

The *Takagi-Sugeno-Kang fuzzy system* is constructed from the following fuzzy IF-THEN rules:

The final output of the Takagi-Sugeno fuzzy system can be inferred as the weighted average of the:

in which the weights are calculated as follows:

$$
w^l=\prod_{i=1}^n \mu_{A_i^l}\left(x_i\right)
$$

This kind of fuzzy systems can be employed to represent nonlinear plant by local linear model. Consider a continuous nonlinear system of the following state space form: Systems

lems can be employed to

del. Consider a continuou

e form:
 $(t) = f(x(t), u(t))$

(t) = $h(x(t), u(t))$

of fuzzy IF-THEN rules Systems

ems can be employed t

lel. Consider a continuou
 f form:
 $(t) = f(x(t), u(t))$
 $(t) = h(x(t), u(t))$

of fuzzy IF-THEN rule

.... and z_k is F_k^l THEN *x***ystems**
x **tems can be employed bodel. Consider a continu** $\dot{x}(t) = f(x(t), u(t))$ **
** $y(t) = h(x(t), u(t))$ **
1 of fuzzy IF-THEN ru Systems**

stems can be employed

odel. Consider a continu
 $\dot{x}(t) = f(x(t), u(t))$
 $y(t) = h(x(t), u(t))$

I of fuzzy IF-THEN ru

I and z_k is F_k^l THEI

$$
\begin{cases}\n\dot{x}(t) = f\left(x(t), u(t)\right) \\
y(t) = h\left(x(t), u(t)\right)\n\end{cases}
$$

The local linear model of fuzzy IF-THEN rules can be written as follows:

 $Ru^{\left(l\right) }:\overline{\left\vert \begin{matrix} IF\ z_{1}\end{matrix}\right\vert}$

Where $Z_1, ..., Z_k$ denote known premise variables that may be functions of the state variables, and $F_1^l, ..., F_k^l$ are fuzzy sets characterized by fuzzy membership functions. 1 1 represent nonlinear

nonlinear system of

can be written as
 $(t) = A_l x(t) + B_l u(t)$
 $(t) = C_l x(t) + D_l u(t)$

bles that may be
 F_k^l are fuzzy sets
 F_k^l are fuzzy sets Systems

ems can be employed to represent nonlinear

tel. Consider a continuous nonlinear system of
 $(t) = f(x(t), u(t))$

(t) = $h(x(t), u(t))$

of fuzzy IF-THEN rules can be written as

.... and z_k is F_k^l THEN $\begin{cases} \dot{x}(t) = A_l x$ *zy* **Systems**
 l zy **Systems**
 *l l consider a continuous nonlinear system c space form:
* $\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t), u(t)) \end{cases}$ *

model of fuzzy IF-THEN rules can be written a

<i>l* **and** **and** z_k **is** $F_k^$ *k k* ent nonlinea

ear system of

the written as
 $\frac{1}{2}x(t) + B_l u(t)$

that may be

the fuzzy set:

the fuzzy set: *x* to represent nonlinear
 x nonlinear system of
 x (*t*) = $A_l x(t) + B_l u(t)$
 y(*t*) = $C_l x(t) + D_l u(t)$

ables that may be

.., F_k^l are fuzzy sets
 $C_{enter, University of Kurdistan}$ *I*^{*III} SIMPLE
 IF THEN IVE SUPPOSE

IF* z_1 *is* F_1^t *and* \ldots *and* z_k *is* F_k^t *THEN rules and be write

IF* z_1 *is* F_1^t *and* \ldots *and* z_k *is* F_k^t *THEN rules and be write

IF* z_1 *is* F_1^t *and \ldots</sup> y*
 y the present nonlinear
 y to the virition as
 y to $\frac{1}{2}$ and $A_l x(t) + B_l u(t)$
 y (*t*) = $C_l x(t) + D_l u(t)$

ables that may be
 w, F_k^l are fuzzy sets
 Center, University of Kurdistan to represent nonlinear
us nonlinear system of
es can be written as
 $\int \dot{x}(t) = A_l x(t) + B_l u(t)$
 $y(t) = C_l x(t) + D_l u(t)$
riables that may be
 \ldots, F_k^l are fuzzy sets to represent nonlinear
us nonlinear system of
es can be written as
 $\int \dot{x}(t) = A_l x(t) + B_l u(t)$
 $y(t) = C_l x(t) + D_l u(t)$
riables that may be
 \ldots, F_k^l are fuzzy sets

