Robust Control Systems

Kharitonov Theorem

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Contents

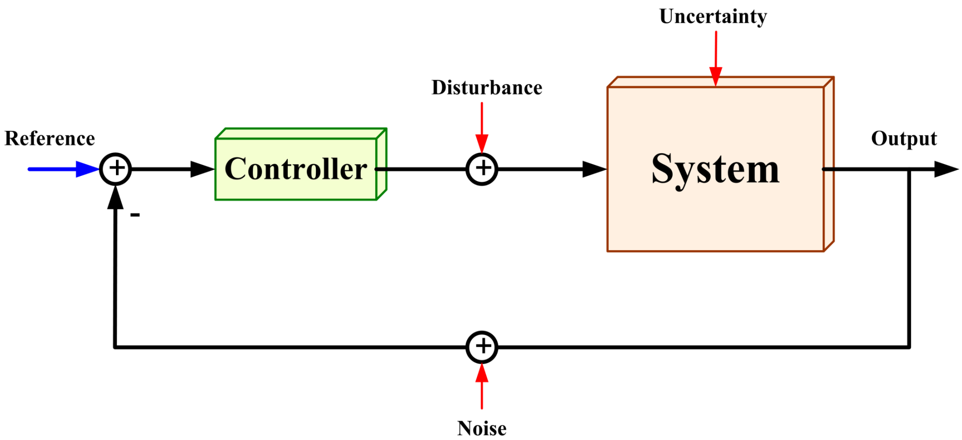
Reference

1. B. R. Barmish, **New Tools for Robustness of Linear Systems**, Macmillan, 1994.

2. H. Bevrani, S. Shokoohi, **Robust Stabilizer Feedback Loop Design for a Radio-Frequency Amplifier**, 2010 IEEE International Conference on Control Applications, Yokohama, Japan, 2010.

3. F. Habibi, A. Hesami, and H. Bevrani, **Robust voltage controller design for an isolated Microgrid using Kharitonov's theorem and D-stability concept**, Int. J. of Electrical Power and Energy Systems, vol. 44, 2013.

Feedback Control System



Control objectives:

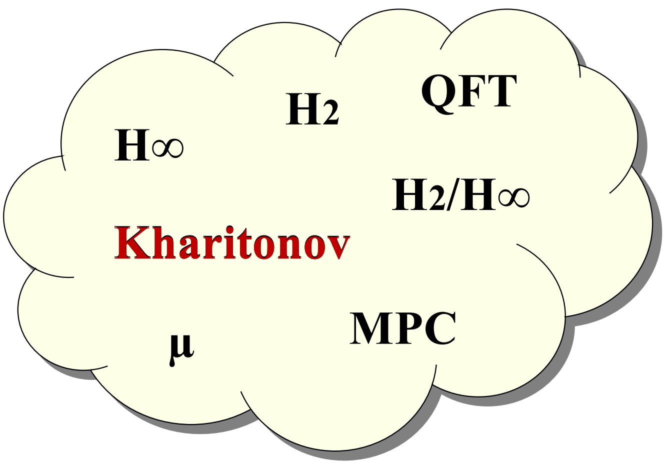
Stability, reference tracking,
disturbance attenuation and noise rejection;
in the presence of system uncertainties and practical constraints

Robustness

- **Nominal stability (NS)** Control system is stable with no model uncertainty
- **Robust stability (RS)** Control system is stable in the face of uncertainty
- **Nominal performance (NP)** Control system meets the performance requirements with no model uncertainty
- **Robust performance (RP)** Control system meets the performance requirements in the face of uncertainty

Robust Control Techniques

Conventional control may fail to meet the control objectives in the presence of uncertainty.



Khartonov's Theorem

- The polynomial:

$$K(s) = a_0 s + a_1 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \cdots$$

with real coefficients is Hurwitz if and only if the following four polynomials are Hurwitz:

$$K_1(s) = \; {a_0}^+ + {a_1}^+ \; s + {a_2}^- s^2 + {a_3}^- s^3 + \cdots$$

$$K_2(s) = \; {a_0}^- + {a_1}^- \; s + {a_2}^+ s^2 + {a_3}^+ s^3 + \cdots$$

$$K_3(s) = \; {a_0}^- + {a_1}^+ \; s + {a_2}^+ s^2 + {a_3}^- s^3 + \cdots$$

$$K_4(s) = \; {a_0}^+ + {a_1}^- \; s + {a_2}^- s^2 + {a_3}^+ s^3 + \cdots$$

- The "-" and "+" show the minimum and maximum bounds.

Definition

Consider the following polynomial

$$p(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0$$

Assume that in the face of uncertain parameters q the polynomial is expressed *as*

$$p(s,q) = \sum_{i=0}^{n} a_i(q) s^i$$

The uncertainty structure is manifested via the coefficient functions a_n(q), a_{n-1}(q), … , a_0(q). Here, each component q_i of q enters only one coefficient.

Definition

Uncertainty Bounding Set

For robustness problems, we often assume a bound Q (*uncertainty bounding set*) for the vector of uncertain parameters *q.*

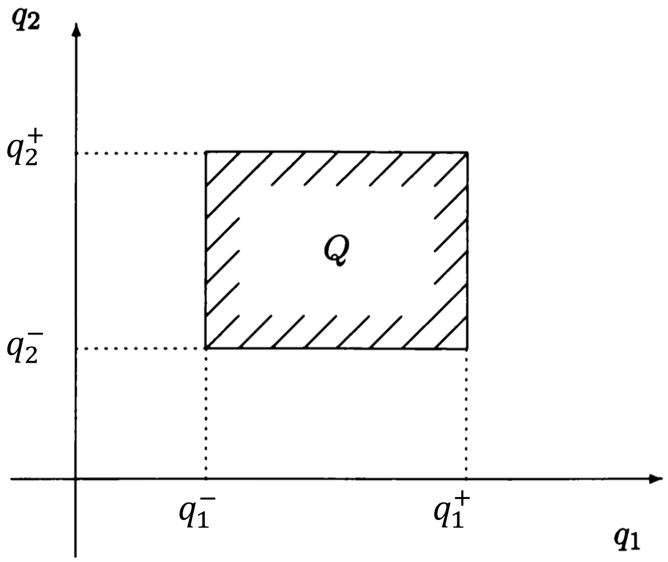
$$p(s,q) = \sum_{i=0}^{n} a_i(q)s^i \qquad q \in Q$$

$$Q = \{q \in \mathbf{R}^{\ell} : q_i^- \leq q_i \leq q_i^+ \text{ for } i = 1, 2, \ldots, \ell\}$$

where q_i^- and q_i^+ are the specified bounds for the *i-th* component Q_i of q.

