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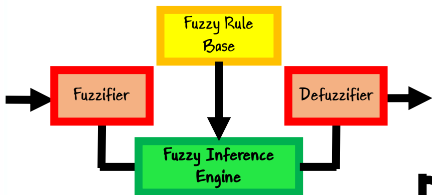
Intelligent Control



Fuzzification and Defuzzification Methods

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Content

- Fuzzifiers
- Defuzzifiers

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Fuzzifier

Fuzzification is the process of converting a crisp input value to a fuzzy value. The ***fuzzifier*** is defined as a mapping from a real-valued point $x^* \in U \subset R^n$ to a fuzzy set A' in U. Three well-known ***fuzzifiers*** are:

- ***Singleton Fuzzifier***
- ***Gaussian Fuzzifier***
- ***Triangular Fuzzifier***

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Singleton Fuzzifier

The ***singleton fuzzifier*** maps a real-valued point $x^* \in U$into a fuzzy singleton A' in U, which has membership value 1 at x^* and 0 at all other points in U; that is,

$$\mu_{A'}(x) = \begin{cases} 1 & x = x^* \\ 0 & o.w. \end{cases}$$

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Gaussian Fuzzifier

The ***Gaussian fuzzifier*** maps a real-valued point $x^* \in U$ into a fuzzy singleton A' in U, which has the following membership function:

$$\mu_{A'}(x) = e^{-\left(\frac{x_1 - x_1^*}{a_1}\right)^2} * \dots * e^{-\left(\frac{x_n - x_n^*}{a_n}\right)^2}$$

where a_i are positive parameters and the t-norm * is usually-chosen as algebraic product or min.

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Gaussian Fuzzifier

The ***triangular fuzzifier*** maps a real-valued point $x^* \in U$ into a fuzzy singleton A' in U, which has the following membership function:

$$\mu_{A'}(x) = \begin{cases} \left(1 - \frac{|x_1 - x_1^*|}{b_1}\right) * \dots * \left(1 - \frac{|x_n - x_n^*|}{b_n}\right) & |x_n - x_n^*| \le b_i \, , \; i = 1, 2, \dots, n \\ 0 & o.w. \end{cases}$$

where b_i are positive parameters and the t-norm * is usually-chosen as algebraic product or min.

Fuzzifier

Note: It can be seen that

- All three fuzzifiers satisfy $\mu_{A'}(x^*) = 1$.
- The singleton fuzzifier greatly simplifies the computations involved in the fuzzy inference engine.
- If the fuzzy sets have Gaussian or triangular membership functions, then the Gaussian or triangular fuzzifier also will simplify some fuzzy inference engines.
- The Gaussian and triangular fuzzifiers can suppress noise in the input, but the singleton fuzzifier cannot.

Fuzzifier

Example: Suppose that the fuzzy rule base consists of M rules in the canonical form and that

$$\mu_{A_i^l}(x_i) = e^{-\left(\frac{x_i - \bar{x}_i^l}{\sigma_i^l}\right)^2}$$

where σ_i^l and $\bar{x}_i^l$ are positive parameters $(1 \le l \le M,\ 1 \le i \le n)$. Show that if we use the Gaussian fuzzifier, algebraic product for the t-norm * , then the product inference engine is simplified to

$$\mu_{B'}(y) = \max_{l=1}^{M}\left[\prod_{i=1}^{n} e^{-\left(\frac{x_{ip}^l - \bar{x}_i^l}{\sigma_i^l}\right)^2} e^{-\left(\frac{x_{ip}^l - x_i^*}{a_i}\right)^2} \mu_{B^l}(y)\right] \quad , x_{ip}^l = \frac{(a_i)^2 \bar{x}_i^l + (\sigma_i^l)^2 x_i^*}{(a_i)^2 + (\sigma_i^l)^2}$$

$$\text{Product Inference Engine}: \mu_{B'}(y) = \max_{l=1}^{M}\left[\sup_{x \in U}\left(\mu_{A'}(x)\prod_{i=1}^{n}\mu_{A_i^l}(x_i)\mu_{B^l}(y)\right)\right]$$

Content

- Fuzzifiers
- Defuzzifiers

Defuzzifier

Defuzzification is the process of converting a fuzzified output into a single crisp value with respect to a fuzzy set. The ***defuzzifier*** is defined as a mapping from fuzzy set B' in $V \subset R$ (which is the output of the fuzzy inference engine) to crisp point $y^* \in V$. The following are the usual ***defuzzifier***:

- ***Center of Gravity Defuzzifier***
- ***Center Average Defuzzifier***
- ***Maximum Defuzzifier***

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Center of Gravity Defuzzifier

The ***center of gravity defuzzifier*** specifies the y^* as the center of the area covered by the membership function of B′ , that is,

$$y^* = \frac{\int_V y\mu_{B'}(y)dy}{\int_V \mu_{B'}(y)dy}$$

Note: The ***advantage*** of the center of gravity defuzzifier lies in its intuitive plausibility. The ***disadvantage*** is that it is computationally intensive.