The final outputs of the fuzzy systems are inferred as follows:

$$
\begin{array}{ll}\n\text{ZZY} & \text{Systems} \\
\text{so the fuzzy systems are inferred as follows:} \\
\begin{bmatrix}\n\dot{x}(t) = \sum_{i=1}^{N} \mu_i(z) \big(A_i x(t) + B_i u(t) \big) \\
y(t) = \sum_{i=1}^{N} \mu_i(z) \big(C_i x(t) + D_i u(t) \big) \\
\text{number of rule, } z = [z_1, ..., z_k]^T \text{ is the vector of fuzzy} \\
\text{is a fuzzy basis function:} \\
\mu_i(z) = \frac{\prod_{i=1}^{k} F_i^i(z_i)}{\sum_{i=1}^{N} \prod_{i=1}^{k} F_i^i(z_i)} \\
\text{denote} & \text{Smat/Mico Grids Research Center, University of Kurdistan} \\
\text{Smat/Mico Grids Research Center, University of Kurdistan} \\
\end{array}
$$

Where N is the number of rule, $z = [z_1, ..., z_k]^T$ is the vector of fuzzy inputs and $\mu_1(z)$ is a fuzzy basis function:

$$
\mu_l(z) = \frac{\displaystyle\prod_{i=1}^k F_i^l\left(z_i\right)}{\displaystyle\sum_{l=1}^N \prod_{i=1}^k F_i^l\left(z_i\right)}
$$

 $\begin{smallmatrix}\mathcal{N}\mathbb{C} & \mathbf{S}\mathbf{M} \end{smallmatrix}$

Example: Consider the following nonlinear system:

$$
\begin{aligned}\n\text{Systems} \\
\text{following nonlinear system:} \\
\int \dot{x}_1 &= -x_1 + x_1 x_2^3 \\
\dot{x}_2 &= -x_2 + (3 + x_2) x_1^3\n\end{aligned}
$$
\nczy local linear model, It is not not possible to find the following properties.

To construct a fuzzy local linear model, It is assumed that $x_1, x_2 \in [-1, +1]$. This nonlinear model can be easily rewritten as follows: Systems

ollowing nonlinear system:
 $\dot{x}_1 = -x_1 + x_1x_2^3$
 $\dot{x}_2 = -x_2 + (3 + x_2)x_1^3$

y local linear model, It is

s nonlinear model can be easil Systems

following nonlinear system:
 $\begin{cases} \n\dot{x}_1 = -x_1 + x_1x_2^3 \\
 \n\dot{x}_2 = -x_2 + (3 + x_2)x_1^3\n\end{cases}$

zzy local linear model, It is assumed that

his nonlinear model can be easily rewritten as Systems

following nonlinear system:
 $\begin{cases} \n\dot{x}_1 = -x_1 + x_1x_2^3 \\
\dot{x}_2 = -x_2 + (3 + x_2)x_1^3\n\end{cases}$

zzy local linear model, It is assumed that

his nonlinear model can be easily rewritten as

$$
\begin{array}{c|c}\n\hline\n\text{TSK Fuzzy Systems} \\
\hline\n\text{Example: Consider the following nonlinear system:} \\
\begin{cases}\n\dot{x}_1 = -x_1 + x_1 x_2^3 \\
\dot{x}_2 = -x_2 + (3 + x_2) x_1^3\n\end{cases}\n\end{array}
$$
\nTo construct a fuzzy local linear model, It is assumed that $x_1, x_2 \in [-1, +1]$. This nonlinear model can be easily rewritten as follows:\n
$$
\dot{x} = \begin{bmatrix}\n-1 & x_1 x_2^2 \\
\hline\n(3 + x_2) x_1^2 & -1\n\end{bmatrix}\n\begin{cases}\nx_1 x_2^2 \\
x_2\n\end{cases}
$$

