ 1 & 103\end{vmatrix}}{1} = -72$ (1)	$\frac{-\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix}}{1} = 0$	$\frac{-\begin{vmatrix}1 & 0\\ 1 & 0\end{vmatrix}}{1} = 0$
s^0	$\frac{-\begin{vmatrix}1 & 103\\ -72 & 0\end{vmatrix}}{-72} = 103$ (2)	$\frac{-\begin{vmatrix}1 & 0\\ -72 & 0\end{vmatrix}}{-72} = 0$	$\frac{-\begin{vmatrix}1 & 0\\ -72 & 0\end{vmatrix}}{-72} = 0$

Unstable with 2 roots at the RHP

Example 2

- **Check the stability of following system for different values of k**



$$M(s) = \frac{k\dfrac{s(3s+1)}{s^3+2s+4}}{1+k\dfrac{s(3s+1)}{s^3+2s+4}} = \frac{ks(3s+1)}{s^3+2s+4+ks(3s+1)}$$

$\Rightarrow$ $s^3 + 3ks^2 + (2+k)s + 4 = 0$ **Characteristic Equation**

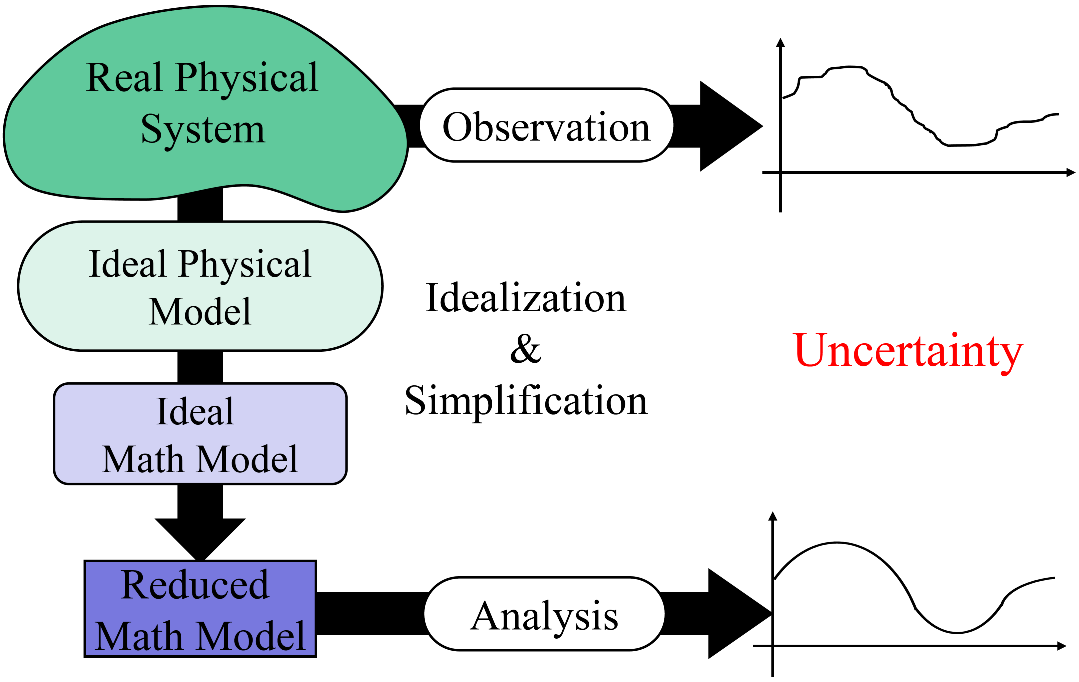
s^3	1	$2+k$
s^2	$3k$	4
s^1	$\frac{3k(2+k)-4}{3k}$	0
s^0	4	0

$\Rightarrow$ $3k > 0$, $\frac{3k^2+6k-4}{3k} > 0$ $\Rightarrow$ For stability: **k>0.528**

