Linear Control Systems

An Introduction on Control Systems

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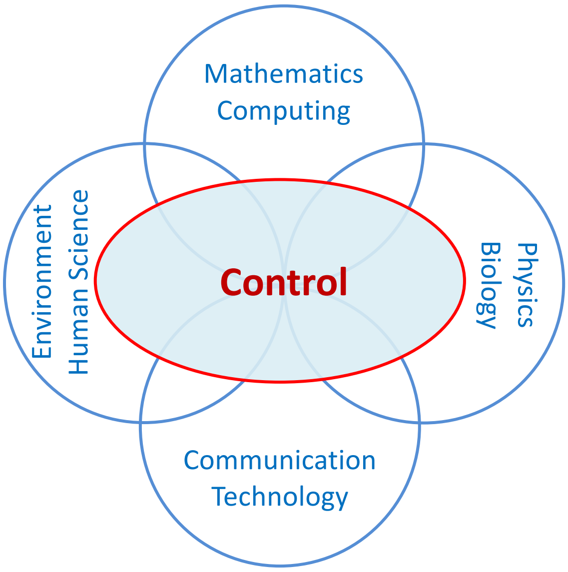
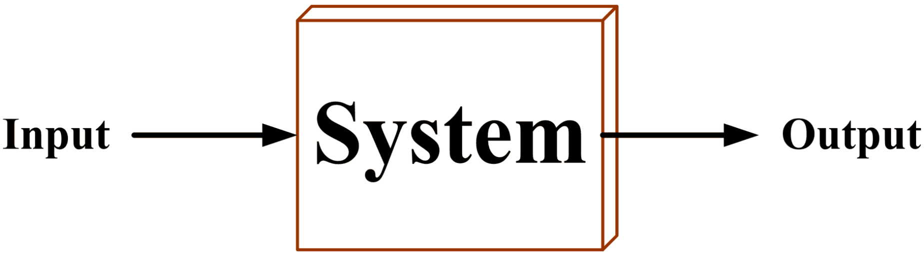
Contents

Control Engineering



System

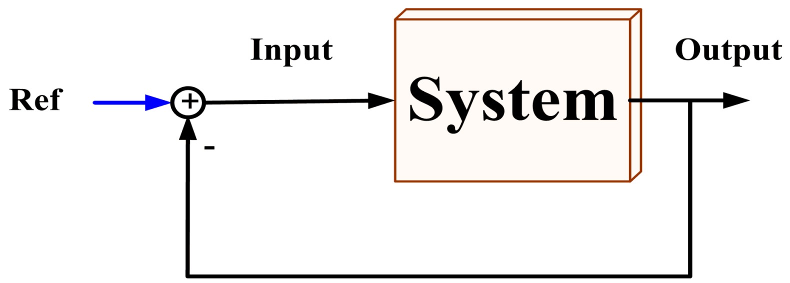
- **Definition:**



- **System:** An interconnection of elements for a desired purpose. A system has **input** and **output**.

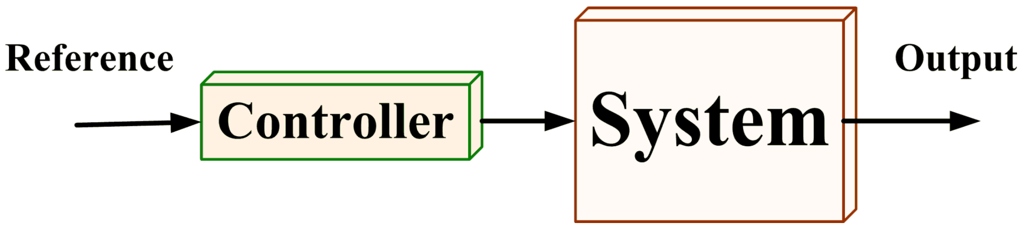
Feedback Control System

- ❑ Uses a measurement of the output signal and feedback it to compare with the desired signal to correct the system output.



- The output is measured
- System output affects the control action
- System can compensate for disturbance

Control System

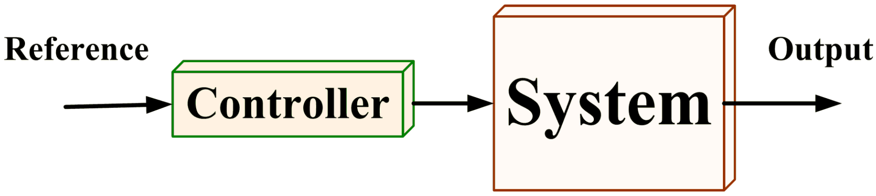


❑ Control:

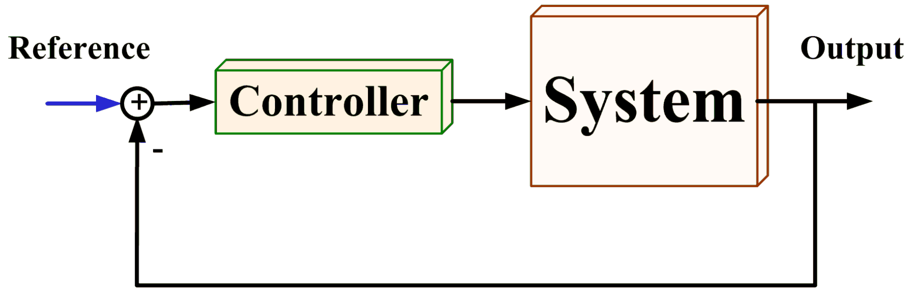
A mechanism to **regulate**, **direct**, **command**, or **govern** a system. This mechanism can be done by a **controller**.