--: To obtain the local linear model of the considered nonlinear system, the following fuzzy IF-THEN rules can be employed:

1 1 2 1 1 *IF z is F and z is G THEN x t A x t* () () 1 *Ru* : 1 1 2 2 2 *IF z is F and z is G THEN x t A x t* () () 2 *Ru* : 1 2 2 1 3 *IF z is F and z is G THEN x t A x t* () () 3 *Ru* : 1 2 2 2 4 *IF z is F and z is G THEN x t A x t* () () 4 *Ru* :

-: Now, calculate the minimum and maximum of the chosen premise variables: SK Fuzzy Systems

Now, calculate the minimum and maximum of the chosen

ise variables:
 $\in [-1,+1]$
 $\leftarrow [-1,+1]$ SK Fuzzy Systems

Now, calculate the minimum and maximum of the chosen

sise variables:
 $\in [-1,+1]$
 $\leftarrow [-1,+1]$
 \longrightarrow
 \leftarrow
 $\min\{z_1 = x_1x_2^2\} = -1$ $\max\{z_1 = x_1x_2^2\} = +1$

 1 2 1, 1 1, 1 *x x*

 $\min \left\{ z_1 = x_1 x_2^2 \right\} = -1$ $\max \left\{ z_1 = x_1 x_2^2 \right\} = +1$ $\left[\min \{z_2 = (3 + x_2)x_1^2\} = 0 \quad \max \{z_2 = (3 + x_2)x_1^2\} = 4\right]$ Eq. the minimum and maximum of the
 $\begin{bmatrix}\n\frac{2}{2} & -1 & \text{max}\left\{z_1 = x_1 x_2^2\right\} = + \\
\frac{2}{2}x_1^2 & = 0 & \text{max}\left\{z_2 = \left(3 + x_2\right)x_1^2\right\} \\
\frac{2}{2} & = -\frac{2}{2} - \frac{2}{2} -$ 1 1 2 1 1 2 2 2 2 2 1 2 2 1 SK Fuzzy Systems
 \vdots Now, calculate the minimum and maximum of the chosen
 $\begin{aligned}\n\mathbf{r} &\in [-1,+1] \\
\mathbf{r} &\in [-1,+1]\n\end{aligned}$
 $\begin{aligned}\n\mathbf{r} &\in [-1,+1] \\
\mathbf{r} &\in [-1,+1]\n\end{aligned}$
 $\begin{aligned}\n\mathbf{r} &\in [-1,+1] \\
\mathbf{r} &\in [-1,+1] \\
\mathbf{r} &\in [-1,+1]\n\$ $\begin{array}{ll}\n & \text{Singic}\n \hline\n & \text{TSK Fuzzy Systems}\n & \text{new, calculate the minimum and maximum of the chosen}\n\end{array}$
 $x_1 \in [-1, +1]$
 $x_2 \in [-1, +1]$
 $\begin{array}{ll}\n & \text{min}\{z_1 = x_1x_2^2\} = -1 \\
 & \text{min}\{z_1 = x_1x_2^2\} = +1\n\end{array}$
 $\begin{array}{ll}\n & \text{min}\{z_2 = (3 + x_2)x_1^2\} = 0 \\
 & \text{max}\{z_2 = (3 + x_2)x_1^2\} = 4\n\end$ Fuzzy Systems
 *z*₁ = $x_1x_2^2$ } = -1
 $\left[\begin{array}{c} 1, +1 \end{array}\right]$
 $\left[\begin{array}{c} 2, +1 \end{array}\right]$
 $\left[\begin{array}{c} 2, -1 \end{array}\right]$
 $\left[\$ *z*
 z $_{2} = (3 + x_{2})x_{1}^{2}$
 z $_{2} = (3 + x_{2})x_{1}^{2}$
 z $_{3} = x_{1}x_{2}^{2}$
 z $_{4} = x_{1}x_{2}^{2}$
 z $_{5} = x_{2}x_{3}^{2}$
 z $_{6} = x_{1}x_{2}^{2}$
 z $_{7} = x_{1}x_{2}^{2}$
 z $_{8} = x_{1}x_{2}^{2}$
 z $_{9} = x_{1}x_{2}^{2}$
 z magnetic and the minimum and maximum of the chosen