Definition

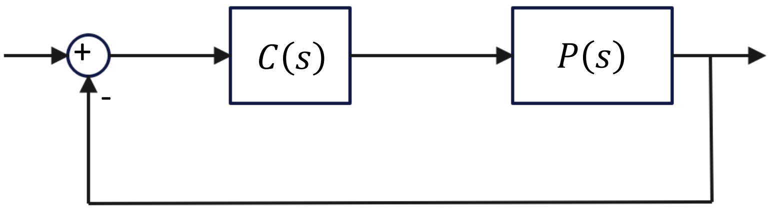
Example



Example

$C(s) = 1$

$P(s,q) = \dfrac{1}{s-q}$



Closed-loop polynomial (characteristic equation):

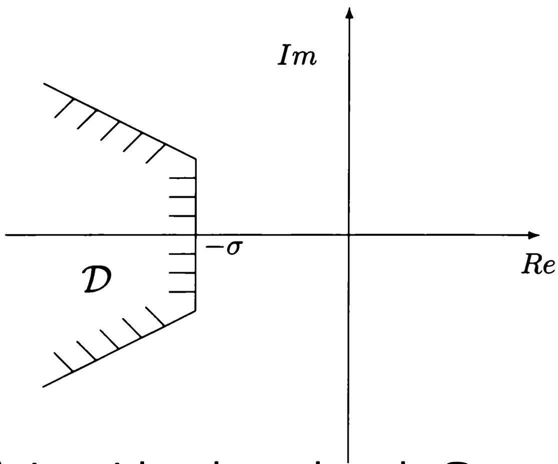
$p(s,q) = s + 1 - q \Rightarrow s = -1 + q$

$p(s,q) \in \mathcal{P}$

$\mathcal{P}$ is robustly stable if and only if $q < 1$

$\mathcal{D}$-Stability Concept

$$p(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0$$



$\mathcal{D}$-Stability:

Let $\mathcal{D} \subseteq \mathcal{C}$ and take $p(s)$ to be a fixed polynomial. Then $p(s)$ is said to be $\mathcal{D}$-*stable* if all its roots lie in the region $\mathcal{D}$.

Robust $\mathcal{D}$-Stability:

A family of polynomials $P = \{p(s,q): q \in Q\}$ is said to be robustly $\mathcal{D}$-stable if for all $q \in Q$, $p(s,q)$ is $\mathcal{D}$-stable.

Definition

Independent Uncertainty Structure:

An uncertain polynomial like $$p(s,q) = \sum_{i=0}^{n} a_i(q)s^i$$

is said to have an *independent uncertainty structure* if each component q_i of q enters into only one coefficient.

Interval Polynomial Family:

A family of polynomials $\mathcal{P} = \{p(.,q): q \in Q\}$ is said to be an *interval polynomial family* if $p(s,q)$ has an independent uncertainty structure.

The Kharitonov Polynomials

The Kharitonov polynomials are four fixed polynomials associated with

$$p(s,q) = \sum_{i=0}^{n} [q_i^-, q_i^+] s^i$$

as follows:

$$\begin{aligned}
K_1(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + q_4^- s^4 + q_5^- s^5 + q_6^+ s^6 + \cdots;\\
K_2(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + q_4^+ s^4 + q_5^+ s^5 + q_6^- s^6 + \cdots;\\
K_3(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + q_4^+ s^4 + q_5^- s^5 + q_6^- s^6 + \cdots;\\
K_4(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + q_4^- s^4 + q_5^+ s^5 + q_6^+ s^6 + \cdots.
\end{aligned}$$

Kharitonov, V. L., "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations," *Differentsial'nye Uravneniya,* vol. 14, pp. 2086-2088, 1978.

Example

The Kharitonov polynomials for

$$p(s,q) = [1,2]s^5 + [3,4]s^4 + [5,6]s^3 + [7,8]s^2 + [9,10]s + [11,12]$$

are:

$$\begin{aligned}
K_1(s) &= 11 + 9s + 8s^2 + 6s^3 + 3s^4 + s^5; \\
K_2(s) &= 12 + 10s + 7s^2 + 5s^3 + 4s^4 + 2s^5; \\
K_3(s) &= 12 + 9s + 7s^2 + 6s^3 + 4s^4 + s^5; \\
K_4(s) &= 11 + 10s + 8s^2 + 5s^3 + 3s^4 + 2s^5.
\end{aligned}$$

Kharitonov Theorem (1978)

An interval polynomial family $p(s,q) \in \mathcal{P}$ with invariant degree is robustly stable if and only if its four Kharitonov polynomials are stable.

Example: The interval polynomial

$$p(s,q) = [0.25, 1.25]s^3 + [2.75, 3.25]s^2 + [0.75, 1.25]s + [0.25, 1.25],$$

is robustly stable if the following polynomials to be stable:

$$\begin{aligned}
K_1(s) &= 0.25 + 0.75s + 3.25s^2 + 1.25s^3; \\
K_2(s) &= 1.25 + 1.25s + 2.75s^2 + 0.25s^3; \\
K_3(s) &= 1.25 + 0.75s + 2.75s^2 + 1.25s^3; \\
K_4(s) &= 0.25 + 1.25s + 3.25s^2 + 0.25s^3.
\end{aligned}$$

Kharitonov, V. L., "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations," *Differentsial'nye Uravneniya,* vol. 14, pp. 2086-2088, 1978.

Kharitonov Rectangle

- Given an interval polynomial $p(s,q)=\sum_{i=0}^{n}[q_i^-,q_i^+]s^i$ at a fixed frequency $\omega=\omega_0$, the $p(j\omega_0,Q)=\{p(j\omega_0,q):q\in Q\}$ is called the Kharitonove rectangle.

- It can be proved that $p(j\omega_0,Q)$ is a rectangle with vertices which are obtained by evaluating the four *fixed* Kharitonov polynomials $K_1(s)$, $K_2(s)$, $K_3(s)$ and $K_4(s)$ at $\omega=\omega_0$.

Kharitonov Rectangle

$$Re\ p(j\omega_0,q)=\sum_{i\ even} q_i(j\omega_0)^i = q_0 - q_2\omega_0^2 + q_4\omega_0^4 - q_6\omega_0^6 + q_8\omega_0^8 - \cdots$$

$$Im\ p(j\omega_0,q)=\frac{1}{j}\sum_{i\ odd} q_i(j\omega_0)^i = q_1\omega_0 - q_3\omega_0^3 + q_5\omega_0^5 - q_7\omega_0^7 + q_9\omega_0^9 - \cdots$$

for $\omega_0 \geq 0$

$$\min_{q\in Q} Re\ p(j\omega_0,q) = q_0^- - q_2^+\omega_0^2 + q_4^-\omega_0^4 - q_6^+\omega_0^6 + q_8^-\omega_0^8 - \cdots = Re\ K_1(j\omega_0)$$

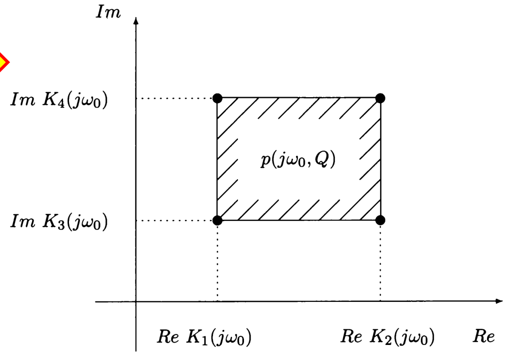
$$\max_{q\in Q} Re\ p(j\omega_0,q) = q_0^+ - q_2^-\omega_0^2 + q_4^+\omega_0^4 - q_6^-\omega_0^6 + q_8^+\omega_0^8 - \cdots = Re\ K_2(j\omega_0)$$

$$\min_{q\in Q} Im\ p(j\omega_0,q) = q_1^-\omega_0 - q_3^+\omega_0^3 + q_5^-\omega_0^5 - q_7^+\omega_0^7 + \cdots = Im\ K_3(j\omega_0)$$