Robustness

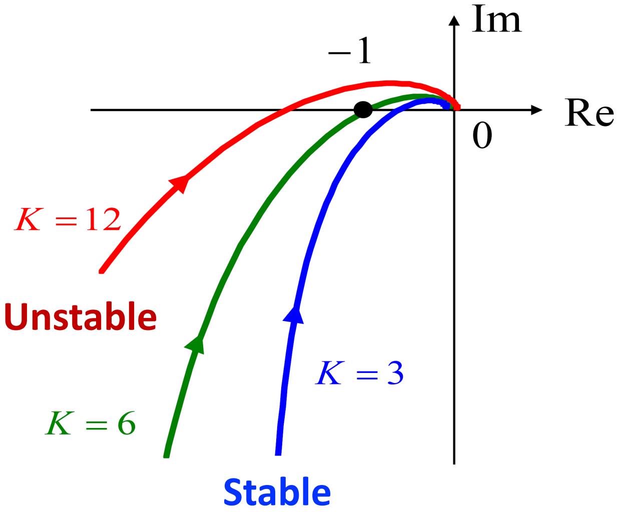
- **Nominal stability (NS)** Control system is stable with no model uncertainty
- **Robust stability (RS)** Control system is stable in the face of uncertainty
- **Nominal performance (NP)** Control system meets the performance requirements with no model uncertainty
- **Robust performance (RP)** Control system meets the performance requirements in the face of uncertainty

System and Model: Uncertainty



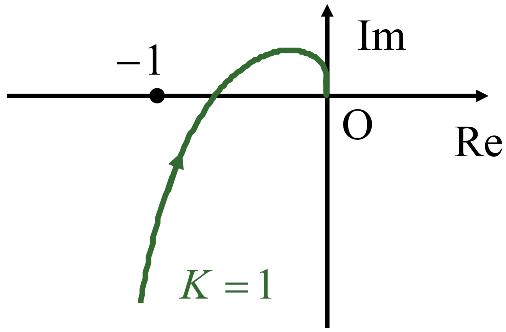
Stability: Nyquist Example

$$L(s) = \frac{K}{s(s+1)(s+2)} \qquad K = 3,\ 6,\ 12$$

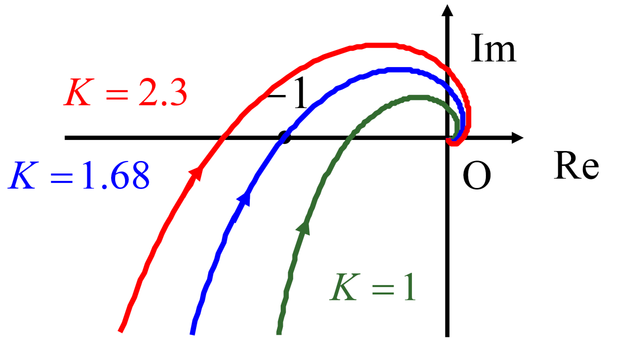


System with Uncertainty

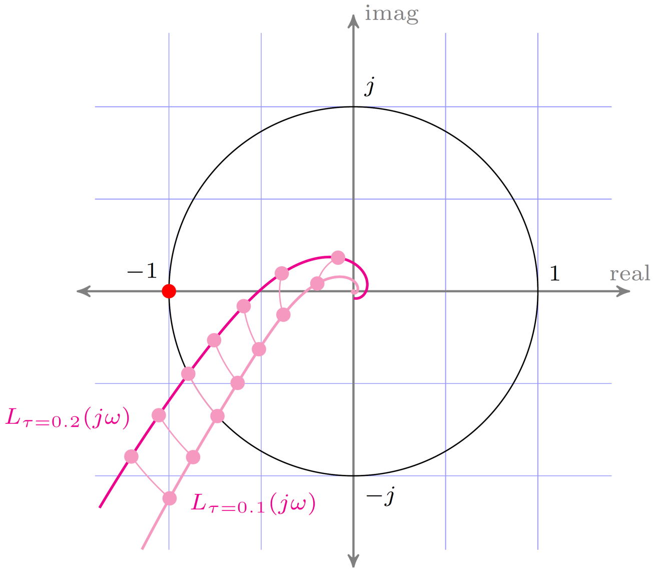
- **Nominal Model:** $L(s) = \frac{K}{s(s+1)(s+2)}$