validate the minimum and maximum of the chosen
 $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
 $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
 $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
 $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
 $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
 $\begin{pmatrix} 3+x_2 \end{pm$

--: Accordingly, the matrices A_1 , A_2 , A_3 and A_4 can be given as:

Consequently, the grades of membership of the premise variables are obtained as follows:

$$
\begin{array}{c}\n\begin{array}{c}\n\hline\n\text{TSK Fuzzy Systems} \\
\hline\n\text{1} & \text{Consequently, the grades of membership of the premise}\n\end{array}\n\end{array}
$$
\n
$$
\begin{array}{c}\n\hline\n\text{1} & \text{Consequently, the grades of membership of the premise}\n\end{array}
$$
\n
$$
\begin{array}{c}\n\hline\n\text{min}\{z_1\} = -1 \\
\hline\n\text{max}\{z_1\} = +1\n\end{array}
$$
\n
$$
\begin{array}{c}\n\hline\n\text{min}\{z_2\} = 0 \\
\hline\n\text{max}\{z_2\} = +4\n\end{array}
$$
\n
$$
\begin{array}{c}\n\hline\n\text{max}\{z_1\} = +4 \\
\hline\n\text{where}\n\end{array}
$$
\n
$$
\begin{array}{c}\n\hline\n\text{max}\{z_1\} = +4 \\
\hline\n\text{min}\{z_1\} = -1 \\
\hline\n\text{min}\{z_1\} = -1 \\
\hline\n\text{min}\{z_2\} = (-4)G_1(z_2) + (0)G_2(z_2)\n\end{array}
$$
\n
$$
\begin{array}{c}\n\hline\n\text{where}\n\end{array}
$$
\n
$$
F_1(z_1) + F_2(z_1) = 1 \\
\hline\n\text{min}\{z_2 = -1, \text{in the following Crits. On the following, the first point is the first point. The second point is the second point. The second point is the second
$$

where

 $F_1(z_1) + F_2(z_1) = 1$ $G_1(z_2)+G_2(z_2)=1$ 1 1

Then, by solving the obtained equations, the membership functions are as follows:

--: It can be checked that the designed fuzzy systems exactly represents the nonlinear system in the interval $[-1, +1] \times [-1, +1]$:

Systems
\n**Systems**
\n
$$
x + y = 0
$$
\n
$$
x + z = 0
$$
\n
$$
y = 0
$$
\n<

with

Systems
\ned that the designed fuzzy syst
\nsystem in the interval [-1, +1] ×
\n
$$
\dot{x}(t) = \sum_{l=1}^{4} \mu_l(z) A_l x(t)
$$
\n
$$
\mu_l(z_1, z_2) = F_1(z_1) G_1(z_2)
$$
\n
$$
\mu_2(z_1, z_2) = F_2(z_1) G_2(z_2)
$$
\n
$$
\mu_3(z_1, z_2) = F_2(z_1) G_1(z_2)
$$
\n
$$
\mu_4(z_1, z_2) = F_2(z_1) G_2(z_2)
$$
\n
$$
\mu_5(\text{SIN}) = \sum_{\text{Smax}/\text{Micro Grids Research Center, University}}
$$

Assignment: Consider the following nonlinear system:

$$
y_{\text{systems}} = \sqrt{\sum_{i=1}^{n} (x_i - x_i)^2}
$$
\n
$$
\begin{cases}\n\dot{x}_1 = -x_1 + x_1 x_2^3 \\
\dot{x}_1 = -x_2 + (3 + x_2) x_1^3\n\end{cases}
$$
\n
$$
[-\alpha, \alpha].
$$

assume that $x_1, x_2 \in [-\alpha, \alpha]$.