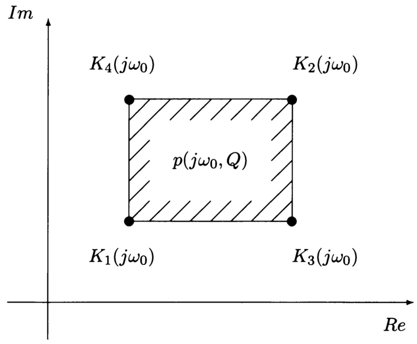
$$\max_{q\in Q} Im\ p(j\omega_0,q) = Im\ K_4(j\omega_0)$$

Kharitonov Rectangle

- The Kharitonov Rectangle for $\omega \geq 0$ gives this rectangle graph:
- Relating the vertices of the rectangle to the Kharitonov polynomials gives simpler graph:



$$
\begin{aligned}
\text{Southwest Vertex} &= Re\ K_1(j\omega_0) + jIm\ K_3(j\omega_0) \\
&= Re\ K_1(j\omega_0) + jIm\ K_1(j\omega_0) \\
&= K_1(j\omega_0); \\
\text{Northeast Vertex} &= Re\ K_2(j\omega_0) + jIm\ K_4(j\omega_0) \\
&= Re\ K_2(j\omega_0) + jIm\ K_2(j\omega_0) \\
&= K_2(j\omega_0); \\
\text{Southeast Vertex} &= Re\ K_2(j\omega_0) + jIm\ K_3(j\omega_0) \\
&= Re\ K_3(j\omega_0) + jIm\ K_3(j\omega_0) \\
&= K_3(j\omega_0); \\
\text{Northwest Vertex} &= Re\ K_1(j\omega_0) + jIm\ K_4(j\omega_0) \\
&= Re\ K_4(j\omega_0) + jIm\ K_4(j\omega_0) \\
&= K_4(j\omega_0).
\end{aligned}
$$

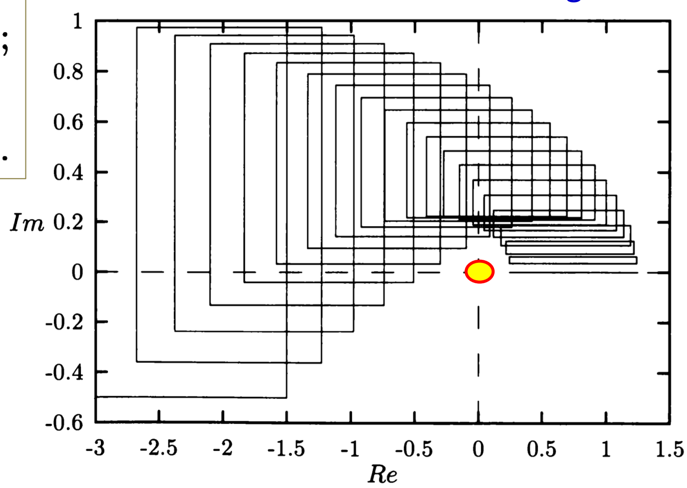


Kharitonov Rectangle: Example

$$p(s,q) = [1,2]s^5 + [3,4]s^4 + [5,6]s^3 + [7,8]s^2 + [9,10]s + [11,12]$$

$$
\begin{aligned}
K_1(s) &= 11 + 9s + 8s^2 + 6s^3 + 3s^4 + s^5; \\
K_2(s) &= 12 + 10s + 7s^2 + 5s^3 + 4s^4 + 2s^5; \\
K_3(s) &= 12 + 9s + 7s^2 + 6s^3 + 4s^4 + s^5; \\
K_4(s) &= 11 + 10s + 8s^2 + 5s^3 + 3s^4 + 2s^5.
\end{aligned}
$$

Motion of Kharitonov Rectangle



Zero Exclusion Condition

Lemma (Zero Exclusion Condition):

Suppose that an interval polynomial family

$$\mathcal{P} = \{p(.,q): q \in Q\}$$

has invariant degree and at least one stable member $p(s, q^0)$. Then $\mathcal{P}$ is robustly stable if and only if $z = 0$ is excluded from the Kharitonov rectangle at all nonnegative frequencies; i.e.,

$$0 \notin p(j\omega, Q)$$

for all frequencies $\omega \geq 0$.

The Zero Exclusion Condition suggests a simple graphical procedure for checking robust stability.

Zero Exclusion Condition: Example

$$p(s,q) = s^6 + [3.95, 4.05]s^5 + [3.95, 4.05]s^4 + [5.95, 6.05]s^3 + [2.95, 3.05]s^2 + [1.95, 2.05]s + [0.45, 0.55].$$

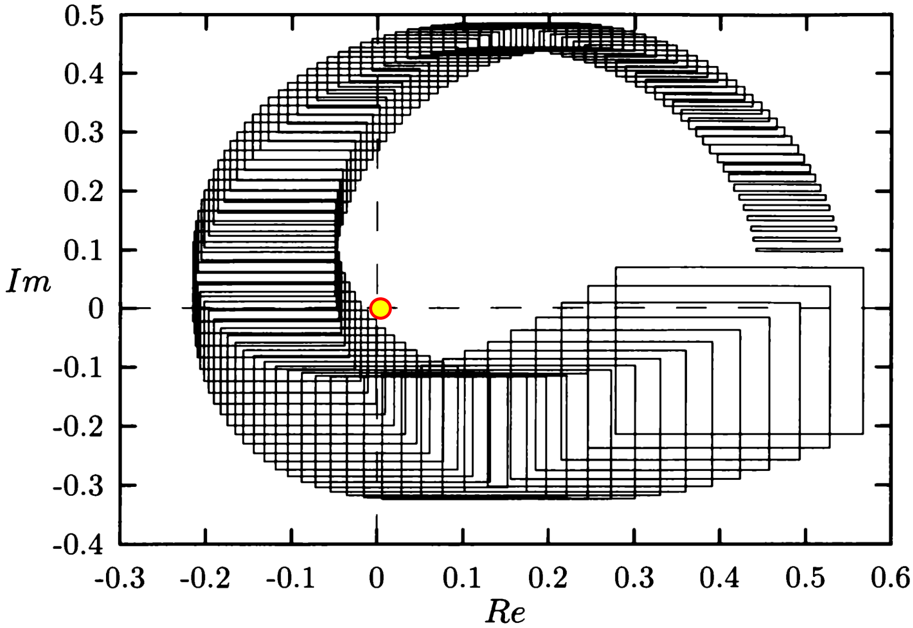
In accordance with Lemma, the first step in the graphical test for robust stability requires that we guarantee that at least one polynomial in $\mathcal{P}$ is stable. Using the midpoint of each interval above, we obtain

$$p(s,q^0) = s^6 + 4s^5 + 4s^4 + 6s^3 + 3s^2 + 2s + 0.5$$

which its roots are stable as:

$s_1 \approx -3.2681,\ s_{2,3} \approx -0.1328 \pm 0.9473j,\ s_{4,5} \approx -0.0731 \pm 0.7190j$ and $s_6 \approx -0.3201$.