- **Uncertain Model:** $\tilde{L}(s) = \frac{K\,e^{-s}}{s(s+1)(s+2)}$



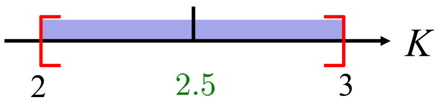
Nyquist Plot: Delay Perturbation



$$G(s) = \frac{5\,e^{-\tau s}}{(s+1)(0.1s+1)}, \quad \tau \in [0.1, 0.2] \qquad K(s) = \frac{0.5s+1}{s}$$

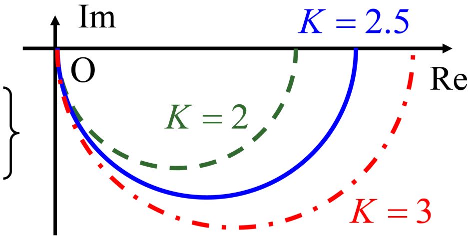
Uncertainty Modeling

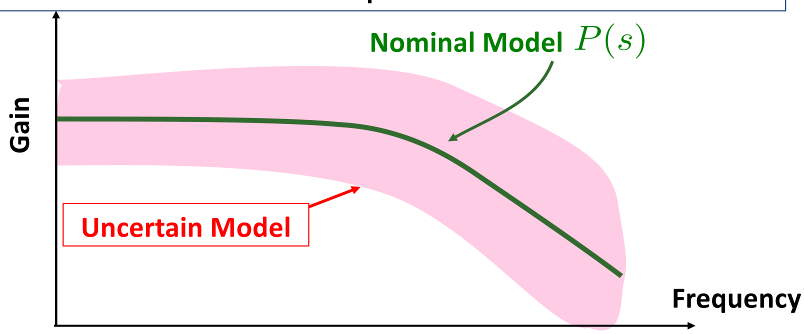
$$P(s) = \frac{K}{Ts+1} \qquad 2 \le K \le 3$$



$T = 2.5; \quad K = 2,\ 2.5,\ 3$

$$P : \left\{ \frac{2}{2.5s+1}, \frac{2.5}{2.5s+1}, \frac{3}{2.5s+1} \right\}$$

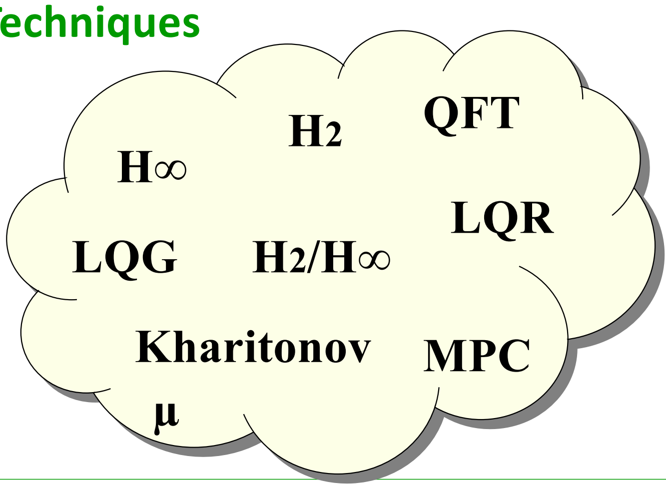




Why Robust Control?

Conventional control fails to meet the specified objectives in new environment.