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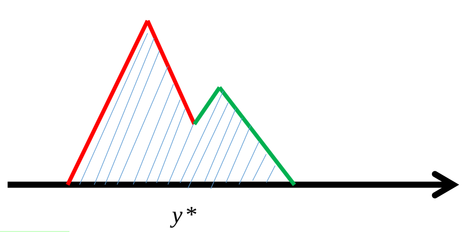
Center of Gravity Defuzzifier

To illustrates the ***center of gravity defuzzifier*** graphically, consider a simple example with M = 2 :



$$y^* = \frac{\int_V y\mu_{B'}(y)dy}{\int_V \mu_{B'}(y)dy}$$

Center Average Defuzzifier

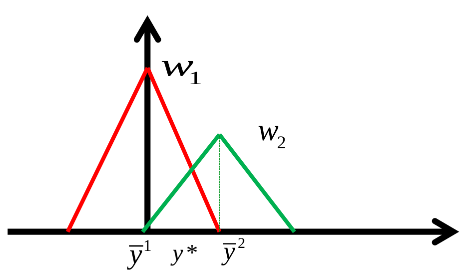
Let $\bar{y}^l$ be the center of the y^{th} fuzzy set and w_l be its height, the ***center average defuzzifier*** determines y^* as

$$y^* = \frac{\sum_{l=1}^{M} \bar{y}^l w_l}{\sum_{l=1}^{M} w_l}$$

Note: The center average defuzzifier is the most commonly used defuzzifier in fuzzy systems and fuzzy control. The ***advantage*** is that it is computationally simple and intuitively plausible.

Center Average Defuzzifier

To illustrates the ***center average defuzzifier*** graphically, consider a simple example with $\mathrm{M} = 2$:



$$y^* = \frac{\sum_{l=1}^{M} \bar{y}^l w_l}{\sum_{l=1}^{M} w_l}$$

M=2 ⇒

$$y^* = \frac{\bar{y}^1 w_1 + \bar{y}^2 w_2}{w_1 + w_2}$$

Maximum Defuzzifier

the ***maximum defuzzifier*** chooses the y^* as the point in V at which $\mu_{B'}(y)$ achieves its maximum value. Define the set

$$hgt(B') = \left\{ y \in V \,\middle|\, \mu_{B'}(y) = \sup_{y \in V} \mu_{B'}(y) \right\}$$

that is, $hgt(B')$ is the set of all points in V at which $\mu_{B'}(y)$ achieves its maximum value. The ***maximum defuzzifier*** defines y^* as an arbitrary element in $hgt(B')$, that is,

$$y^* = \text{any point in } hgt(B')$$

If $hgt(B')$ contains a single point, then y^* is uniquely defined.

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Maximum Defuzzifier

If $hgt(B')$ contains more than one point, then we can define the following ***maximum defuzzifier***:

- ***Smallest of Maxima Defuzzifier:*** $y^* = \inf\{y \in hgt(B')\}$
- ***Largest of Maxima Defuzzifier:*** $y^* = \sup\{y \in hgt(B')\}$
- ***Mean of Maximum Defuzzifier:*** $y^* = \frac{\int_{hgt(B')} y\,dy}{\int_{hgt(B')} dy}$

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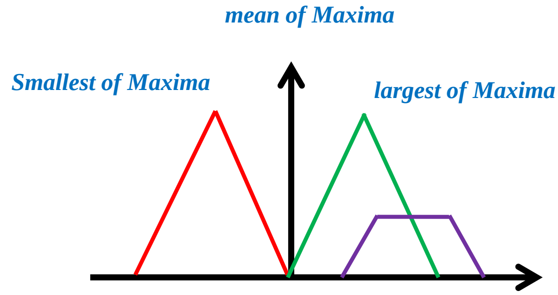
Maximum Defuzzifier

To illustrates the ***maximum defuzzifier*** graphically, consider the following simple example :



Note: The ***advantage*** of the maximum defuzzifier is intuitively plausible and computationally simple. But small changes in B′ may result in large changes in y^*.

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Defuzzifiers Comparison

maximum	center average	center of gravity	
yes	yes	yes	Plausibility
yes	yes	No	Computational simplicity
No	yes	yes	Continuity

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Defuzzifier

Example: Consider a two-input-one-output fuzzy system that is constructed from the following two rules:

$$\begin{cases} Ru_1 : IF \ \ x_1 \text{ is } A_1 \ \ and \ \ x_2 \text{ is } A_2 \ \ THEN \ \ y \text{ is } A_1 \\ Ru_2 : IF \ \ x_1 \text{ is } A_2 \ \ and \ \ x_2 \text{ is } A_1 \ \ THEN \ \ y \text{ is } A_2 \end{cases}$$

where A_1 and A_2 are fuzzy sets in R with membership functions:

$$\mu_{A_1}(u) = \begin{cases} 1-|u| & ; -1 < u < +1 \\ 0 & ; \ o.w. \end{cases}$$

$$\mu_{A_2}(u) = \begin{cases} 1-|u-1| & ; 0 < u < +2 \\ 0 & ; \ o.w. \end{cases}$$

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Defuzzifier

Example:

Suppose that the input to the fuzzy system is $(x_1^*, x_2^*) = (0.3, 0.6)$ and we use the singleton fuzzifier. Determine the output of the fuzzy system y^* by using product inference engine.

Solution:

Singleton Fuzzifier and Product Inference Engine →

$$\mu_{B'}(y) = \max_{l=1}^{M} \left[\prod_{i=1}^{n} \mu_{A_i^l}(x_i^*) \mu_{B^l}(y) \right]$$

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