Zero Exclusion Condition: Example



A zoomed view of the plot using 100 evenly spaced frequencies in the range of $0 \leq \omega \leq 1$.

Low-order Interval Polynomials ($\leq$ 5)

Anderson, Jury and Mansour (1987):

Less than four Kharitonov polynomials are needed for robust stability testing when an interval polynomial has order of 5 or less :

Order 3 (and $q_0^- > 0$): Only it is enough to test $K_3(s)$.

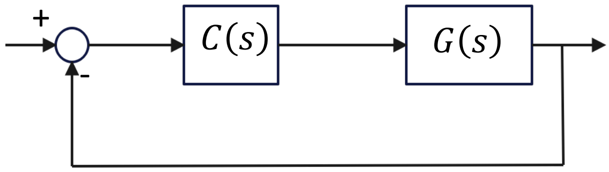
Order 4 (and $q_0^- > 0$): Only it is enough to test $K_2(s)$ and $K_3(s)$.

Order 5: Only it is enough to test $K_2(s)$, $K_3(s)$. and $K_4(s)$.

Anderson, B. D. 0., E. I. Jury and M. Mansour, "On robust Hurwitz polynomials," *IEEE Transactions on Automatic Control,* vol. AC-32, pp. 909-913, 1987.

Control Design Example

$$G(s) = \frac{n_2 s^2 + n_1 s + n_0}{s^3 + d_2 s^2 + d_1 s + d_0}, \qquad C(s) = k$$



where $n_0 \in [1, 2.5], \quad n_1 \in [1, 6], \quad n_2 \in [1, 7],$
$d_2 \in [-1, 1], \quad d_1 \in [-0.5, 1.5], \quad d_0 \in [1, 1.5].$

Solution:

The closed-loop polynomial (characteristic equation):

$$p(s) = s^3 + (d_2 + kn_2)\, s^2 + (d_1 + kn_1) s + (d_0 + kn_0)$$

$$p(s) = q_3 s^3 + q_2 s^2 + q_1 s + q_0$$

⇨

$q_3 = 1$

$q_2 \in [q_2^-, q_2^+] = [-1 + k, 1 + 7k]$

$q_1 \in [q_1^-, q_1^+] = [-0.5 + k, 1.5 + 6k]$

$q_0 \in [q_0^-, q_0^+] = [1 + k, 1.5 + 2.5k]$

Control Design Example

Since, the polynomial order is 3 (and $q_0^- > 0$): Only it is enough that $K_3(s)$ to be Hurwitz.

$$K_3(s) = q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3$$

⇨ $$K_3(s) = [1.5 + 2.5k] + [-0.5 + k]s + [-1 + k]s^2 + s^3$$

The closed-loop polynomial (characteristic equation):

s^3	1	$-0.5 + k$	
s^2	$-1 + k$	$1.5 + 2.5k$	⇨ $k > 1$
s^1	$\frac{k^2 - 4k - 1}{-1 + k}$		⇨ $k < 2 - \sqrt{5}, \; k > 2 + \sqrt{5}$
s^0	$1.5 + 2.5k$		⇨ $k > \frac{-3}{5}$

⇨ $k \in (2 + \sqrt{5}, +\infty)$

Example: Robust Voltage Regulator for an Inverter

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Robust voltage controller design for an isolated Microgrid using Kharitonov's theorem and D-stability concept

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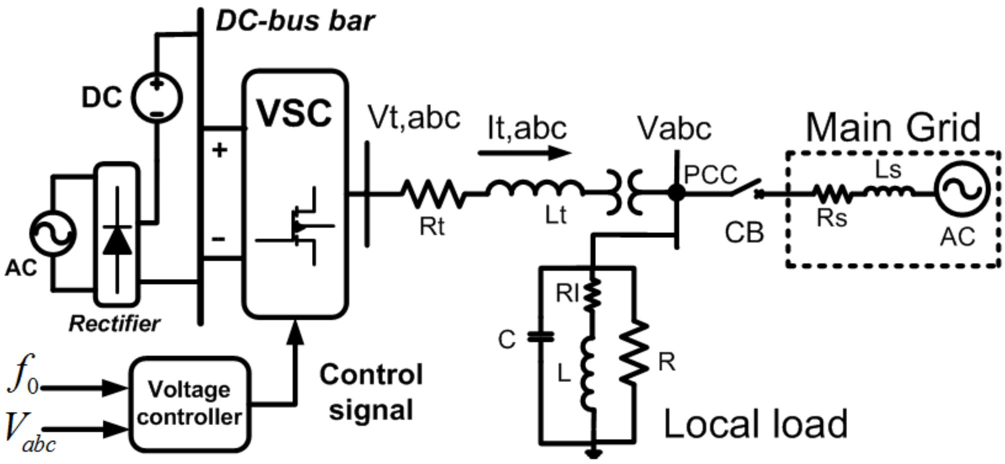
Case Study: A Grid Connected Converter



Dynamic Model: Open-loop System

$$\frac{v_d}{v_{td}} = \frac{N(s)}{D(s)} = \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

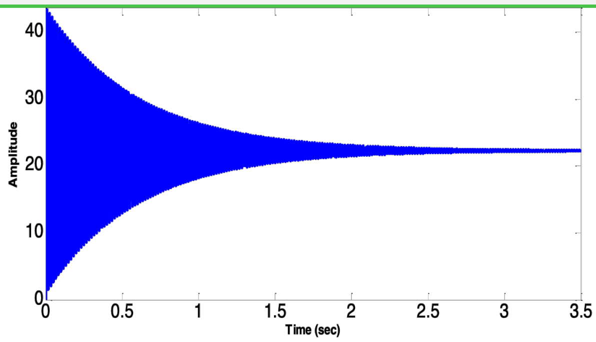
⇓

$$g_n(s) = \frac{7.778e7 s^2 + 1.101e6 s + 2.462e14}{s^4 + 144.2 s^3 + 7.789e7 s^2 + 2.777e8 s + 1.105e13}$$

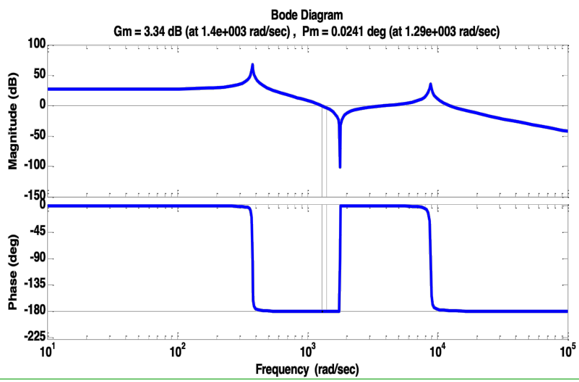
Open-loop system order is **4**.