Control Systems

- **Open-loop Control (Feedforward Control)**



- **Closed-loop Control (Feedback Control)**



Feedforward Control Specifications

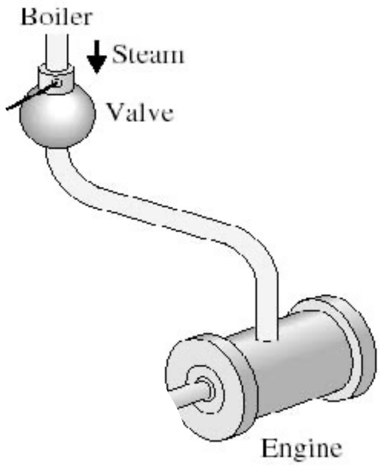
Feedforward Control

- Compute control input without continuous measurement
  - Simple
  - Need to know **EVERYTHING ACCURATELY**

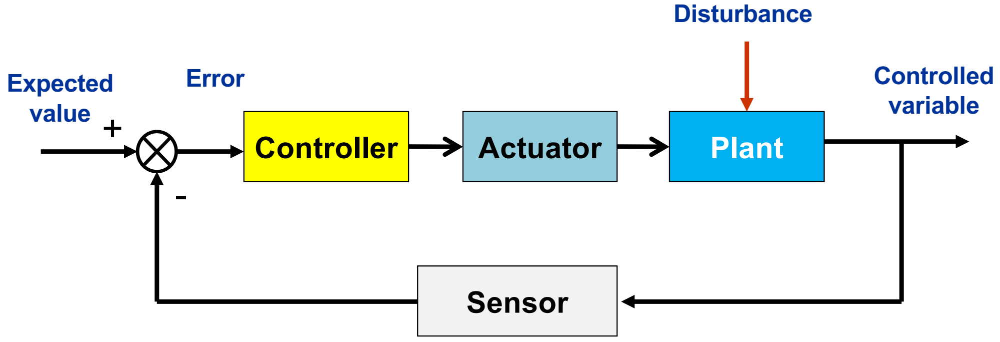
- Feedforward control fails when
  - We don’t know everything
  - We make errors in estimation/modeling
  - Things change