- **Robust Control Techniques**



Khartonov's Theorem

- The polynomial:

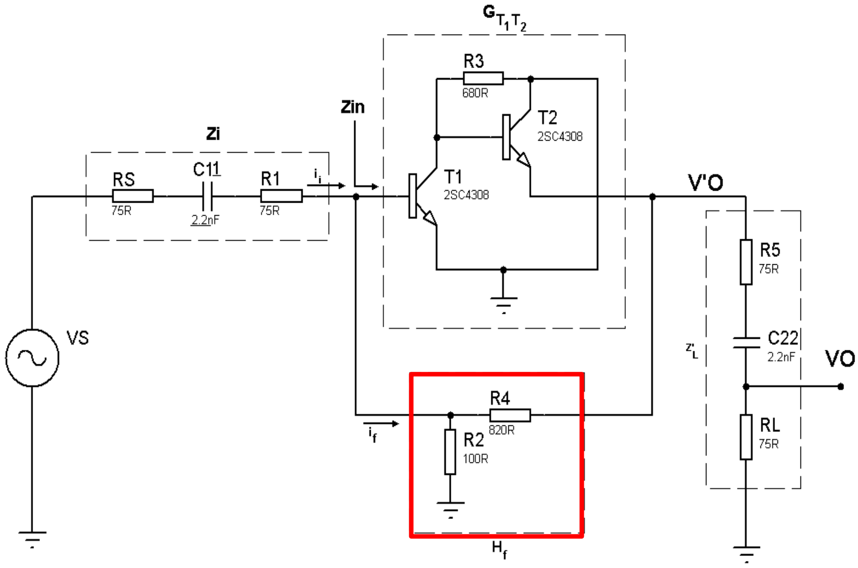
$$K(s) = a_0 s + a_1 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \cdots$$

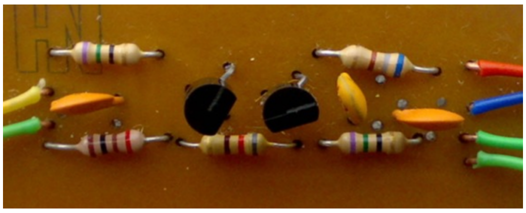
with real coefficients is Hurwitz if and only if the following four polynomials are Hurwitz:

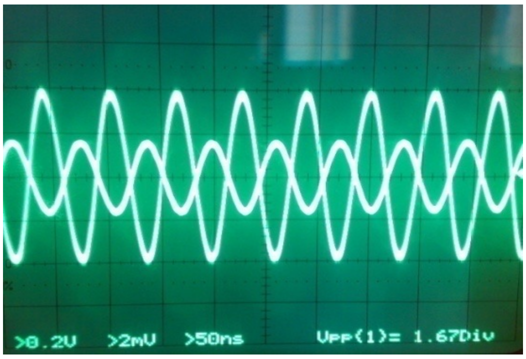
$$K_1(s) = a_0{}^{+} + a_1{}^{+} s + a_2{}^{-} s^2 + a_3{}^{-} s^3 + \cdots$$
$$K_2(s) = a_0{}^{-} + a_1{}^{-} s + a_2{}^{+} s^2 + a_3{}^{+} s^3 + \cdots$$
$$K_3(s) = a_0{}^{-} + a_1{}^{+} s + a_2{}^{+} s^2 + a_3{}^{-} s^3 + \cdots$$
$$K_4(s) = a_0{}^{+} + a_1{}^{-} s + a_2{}^{-} s^2 + a_3{}^{+} s^3 + \cdots$$

- The "-" and "+" show the minimum and maximum bounds.