Open-loop System Response

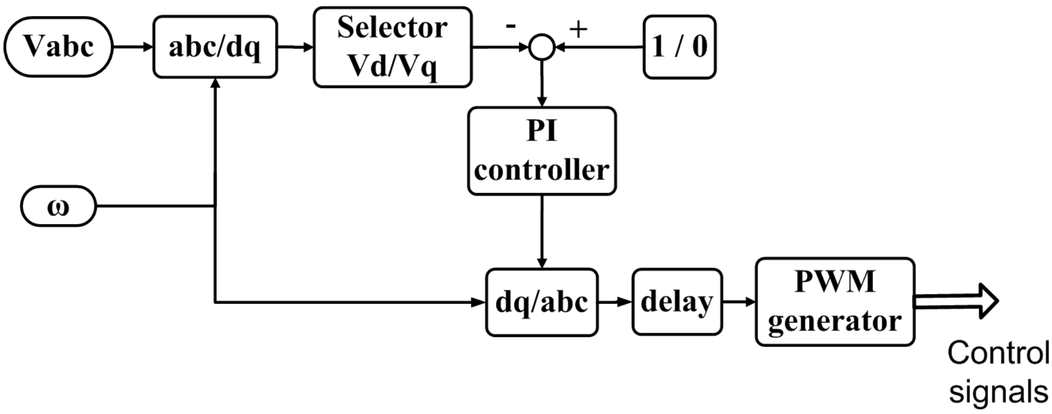
Step response



Bode diagram



Control Framework



Kharitonov's Theorem

- The polynomial

$$K(s) = c_0 + c_1 s + c_2 s^2 + c_3 s^3 + c_4 s^4 + \dots$$

with real coefficients is Hurwitz if and only if the following four extreme polynomials are Hurwitz:

$$K_1(s) = c_0^+ + c_1^+ s + c_2^- s^2 + c_3^- s^3 + \dots$$
$$K_2(s) = c_0^- + c_1^- s + c_2^+ s^2 + c_3^+ s^3 + \dots$$
$$K_3(s) = c_0^- + c_1^+ s + c_2^+ s^2 + c_3^- s^3 + \dots$$
$$K_4(s) = c_0^+ + c_1^- s + c_2^- s^2 + c_3^+ s^3 + \dots$$

- The "-" and "+" show the min and max bounds.

Tuning of a PI parameters

$$C(s) = K_p + \frac{K_i}{s}$$

- Characteristic equation of the closed-loop system (5th order):

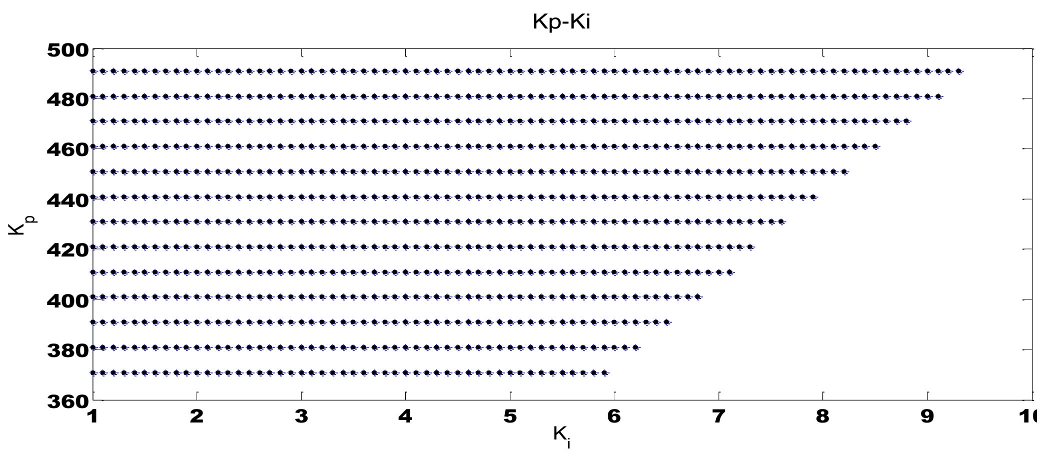
$$K_{closed\_loop}(s) = s^5 + c_4 s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0$$

$$\begin{aligned} c_4 &= a_3 \\ c_3 &= a_2 + b_2 K_p \\ c_2 &= a_1 + b_2 K_i + b_1 K_p \\ c_1 &= a_0 + b_1 K_i + b_0 K_p \\ c_0 &= b_0 K_i \end{aligned}$$

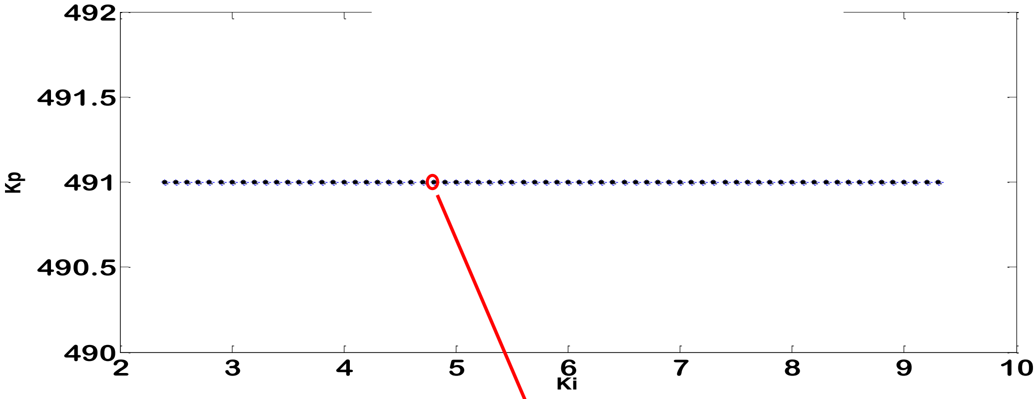
- In this case (5th order), it is enough to apply Hurwitz criteria on 3 polynomials: K_1, K_3, K_4

Tuning of a PI parameters

- ±10% change is considered for the system parameters.
- Acceptable values of K_p and K_i to stabilize closed system are graphically shown below.