Feedback Control Example

- **Watt's centrifugal speed regulator (1976)**



Block Diagram of a Feedback Control System



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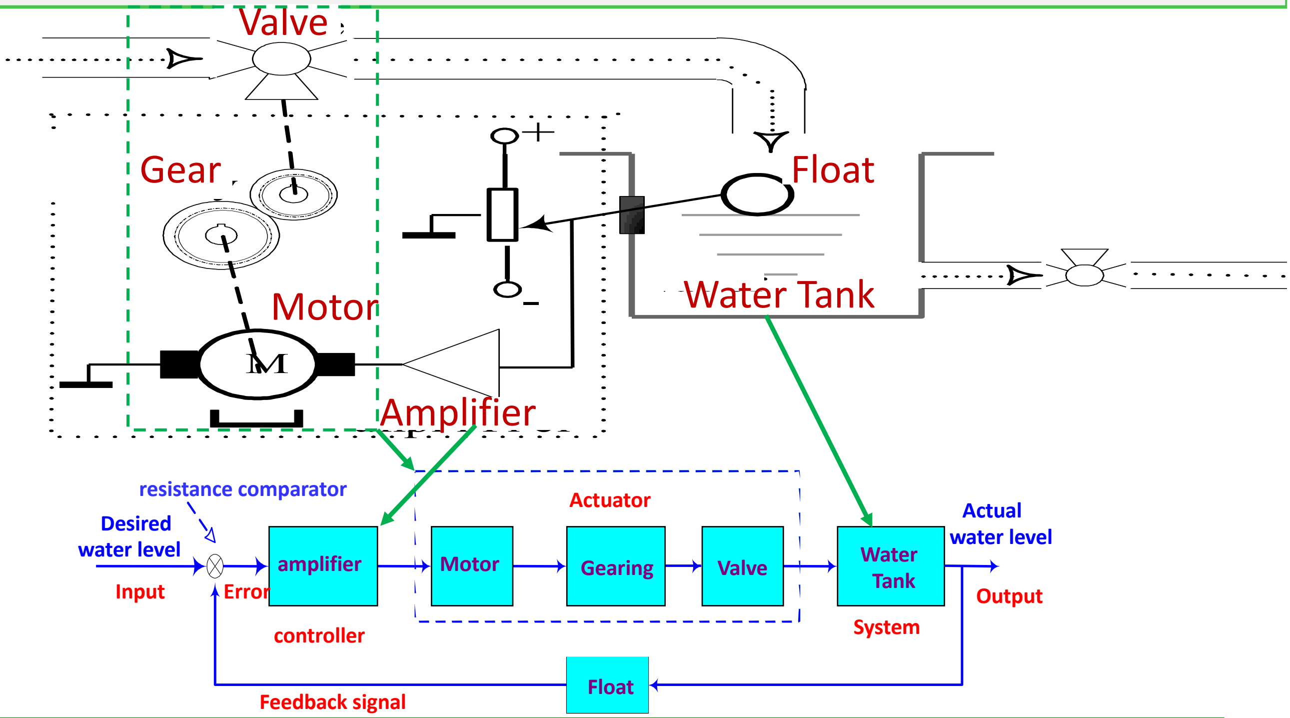
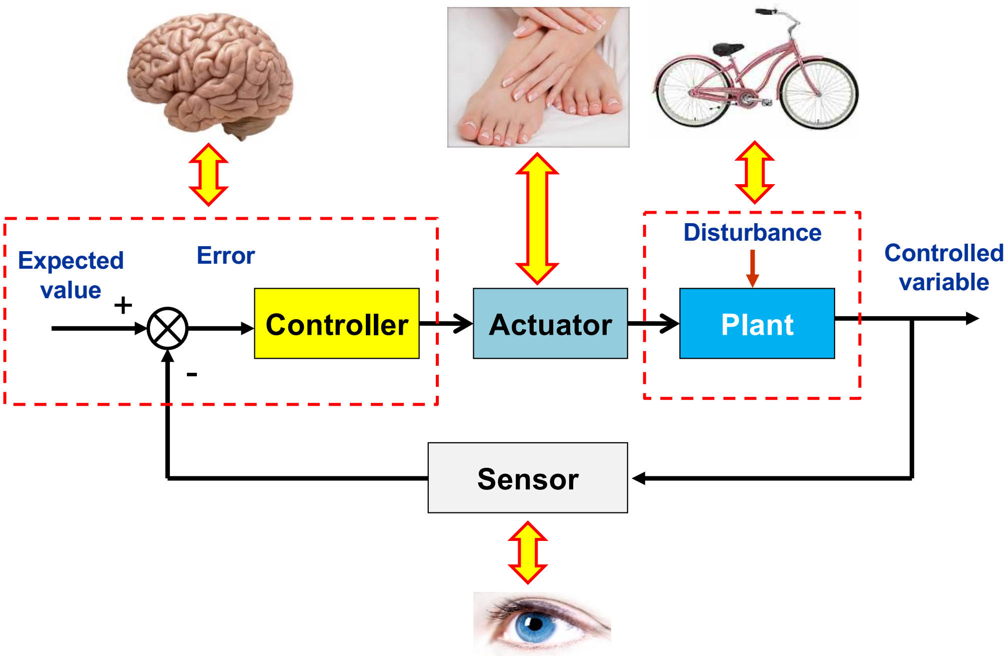
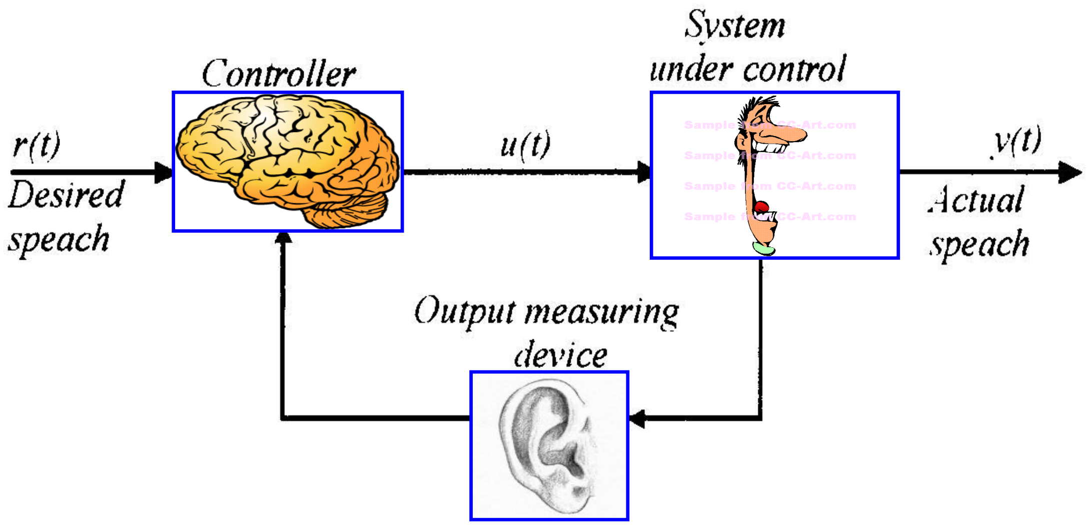


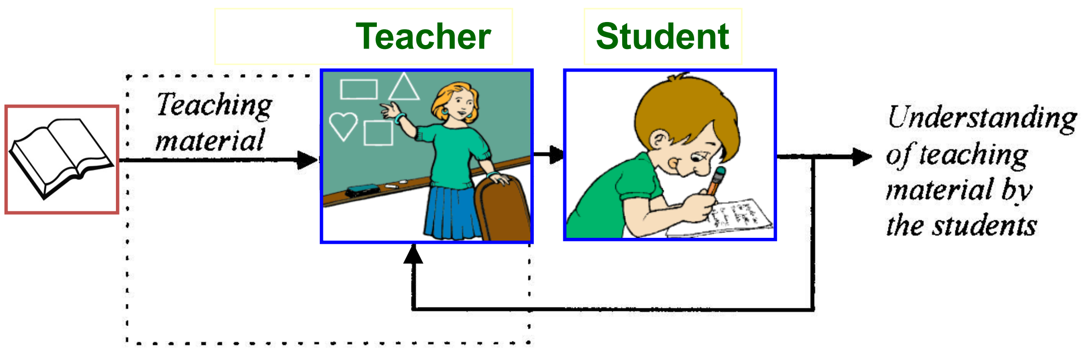
Block Diagram of a Feedback Control System