Axx Smot

To stabilize systems represented by T-S fuzzy systems of the following form:

$$
Ru^{(l)}
$$
 : IF_{z_1} is F_1^l and ... and z_k is F_k^l THEN $\dot{x}(t) = A_l x(t) + B_l u(t)$

the parallel distributed compensator controller is defined as:

Parallel distributed Components	W
To stabilize systems represented by T-S fuzzy systems of the following form:	
$Ru^{(l)}: \overline{IF_{z_1} \text{ is } F_1^l \text{ and } ... \text{ and } z_k \text{ is } F_k^l \text{ THEN } \dot{x}(t) = A_l x(t) + B_l u(t)}$	
the parallel distributed compensator controller is defined as:	
$Ru^{(m)}: \overline{IF_{z_1} \text{ is } F_1^m \text{ and } ... \text{ and } z_k \text{ is } F_k^m \text{ THEN } u(t) = K_m x(t)}$	

By applying the parallel distributed compensator control law to local linear model, the closed loop system is obtained by:

Qistribution
distributed Components
the parallel distributed compensator
del, the closed loop system is obtained by:

$$
\dot{x}(t) = \sum_{m=1}^{N} \sum_{l=1}^{N} \mu_m(z) \mu_l(z) (A_l + B_l K_m) x(t)
$$

represented as follows:

$$
(\mu_l(z))^2 G_{ll} x(t) + \sum_{l=1}^{N} \sum_{l=1}^{N} \mu_m(z) \mu_l(z) (G_{lm} + G_{ml}).
$$

which can be represented as follows:

$$
- \text{Var} \text{ single} \longrightarrow \text{single} \longrightarrow \text{single} \longrightarrow \text{single} \longrightarrow \text{single} \longrightarrow \text{triangle} \longrightarrow \text{value} \longrightarrow \text{value
$$

with

$$
G_{lm} = A_l + B_l K_m
$$

Theorem: The equilibrium of closed loop fuzzy system is asymptotically stable if there exist a symmetric matrix *P>0* such that:

 $G_{II}^T P + PG_{II} < 0$

$$
\begin{array}{ll}\n\text{Example 1} & \text{distributed Components} \\
\text{Time: The equilibrium of closed loop fuzzy system isuptotically stable if there exist a symmetric matrix } P > 0 \text{ such that:} \\
& G_{ll}^T P + PG_{ll} < 0 \\
& \left(\frac{G_{lm} + G_{ml}}{2} \right)^T P + P \left(\frac{G_{lm} + G_{ml}}{2} \right) \leq 0 \quad, \, l < m \\
\mu_m \left(z(t) \right) \mu_l \left(z(t) \right) \neq 0, \forall t \text{ and } l, m = 1, 2, \ldots, N.\n\end{array}
$$
\nExample 2: The following problem is a Bayesian distribution. The first term is not a factor of the number of nodes. The first term is the number of nodes, the number of nodes is the number of nodes. The first term is the number of nodes, the number of nodes is the number of nodes. The first term is the number of nodes, the number of nodes is the number of nodes. The first term is the number of nodes, the number of nodes is the number of nodes. The first term is the number of nodes, the number of nodes is the number of nodes, and the number of nodes is the number of nodes. The first term is the number of nodes, the number of nodes is the number of nodes, and the number of nodes is the number

For $\mu_{m}(z(t)) \mu_{l}(z(t)) \neq 0, \forall t$ and $l, m = 1, 2, ..., N$.