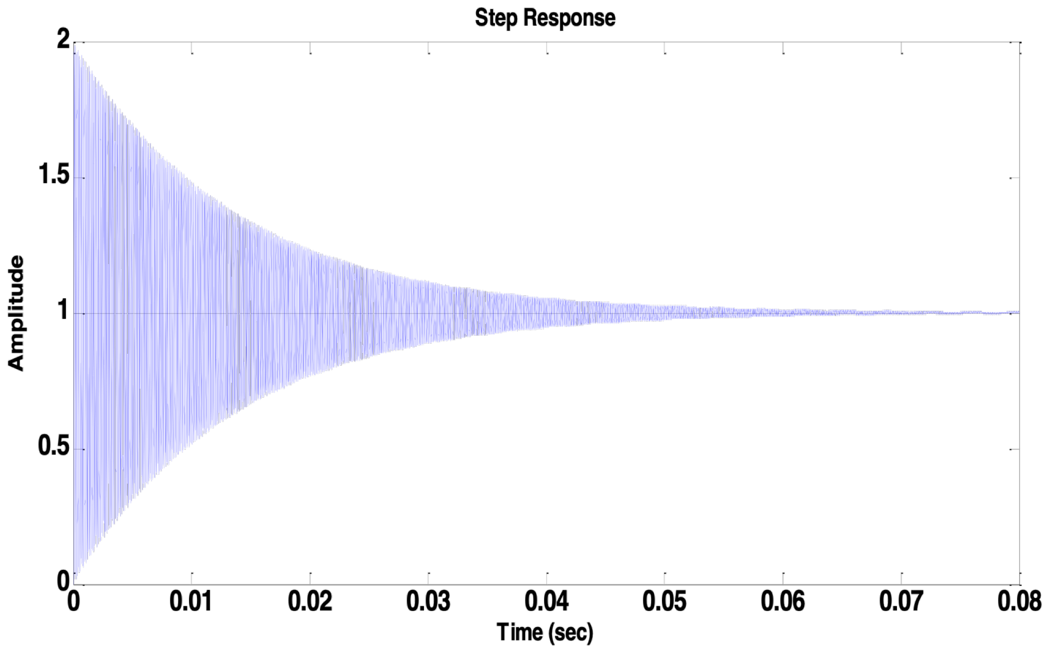


Tuning Using D-Stability Concept



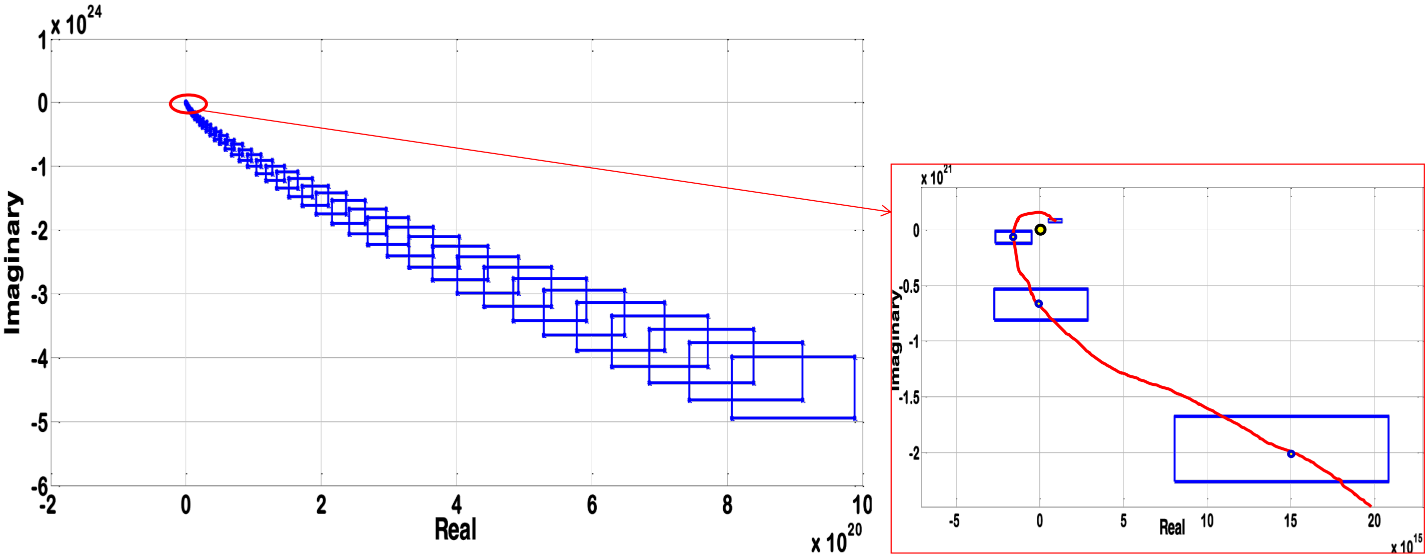
Controller Parameter	Value
K_p	491
K_i	4.9

Closed-loop System Response



Kharitonov's Rectangles

$$K_1(s), K_2(s), K_3(s), K_4(s): \quad s = j\omega, \qquad 0 \le \omega \le 50\ kHz$$

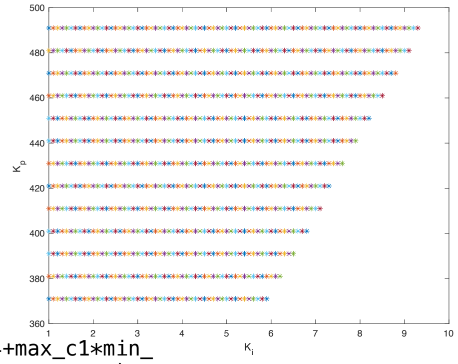


MATLAB Code

```
clear;clc;
% initial values of system parameters
Rt=1.5e-3; Lt=300e-6; Sb=2.5e6; Vll=600; Fsw=1980; Vdc=1500; f=60; W0=(2*pi*60);
R=176; L=0.2119; C=42.855e-6; Rl=0.3515; % load
%Coefficients of open-loop system transfer function
% Den Coefficients
a4a=Lt*R*L^2*C;
a3=(Lt*L^2+Rt*R*C*L^2+2*2*Rl*R*L*C*Lt)/a4a;
a2=(Lt*R*L+R*L^2+Rt*L^2+2*Rl*Rt*R*C*L+2*Rl*Lt*L+Lt*R*C*Rl^2)/a4a;
a1=(Rt*R*L+2*Rl*R*L+Rt*Rl^2*R*C+2*Rt*Rl*L+Rt*R*L^2*C*W0^2+Lt*Rl^2+Lt*L^2*W0^2-2*Rl*R*L*C*W0^2*Lt)/a4a;
a0=(Rt*Rl*R+R*L^2*W0^2+Rt*L^2*W0^2+Rl^2*R+Rl^2*Rt+R*L*W0^2*Lt-Rt^2*R*C*W0^2*Lt-R*L^2*C*W0^4*Lt)/a4a;
a4=1;
% Num Coefficients
b2=(R*L^2)/a4a;
b1=(R*L^2*2*Rt/L)/a4a;
b0=(R*L^2*((W0^2+Rt^2)/L^2))/a4a;
t_f=tf([b2 b1 b0],[a4 a3 a2 a1 a0]); %Open-loop system
% ---------------------------------------
% Application of kharitonov theorem
% Parameter variation ranges
max_a3=a3*1.1;min_a3=a3*.9;
max_a2=a2*1.1;min_a2=a2*.9;
max_a1=a1*1.1;min_a1=a1*.9;
max_a0=a0*1.1;min_a0=a0*.9;
max_b0=b0*1.1;min_b0=b0*.9;
max_b1=b1*1.1;min_b1=b1*.9;
max_b2=b2*1.1;min_b2=b2*.9;
figure(1); % kp & ki's diagram
```