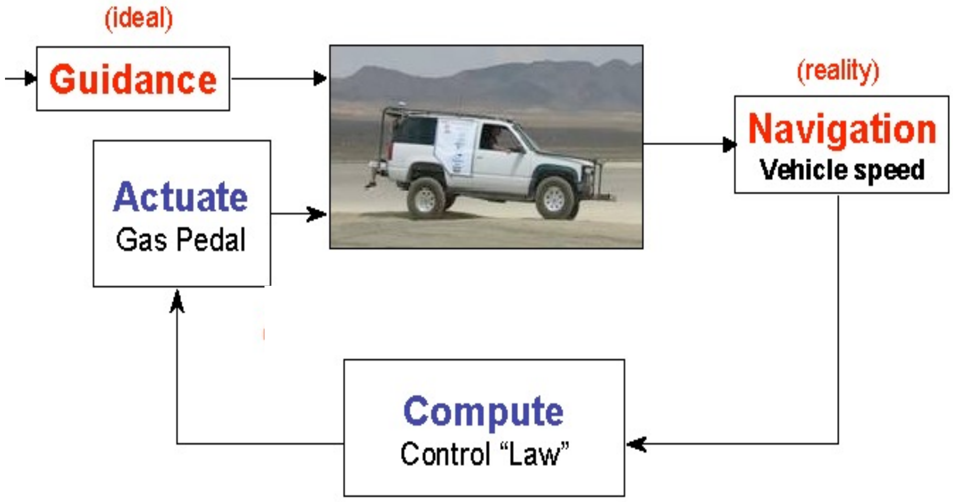
Feedback



Feedback



Feedback

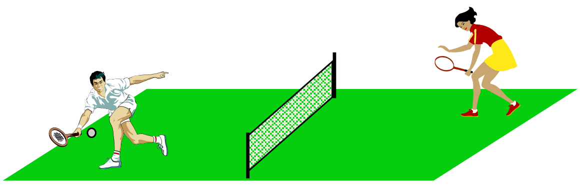


Main Elements of a Control System

- **Better Sensors**
  Provide better *Vision*

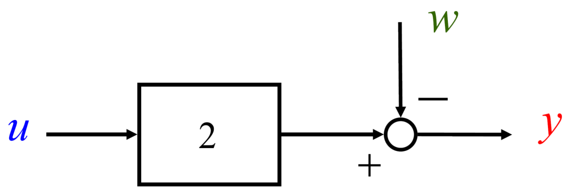
- **Better Actuators**
  Provide more *Muscle*

- **Better Control (Computing)**
  Provides more finesse by combining *sensors* and *actuators* in more intelligent ways

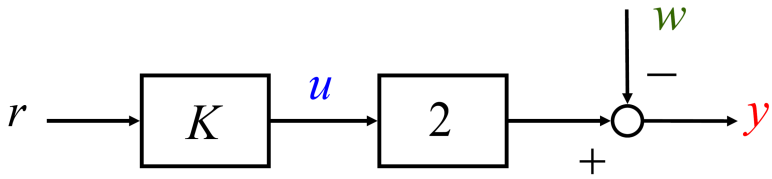


Open-Loop and Feedforward

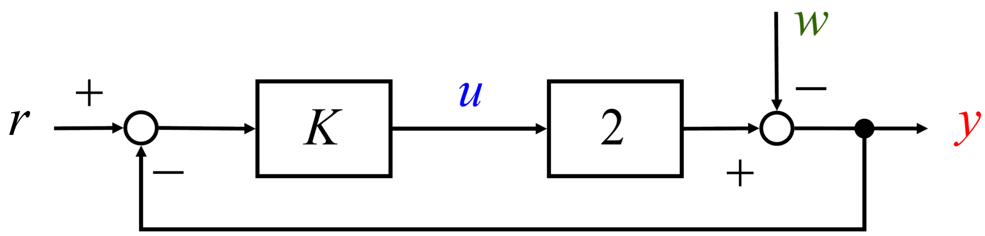
$$y = 2u - w$$



$$\begin{cases} y = 2u - w \\ u = Kr \end{cases}$$



Feedback



$$\begin{cases} y = 2u - w \\ u = K(r - y) \end{cases}$$

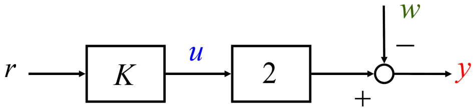
$$y = 2\,K(r - y) - w$$

$$(1 + 2K)y = 2Kr - w$$

$$y = \frac{2K}{1+2K} r - \frac{1}{1+2K} w$$

Feedback vs Feedforward

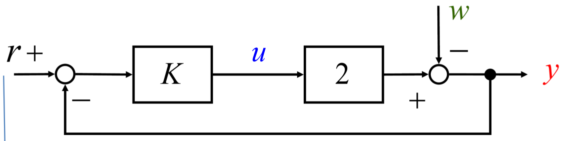
$$r = 5 \ \ [\text{m/s}]$$
$$w = 2 \ \ [\text{m/s}]$$