Example: RF Amplifier Feedback Loop

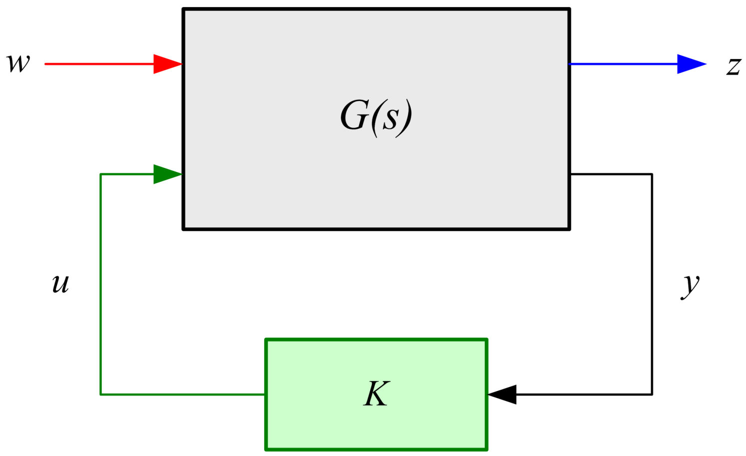






H∞ Control

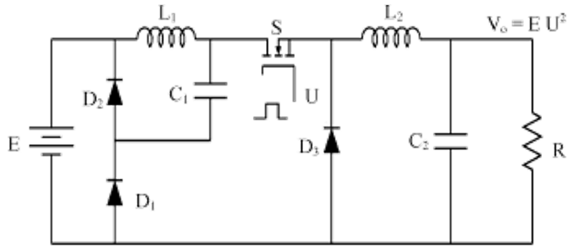
Find an admissible control law $u = K(s)y$ such that: $\|T_{zw}(s)\|_\infty < \gamma$



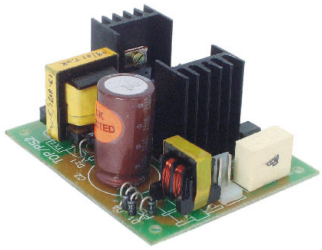
hinfsyn function in MATLAB

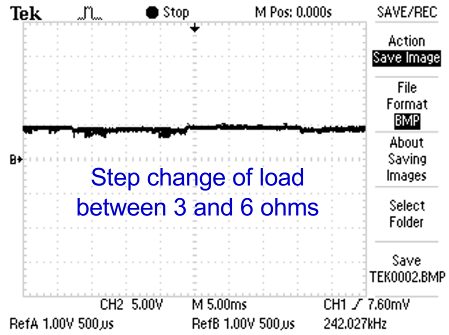
γ is the optimal H∞ performance index

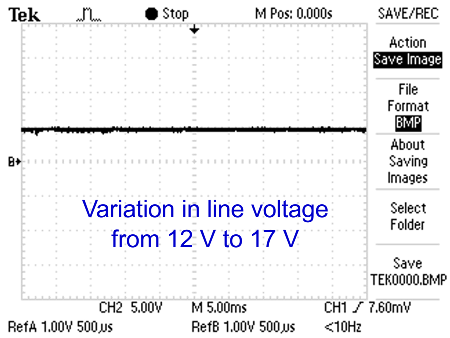
Example: Quadratic Buck Converter Control