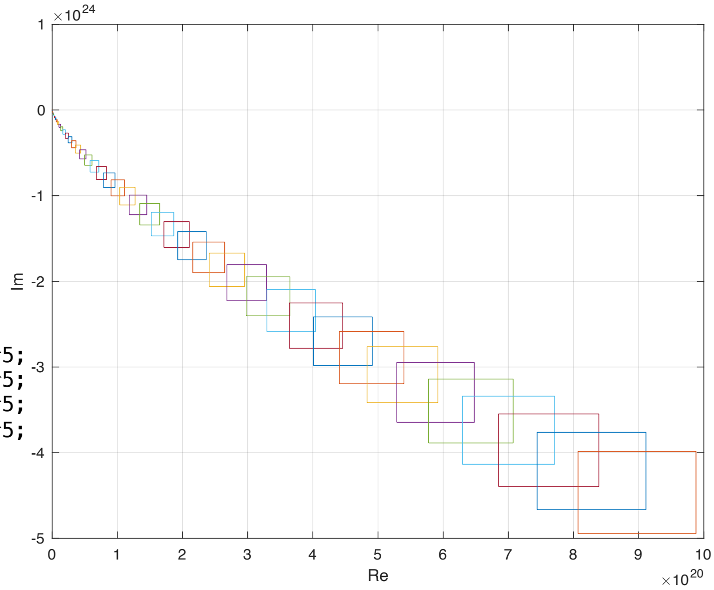
MATLAB Code

```
for kp=1:10:500
for ki=1:.1:10
min_c0=min_b0*ki; max_c0=max_b0*ki;
min_c1=min_a0+min_b1*ki+min_b0*kp;
max_c1=max_a0+max_b1*ki+max_b0*kp;
min_c2=min_a1+min_b2*ki+min_b1*kp;
max_c2=max_a1+max_b2*ki+max_b1*kp;
min_c3=min_a2+min_b2*kp;
max_c3=max_a2+max_b2*kp;
min_c4=min_a3; max_c4=max_a3;
if min_c2<min_c3*max_c4
if -(min_c2)^2+min_c2*min_c3*max_c4-(max_c4)^2*max_c1>-max_c0*max_c4
if -
(max_c4)^3*(max_c1)^2+2*max_c0*max_c1*(max_c4)^2+max_c0*min_c2*min_c3*max_c4+max_c1*min_
c2*min_c3*(max_c4)^2-max_c1*max_c4*(min_c2)^2>(max_c0)^2*max_c4+max_c0*(min_c3*max_c4)^2
if max_c2<min_c3*min_c4
if -(max_c2)^2+max_c2*min_c3*min_c4-(min_c4)^2*max_c1>-min_c0*min_c4
if -
(min_c4)^3*(max_c1)^2+2*min_c0*max_c1*(min_c4)^2+min_c0*max_c2*min_c3*min_c4+max_c1*max_
c2*min_c3*(min_c4)^2-max_c1*min_c4*(max_c2)^2>(min_c0)^2*min_c4+min_c0*(min_c3*min_c4)^2
if min_c2<max_c3*max_c4
if -(min_c2)^2+min_c2*max_c3*max_c4-(max_c4)^2*min_c1>-max_c0*max_c4
if -
(max_c4)^3*(min_c1)^2+2*max_c0*min_c1*(max_c4)^2+max_c0*min_c2*max_c3*max_c4+min_c1*min_
c2*max_c3*(max_c4)^2-min_c1*max_c4*(min_c2)^2>(max_c0)^2*max_c4+max_c0*(max_c3*max_c4)^2
plot(ki,kp,'*');hold on;xlabel('K_{i}');ylabel('K_{p}')
end;end;end;end;end;end;end;end;end;end;end
```



MATLAB Code

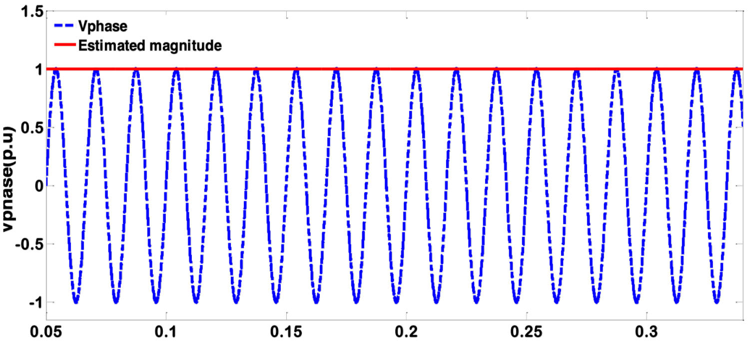
```
% Robust stability evaluation
kp=491;ki=9.4; % PI controller
min_c0=min_b0*ki;
max_c0=max_b0*ki;
min_c1=min_a0+min_b1+min_b0*kp;
max_c1=max_a0+max_b1+max_b0*kp;
min_c2=min_a1+min_b2*ki+min_b1*kp;
max_c2=max_a1+max_b2*ki+max_b1*kp;
min_c3=min_a2+min_b2*kp;
max_c3=max_a2+max_b2*kp;
min_c4=min_a3;
max_c4=max_a3;
figure(2); % Kharitonov rectancles
for w=0:1000:50000
s=j*w;
k1=min_c0+min_c1*s+max_c2*s^2+max_c3*s^3+min_c4*s^4+s^5;
k2=max_c0+max_c1*s+min_c2*s^2+min_c3*s^3+max_c4*s^4+s^5;
k3=max_c0+min_c1*s+min_c2*s^2+max_c3*s^3+max_c4*s^4+s^5;
k4=min_c0+max_c1*s+max_c2*s^2+min_c3*s^3+min_c4*s^4+s^5;
g=[k1 k3 k2 k4 k1];
plot(g);
hold on
xlabel('Re')
ylabel('Im')
grid
end
```



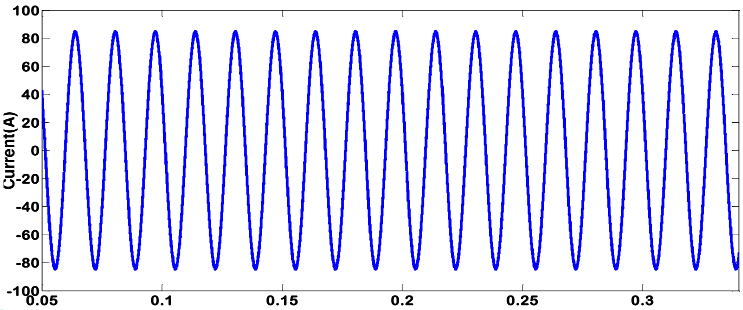
Simulation Results

- Before islanding

v_{out}



i_{out}

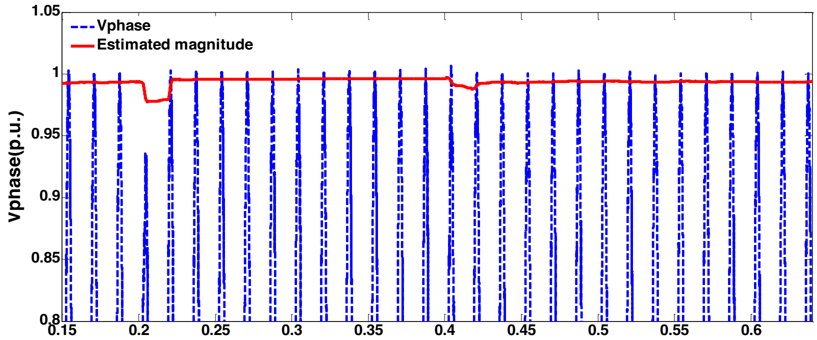


Simulation Results

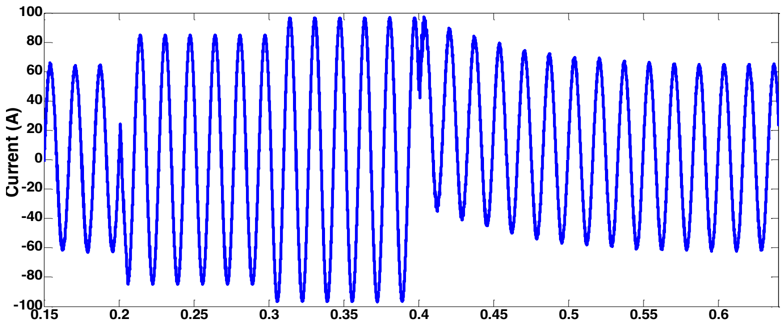
- Islanding at 0.2 s, and reconnecting at 0.4 s.