$$K = \frac{1}{2}$$

$$y = r - w$$
$$= 5 - 2$$
$$= 3 \ \ [\text{m/s}]$$

Deviation of 40% from target value



$$K = 100$$

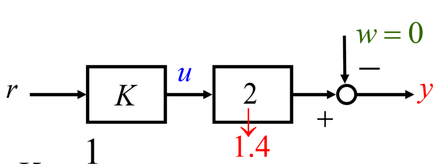
$$y = \frac{200}{201} r - \frac{1}{201} w$$
$$= \frac{200}{201} \cdot 5 - \frac{1}{201} 2$$
$$\approx 4.965\cdots$$
$$\approx 4.97 \ \ [\text{m/s}]$$

The error from the target value is within 1%

Feedback vs Feedforward

$$r = 5, \ \ w = 0$$

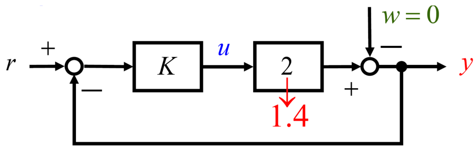
Plant gain: 1.4



$$K = \frac{1}{2}$$

$$y = 1.4 \cdot \frac{r}{2} = 0.7r$$
$$= 3.5 \ [\text{m/s}]$$

Deviation of 30% from target value



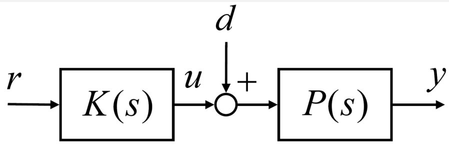
$$K = 100$$

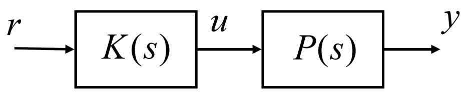
$$141y = 140r$$
$$y = \frac{140}{141} r = \frac{140 \times 5}{141}$$
$$\approx 4.9645\cdots$$
$$\approx 4.96 \ [\text{m/s}]$$

The error from the target value is within 1%

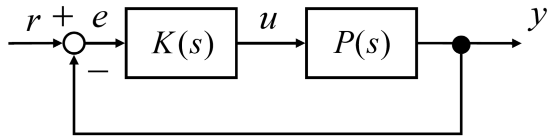
Feedback vs Feedforward

$$P(s)=\frac{A}{\tau s+1} \qquad K(s)=K \qquad (d=0)$$





$$y(s)=P(s)K(s)r(s)$$
$$=\frac{A}{\tau s+1}\cdot K\cdot r(s)$$
$$=\frac{AK}{\tau s+1}r(s)$$

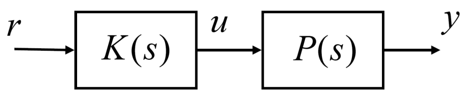


$$\begin{cases} y(s)=P(s)K(s)e(s) \\ e(s)=r(s)-y(s) \end{cases}$$

$$\left(1+P(s)K(s)\right)y(s)$$
$$=P(s)K(s)r(s)$$

$$y(s)=\frac{P(s)K(s)}{1+P(s)K(s)}r(s)$$
$$=\frac{AK}{\tau s+1+AK}r(s)$$

Feedback vs Feedforward



$$K=\frac{1}{A}$$

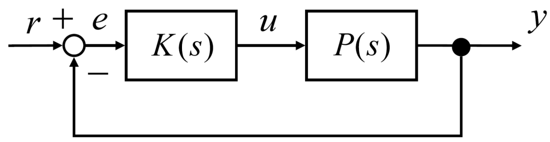
$$y(s)=\frac{AK}{\tau s+1}r(s)=\frac{1}{\tau s+1}r(s)$$