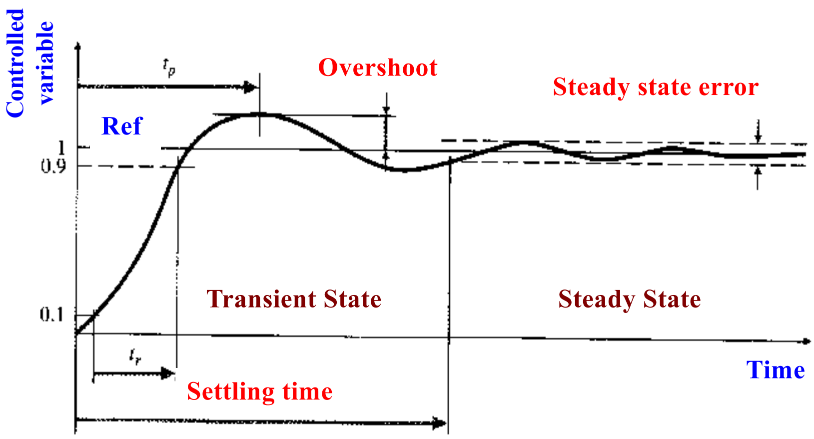
$$\begin{pmatrix} \frac{di_{l1}}{dt} \\ \frac{di_{l2}}{dt} \\ \frac{dv_{c1}}{dt} \\ \frac{dv_{c2}}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-1}{l_1} & 0 \\ 0 & 0 & \frac{u}{l_2} & \frac{-1}{l_2} \\ \frac{1}{c_1} & \frac{-u}{c_1} & 0 & 0 \\ 0 & \frac{1}{c_2} & 0 & \frac{-1}{RC_2} \end{pmatrix} \begin{pmatrix} i_{l1} \\ i_{l2} \\ Vc1 \\ Vc2 \end{pmatrix} + \begin{pmatrix} \frac{u}{l_1} \\ 0 \\ 0 \\ 0 \end{pmatrix} e(t)$$

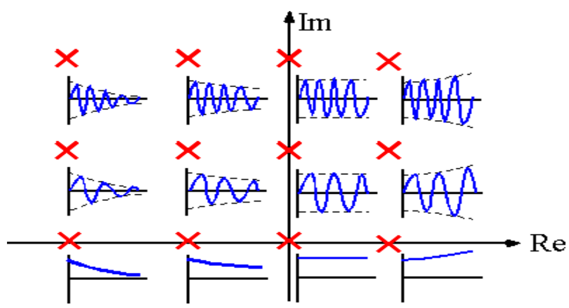




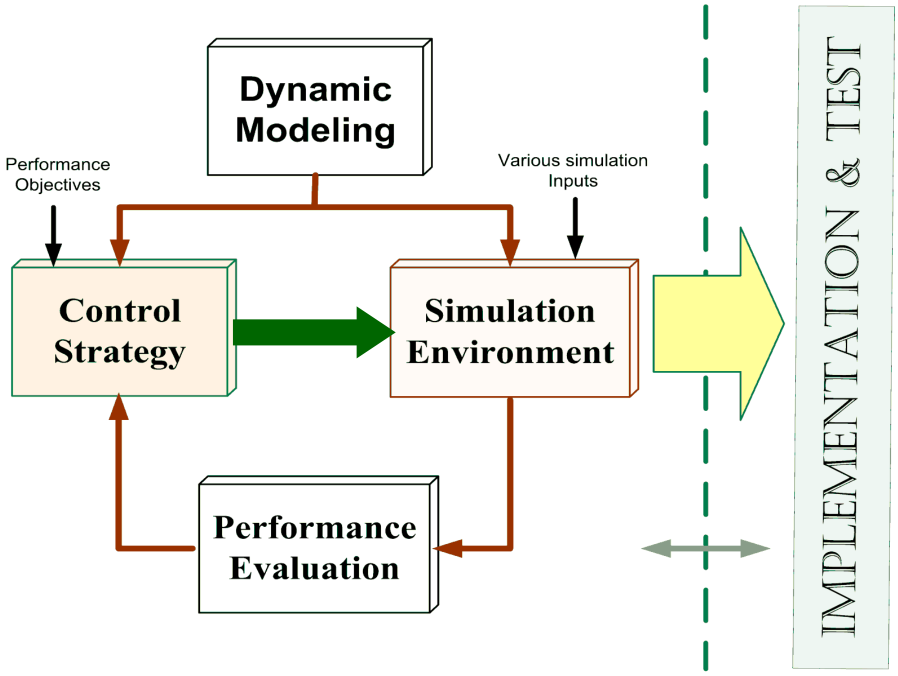


Stability and Performance Characteristics





Control Design Procedure



Thank You!