- Step load disturbance at 0.3 s

v_{out}

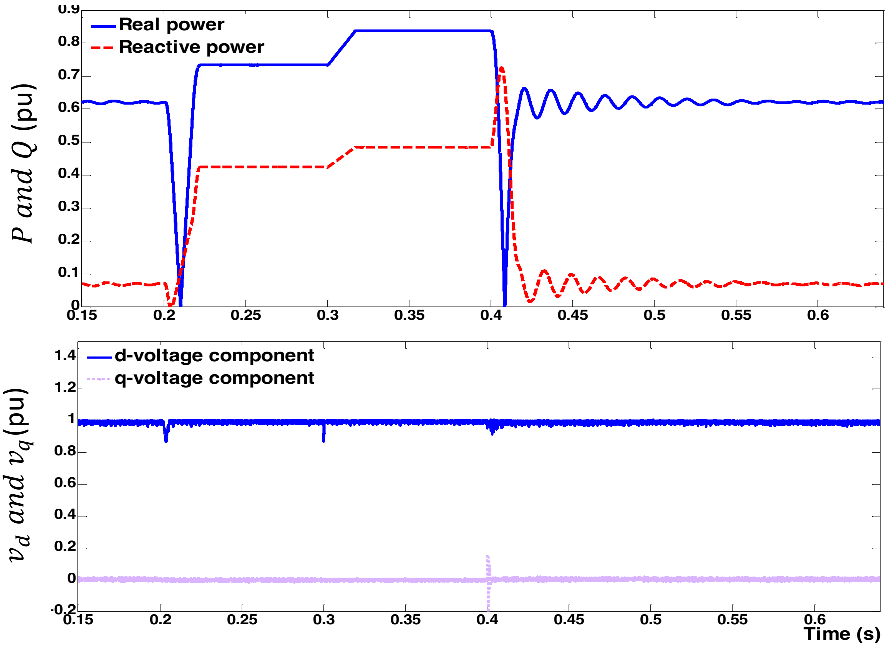


i_{out}



Simulation Results

- Islanding at 0.2 s, and reconnecting at 0.4 s.
- Step load disturbance at 0.3 s



Example: RF Amplifier Feedback Loop

2010 IEEE International Conference on Control Applications
Part of 2010 IEEE Multi-Conference on Systems and Control
Yokohama, Japan, September 8-10, 2010

Robust Stabilizer Feedback Loop Design For A Radio-Frequency Amplifier

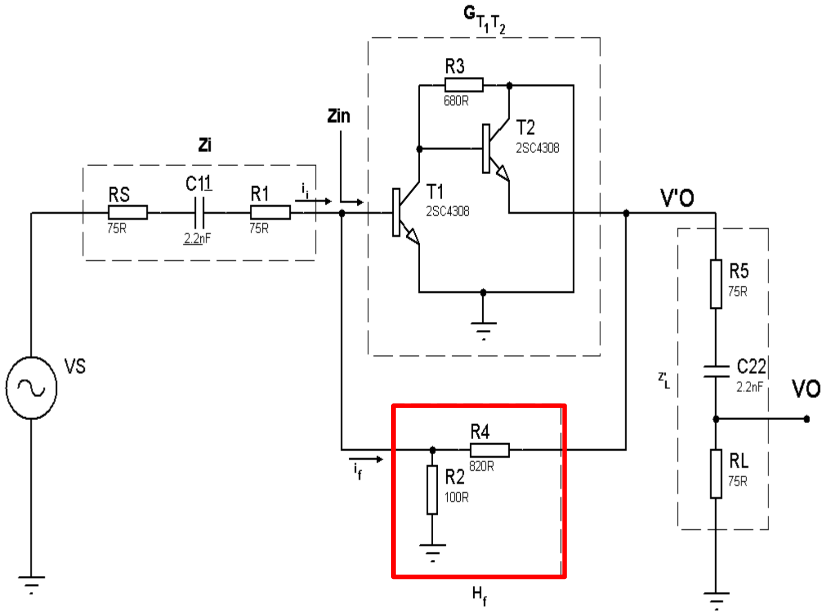
Hassan Bevrani, *Senior Member*, *IEEE,* and Shoresh Shokoohi

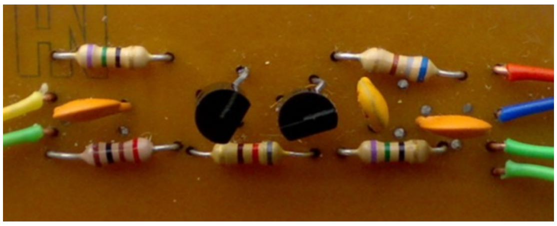
***Abstract*—In this paper, a robust proportional-integral-derivative (PID) feedback compensator is designed for a low power radio-frequency (RF) amplifier, using Kharitonov's theorem. The robust D-stability concept is used to achieve robust performance by clustering the characteristic polynomial equation roots of the closed loop system in a specified angular region. Then, the controller is realized and the obtained results demonstrate a desirable amplification over the operating frequency band. It is shown that the designed feedback compensator guarantees the robust stability and robust performance for a wide range parameter variation.**

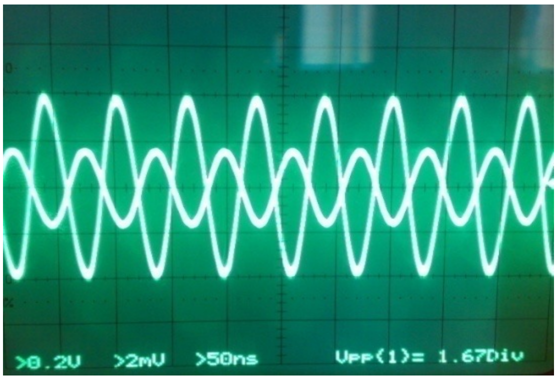
Negative feedback can widen the bandwidth of amplifier and improve the matching at its input and output. However, the gain of the amplifier is reduced and it may adversely affect the noise figure unless another relatively low-noise amplifier is added in the forward path before the original amplifier [4].

In the present work a robust technique based on Kharitonov's theorem [5], is used to obtain an admissible proportional-integral-derivative (PID) feedback compensator which guarantee satisfactory operation of the system under realistic operating conditions. The D-stability concept is used

RF Amplifier Feedback Loop







Project: Report 2

For your selected system:

1) **Design a conventional P/PI controller,**
2) **Determine a deviation range for one or more parameters of system, such that the designed P/PI controller cannot satisfy the control objective.**
3) **Find the P/PI parameter(s) using Kharitonov theorem,**
4) **Analyze the closed-loop system for both P/PI controllers,**

Deadline: The day before next Meeting

Please only use this email address: bevranih18@gmail.com

Thank You!