$$y(t)\approx r(t) \quad (t\to\infty)$$

$$\tilde{A}=1.4A$$

$$\tilde{y}(s)=\frac{\tilde{A}K}{\tau s+1}r(s)=\frac{1.4}{\tau s+1}r(s)$$

$$\tilde{y}(t)\approx 1.4r(t)=(1.4y(t))$$



$$y(s)=\frac{AK}{\tau s+1+AK}r(s)$$

$$K\to\infty$$

$$\frac{AK}{\tau s+1+AK}\approx\frac{AK}{AK}=1$$

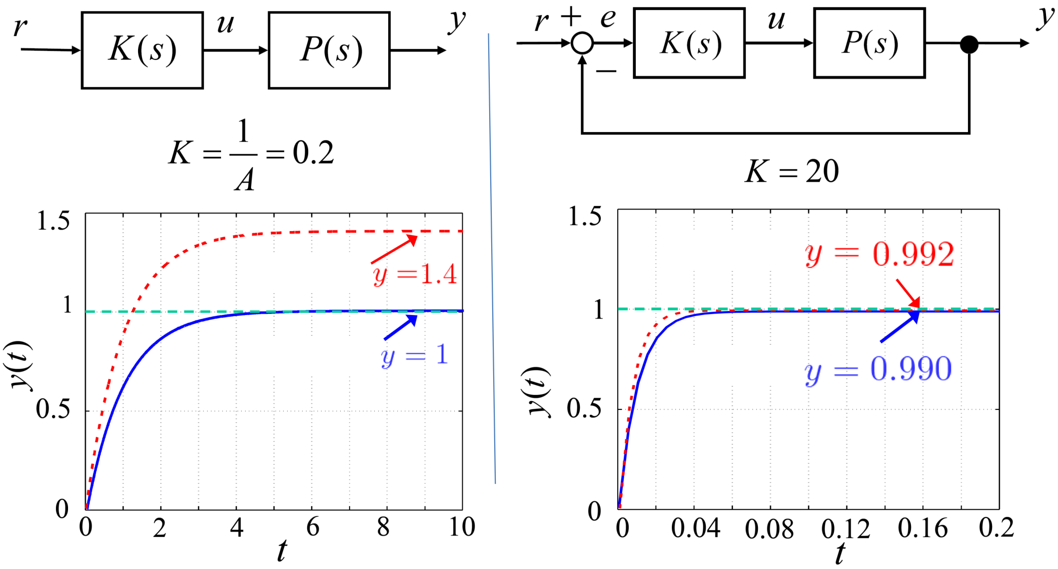
$$\therefore \quad y(t)\approx r(t)$$

Feedback vs Feedforward

$P(s) = \frac{A}{\tau s + 1} \qquad K(s) = K$

$\tau = 1, \quad A = 5$

$A \rightarrow \tilde{A} = 7$

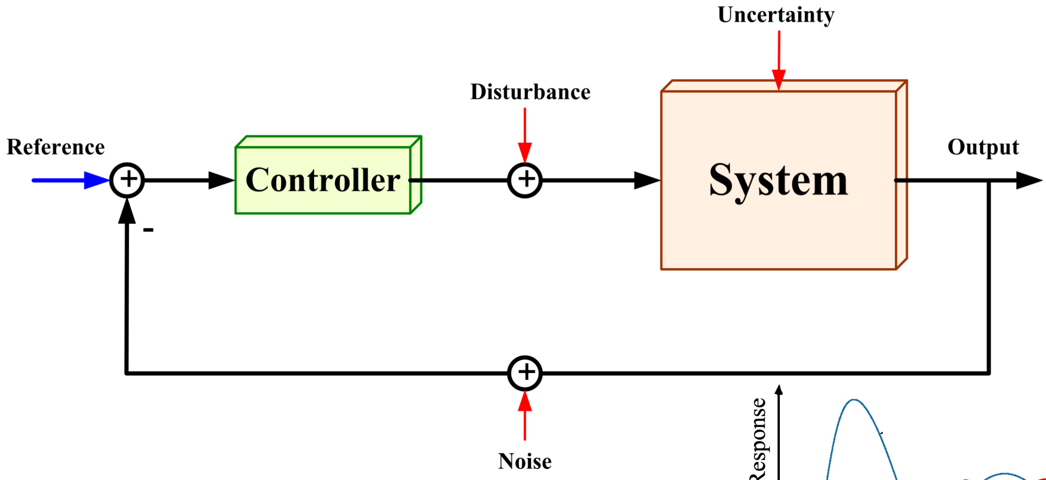


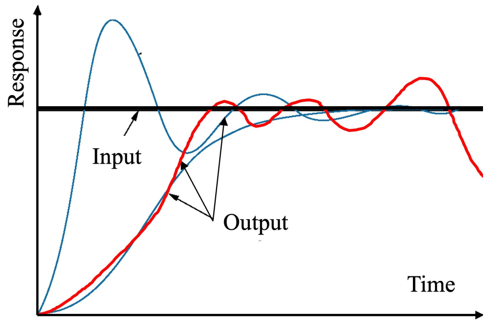
Power of Feedback

- ✓ **Stabilize unstable systems**
- ✓ **Good performance from poor components**
- ✓ **Attenuate disturbance impacts**
- ✓ **Provide degrees of freedom**
- ✓ **Attenuate parameter variation impacts**
- ✓ **Shape behavior**

- **Main drawback**

Control System Objectives

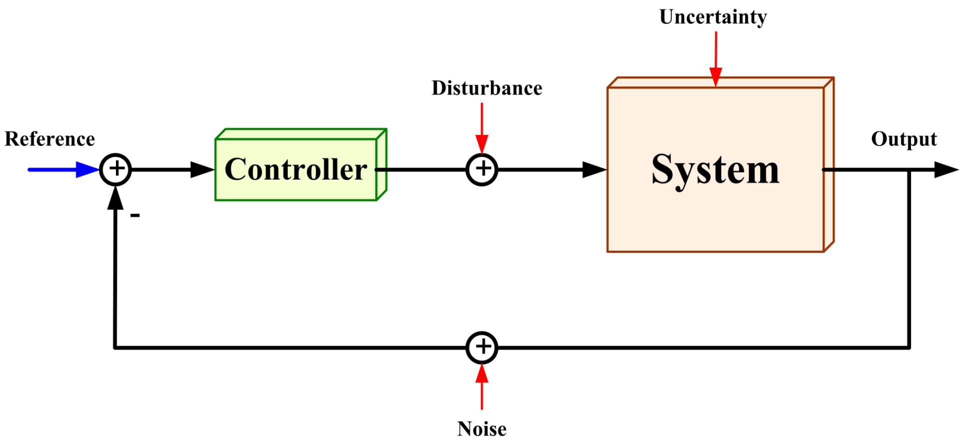




Control objectives:

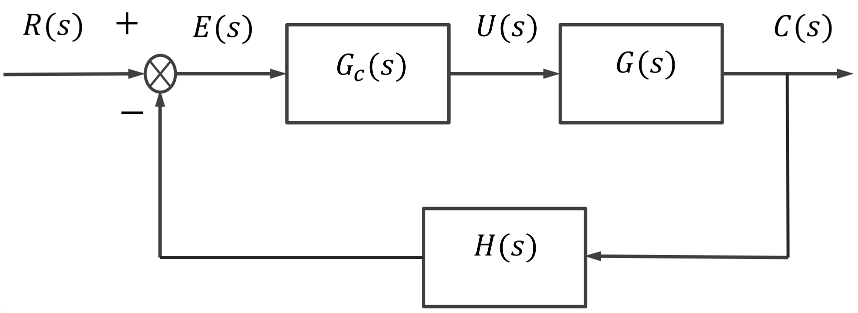
- **Stability**
- **Performance** **(reference tracking, disturbance/noise rejection, rapidness; etc.)**

Feedback Control System



Control objectives: 1. Stability, and 2. Performance
in the presence of noise, disturbance, and uncertainties.

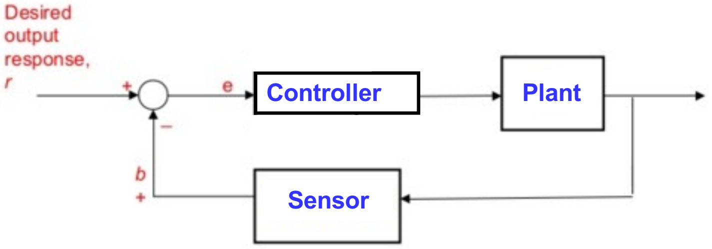
Terminology



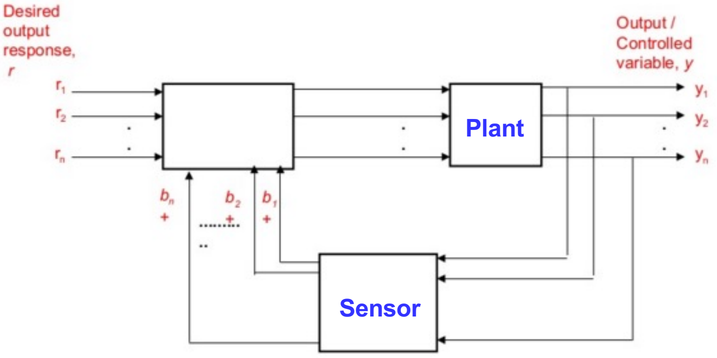
- $G(s)$: system, plant
- $G_c(s)$: Controller
- $H(s)$: feedback component
- $R(s)$: reference input, desired output
- $U(s)$: control signal
- $C(s)$: Controlled output
- $E(s)$: Error signal

Feedback Control Systems: Types

- **Single input single output (SISO)**



- **Multi input Multi output (MIMO)**



Feedback Control Systems: Types

- ❑ **Linear**
- ❑ **Nonlinear**

- ❑ **Continuous** **(system variables are function of a continuous time *t*)**
- ❑ **Discrete** **(system variables are function of a discrete time *t*)**

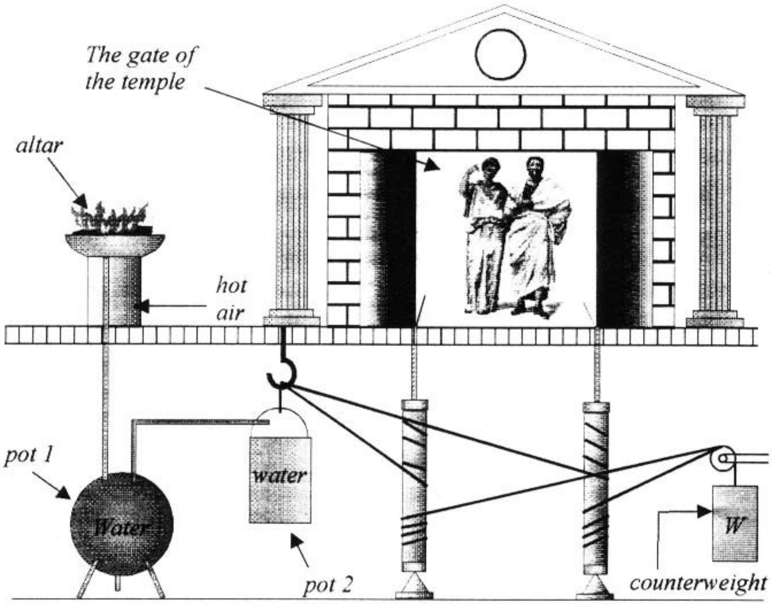
- ❑ **Time Varying** **(system parameters are varying with time)**
- ❑ **Time Invariant** **(system parameters are stationary with respect to time)**