According to the theorem, the following linear matrix inequalities (LMIs) should be solved to design parallel distributed compensator:

$$
-XAlT - AlX + MlTBlT + BlMl > 0
$$

Parallel distributed Components	×
According to the theorem, the following linear matrix inequalities (LMIs) should be solved to design parallel distributed compensator:	
$-XA_{l}^{T} - A_{l}X + M_{l}^{T}B_{l}^{T} + B_{l}M_{l} > 0$	
$-XA_{l}^{T} - A_{l}X - XA_{m}^{T} - A_{m}X + M_{m}^{T}B_{l}^{T} + B_{l}M_{m} + M_{l}^{T}B_{m}^{T} + B_{m}M_{l} \ge 0$	
$XA_{l}^{T} - A_{l}X - XA_{m}^{T} - A_{m}X + M_{m}^{T}B_{l}^{T} + B_{l}M_{m} + M_{l}^{T}B_{m}^{T} + B_{m}M_{l} \ge 0$	
Then the controller gains is obtained:	
$P = X^{-1} \qquad and \qquad K_{l} = M_{l}X^{-1}$	
13	B. Disgraduate

Then the controller gains is obtained:

$$
P = X^{-1} \qquad and \qquad K_l = M_l X^{-1}
$$

Inverted Pendulum System

The dynamic equations of the nonlinear inverted pendulum system are as follows:

$$
4x - 2
$$
\n
$$
4x + 1
$$
\nTherefore, $x_1(t) = x_2(t)$

\nwhere $a = 1/(M+m)$; $x_1(t)$ and $x_2(t)$ are respectively. The equation $x_1(t) = \frac{e^{\sin(x_1(t))} - \frac{am(x_2(t)\sin(2x_1(t))}{2}}{\frac{4l}{3} - \frac{amt\cos^2(x_1(t))}{3}} + \frac{a\cos(x_1(t))}{\frac{4l}{3} - \frac{amt\cos^2(x_1(t))}{3}} + \frac{a\cos(x_1(t))}{\frac{4l}{3} - \frac{amt\cos^2(x_1(t))}{3}}$ where $a = 1/(M+m)$; $x_1(t)$ and $x_2(t)$ are respectively the angle and the angular velocity of the pendulum.

\nSo that *M* is the constant Center, University of Kurdman. The Lagrange function is the constant Center, University of Kurdman. The Lagrange function is the constant Center, University of Kurdman. The Lagrange function is the constant Center, University of Kurdman. The Lagrange function is the constant Center, University of Kurdman. The Lagrange function is the constant Center.

where $a = \frac{1}{2}$ $(x_{(M+m)}; x_1(t)$ and $x_2(t)$ are respectively the angle and the angular velocity of the pendulum.

.

Inverted Pendulum System

It can be shown that this nonlinear system can be approximated by the following two rules:

$$
\frac{1}{n \text{Vert of } P}
$$
\nIt can be shown that this nonlinear system can be approximated by the following two rules:\n
$$
\frac{R u^{(1)} \cdot 1}{R u^{(1)} \cdot 1} \cdot \frac{I F x_1 \text{ is about 0.} \quad \frac{I F x_1 \text{ is about 1.} \quad \frac{I F x_1
$$

Inverted Pendulum System

Accordingly, by solving the following linear matrix inequality:

Inverted Pendulum System
Accordingly, by solving the following linear matrix inequality:
$-XA_1^T - A_1X + M_1^TB_1^T + B_1M_1 > 0$
$-XA_2^T - A_2X + M_2^TB_2^T + B_2M_2 > 0$
$-XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_1^T + B_1M_2 + M_1^TB_2^T + B_2M_1 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_1^T + B_1M_2 + M_1^TB_2^T + B_2M_1 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_1^T + B_1M_2 + M_1^TB_2^T + B_2M_1 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_1^T + B_1M_2 + M_1^TB_2^T + B_2M_1 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_2^T - B_2M_2 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_2^T - B_2M_2 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_2^T + B_2M_2 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_2^T + B_2M_2 \ge 0$
$XA_1^T - A_1X - XA_2^T - A_2X + M_2^TB_1^T + B_1M_2 + M_1^TB_2^T + B_2M_1 \ge 0$

Finally, simulation result is as foolows:

Axx Sm<mark>e</mark>t

$\sqrt{2}$

Thanks