Some Advanced Control Techniques

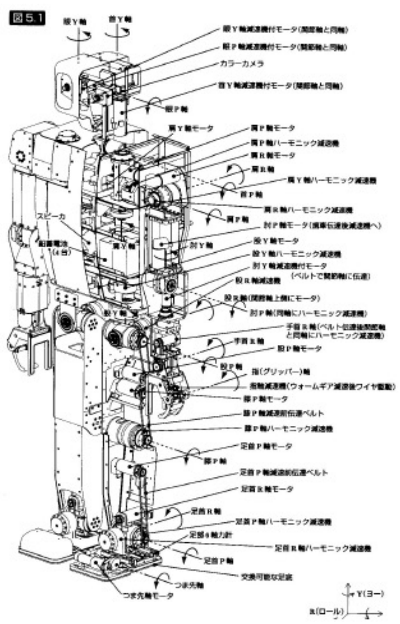
- Robust control
- Adaptive control
- Stochastic control
- Intelligent control
- Optimal control

Control History

- **The regulator of Heron of Alexandria**



Control History



Control History

- **Control appeared in the industries that emerged in the 19th and 20th centuries: steam power, electric power, ships, aircrafts, chemicals, telecommunication.**

- **In the 1940s it appeared as a separate engineering discipline, and it has developed rapidly ever since. Academic positioning difficult since it fits poorly into the ME, EE, ChemE framework. Today applications everywhere.**

Control History

- **1868** first article of control on governors (Maxwell)
- **1877** Routh stability criterion
- **1892** Lyapunov stability condition
- **1895** Hurwitz stability condition
- **1932** Nyquist
- **1945** Bode
- **1947** Nichols
- **1948** Root locus
- **1949** Wiener optimal control research
- **1955** Kalman filter and controlbility observability analysis

Control History

- **1956** Artificial Intelligence
- **1957** Bellman optimal and adaptive control
- **1962** Pontryagin optimal control
- **1965** Zadeh Fuzzy set
- **1972** Multi-variable optimal control and Robust control
- **1981** Doyle Robust control theory
- **1990** Neuro-Fuzzy
- **2000** More intelligent control
- **2010** Wide-area & distributed controls
- **2020** Data-driven & Intelligent controls

Historic Turning Points

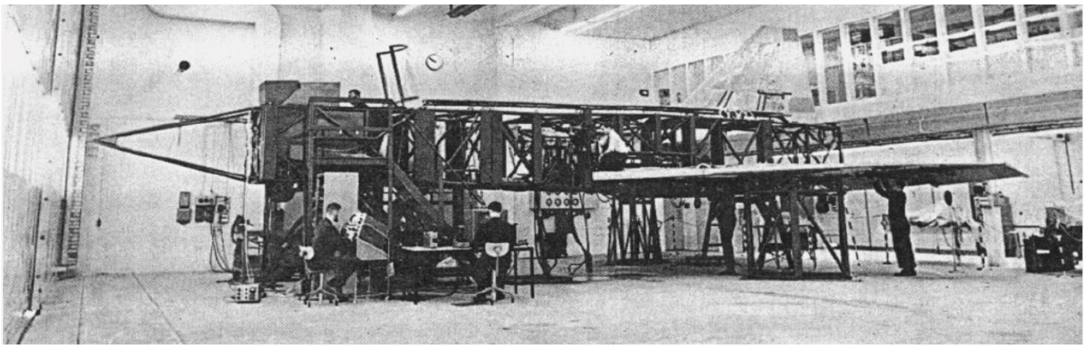
- **1945:** Drivers (gun control, radar), Modeling block diagram, transfer functions, simulation and Theorems
- **1965:** Computational tools, Kalman filter, Nonlinear and stochastic, LQG and H_∞ (optimal control)
- **1985:** Digital control, Robust control
- **2010:** Wide-area control, Distributed control systems
- **2020:** Data-driven control, AI Control

Historical Control Example

- **Flight Control**

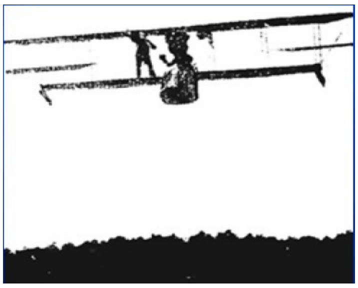
Problem: How to fly in a stabilized condition?

Solution: Stabilization using Feedback



Continue

1. **The Wright Brothers1903**
2. **Sperry's Autopilot 1912**
3. **V1 and V2 1942**
4. **Robert E. 1947**
5. **Sputnik1957**
6. **Apollo 1969**
7. **Mars Pathfinder 1997**
8. **UAVs 2020**

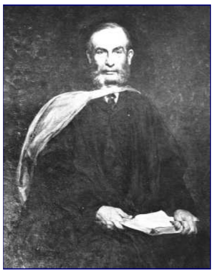


UAV: Unmanned Aerial Vehicle

Control History

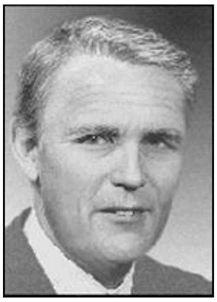

Nyquist

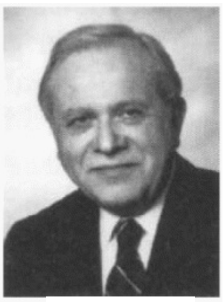

Lyapunov


Routh


Maxwell


Laplace


Kalman


Zames


Zadeh


Doyle


Nichols

Thank You!